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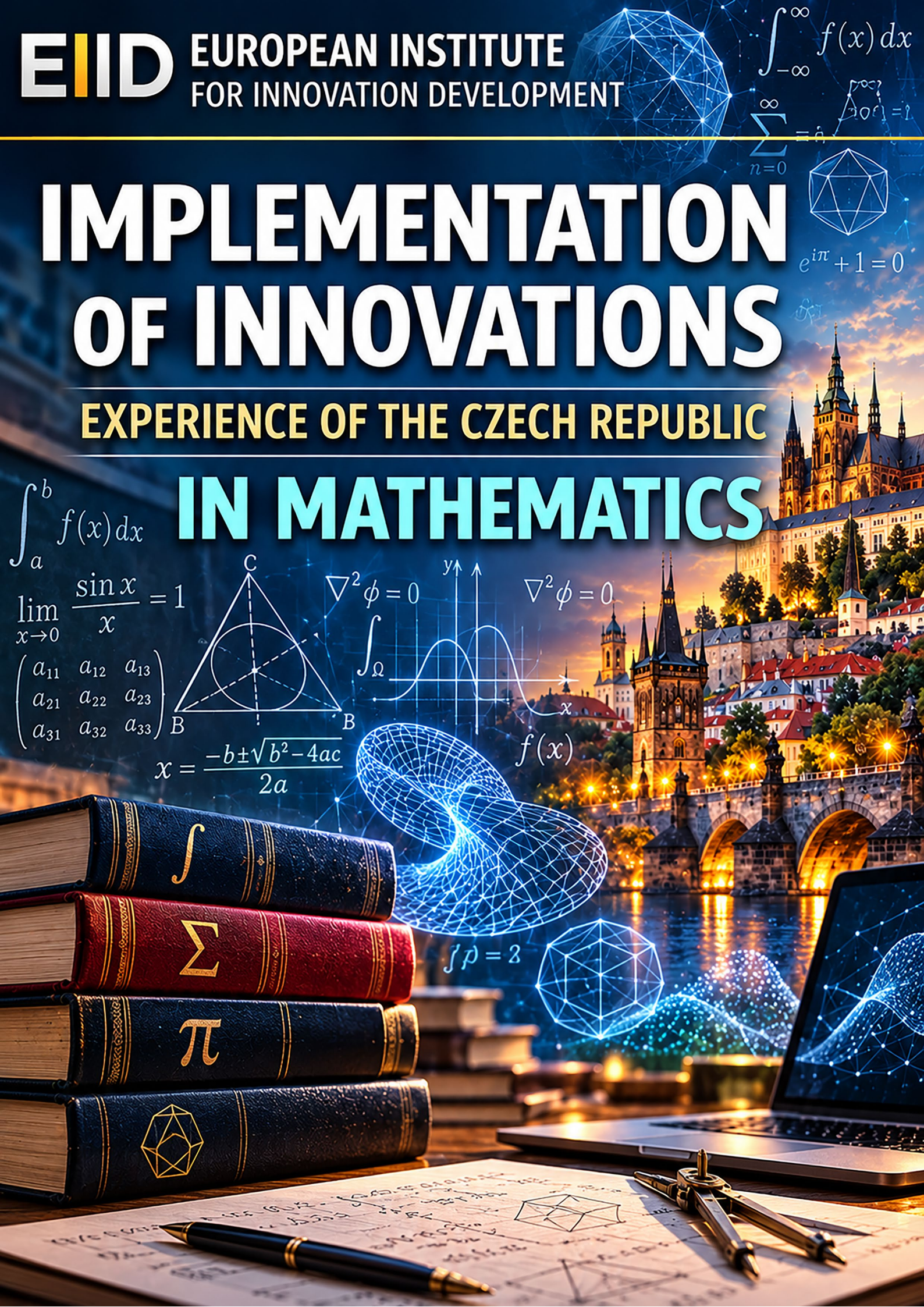
IN MATHEMATICS



EUROPEAN INSTITUTE FOR INNOVATION DEVELOPMENT



Implementation of Innovations in Mathematics

Experience of the Czech Republic

Dr. Alexander Buychik

Ostrava–Opava, Czech Republic

2026

Imprint
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Introduction

Mathematics in Contemporary Czechia: Scientific Traditions, Intellectual Transformation, European Integration, and Innovation between 1993 and 2025

The history of mathematics in the Czech Republic between 1993 and 2025 cannot be understood as the sudden appearance of a new national discipline. Mathematical ideas do not observe political borders with the regularity of administrative institutions, and their intellectual lineages frequently extend across centuries. The establishment of an independent Czech state nevertheless altered the conditions under which mathematical research was supported, organised, taught and recognised. Research communities inherited from Czechoslovakia entered a period of political openness, institutional adjustment and increasingly demanding international comparison. In the decades that followed, established traditions in mathematical analysis, algebra, geometry, logic and discrete mathematics interacted with rapidly developing computational methods. The meaning of mathematical research itself widened as algorithms, stochastic models and data-intensive investigations became indispensable to neighbouring sciences. At the same time, the defining standards of mathematical argument remained proof, conceptual coherence, precision and the possibility of independent verification. Contemporary Czech mathematics therefore offers an instructive history of continuity under changing social and technological circumstances. This book examines that history through scientific ideas, institutions, people and the evolving connections between fundamental knowledge and application.

A useful account must begin by distinguishing the history of the Czech state from the much longer history of mathematical scholarship in the Czech lands. Prague and Brno occupied important positions within the intellectual geography of Central Europe well before the twentieth century. University teaching, learned societies, technical education and scientific publishing created settings in which mathematical knowledge could circulate between theoretical inquiry and practical problems. The development of modern research specialisms gradually gave rise to professional communities concerned with analysis, geometry, probability, mathematical physics and later mathematical logic. After the formation of Czechoslovakia, these communities worked within a shared institutional space that also included Slovak mathematicians and universities. The second half of the twentieth century brought an extensive institutional network, although political controls and restrictions on academic life affected its development. These conditions did not prevent the accumulation of mathematical expertise, scholarly journals, seminars or enduring research relationships. By 1993, the new republic inherited a substantial body of disciplinary knowledge rather than an unformed scientific landscape. Understanding this inheritance helps explain both the resilience of Czech mathematics and the particular challenges of its subsequent transformation. The years before independence are therefore treated here as indispensable background, while the principal subject remains the period from 1993 to 2025.

Charles University provides a particularly visible example of the institutional depth behind this continuity. Its Faculty of Mathematics and Physics was established in 1952, several decades before the political transformations with which this study begins. The faculty developed an environment in which teaching and research were conducted across different branches of mathematics, physics and later computer science. Such an environment matters because advanced mathematical work is transmitted through more than published theorems alone. Seminars, doctoral supervision, informal criticism and successive generations of students preserve techniques of reasoning that may otherwise be difficult to reconstruct. Mathematics also develops through the productive coexistence of specialisation and conversation across research fields. The relationship between abstract structures and computational problems can be especially fruitful when specialists share an academic setting. After 1993, long-established university traditions consequently provided a platform from which new international relationships and research priorities could emerge. The faculty illustrates the broader principle that innovation often depends on the stability of institutions able to sustain intellectual exchange.

The transformation that made an independent Czech scientific system possible began before the formal dissolution of Czechoslovakia. Political change after 1989 widened academic freedom and enabled researchers to pursue international contacts under substantially different conditions. For mathematics, the effects were simultaneously intellectual and organisational. Scholars could participate more freely in conferences, visit foreign departments and enter collaborative networks that had previously been constrained. Access to recent literature and to different research cultures offered opportunities to revise established questions and acquire new methodologies. Yet openness also introduced stronger pressures for publication, international visibility and competition for limited resources. The resulting transition cannot adequately be described either as a simple liberation of research or as a replacement of one institutional model by another. It was instead a difficult process of preserving valuable scientific capabilities while changing the mechanisms by which those capabilities were assessed and maintained. By 1993, Czech mathematicians were entering a national system already shaped by several years of wider political and academic reform. The distinction between the turning point of 1989 and the institutional watershed of 1993 is essential to an accurate chronology.

On 1 January 1993, the institutional framework for mathematics changed with the creation of the independent Czech Republic. The former Czechoslovak Academy of Sciences had ceased to exist at the end of 1992, and its Czech successor began operating under the legislation adopted for the new state. The Czech Academy of Sciences (CAS) became a principal setting for specialised fundamental research, including mathematical investigation. Continuity of research groups and accumulated expertise did not mean that organisational expectations remained unchanged. Institutes faced altered financial arrangements, developing governance structures and increasing attention to international evaluation. Their scientific priorities had to be presented within a national policy environment balancing long-term scholarship against immediate economic and social needs. This balance was especially delicate for mathematics, where the significance of a discovery may become evident only well after its original formulation. A theorem can transform an entire field without yielding an immediate technological product,

while an abstract method may later find unexpected applications. The post-1993 system therefore had to create conditions in which mathematical autonomy and public accountability could coexist.

The Institute of Mathematics of the Czech Academy of Sciences occupied a central place within this new framework. Its mission encompasses mathematical research and the maintenance of an environment in which specialised ideas can develop over long periods. Contemporary descriptions of the institute identify fields such as abstract analysis, algebra, geometry, mathematical physics, differential equations and constructive analytical methods. These directions differ in their objects of investigation, but they share an investment in rigorous reasoning and the development of mathematical structures. An academic institute has a distinctive role in such work because it can support research programmes whose duration does not coincide with an undergraduate curriculum. At the same time, its scientific vitality depends upon active relations with universities, visiting researchers and international collaborators. The institute's institutional form changed as Czech legislation developed, and its annual reports document activities within the public research framework. This history offers a useful way to examine the relationship between administrative structures and the independence of theoretical investigation. It also shows why the development of mathematics should not be measured solely through laboratories, physical instruments or immediately quantifiable inventions. Long-lived mathematical research groups themselves constitute an important form of national scientific infrastructure.

The research environment of the 1990s was further transformed by the growing importance of competition-based funding. The Czech Science Foundation, established in 1993, became a major public institution supporting basic research through evaluated project proposals. For mathematical scientists, competitive funding provided opportunities to sustain projects, support younger colleagues and develop international partnerships. Such programmes also required researchers to articulate questions, methods and expected contributions in a form intelligible to expert assessment. The consequences were not unambiguously positive or negative, because mathematics contains research trajectories that cannot be reliably scheduled in advance. Deep conceptual breakthroughs may follow years of unsuccessful approaches, whereas some projects yield a sequence of more readily demonstrable partial results. A mature funding system must therefore distinguish intellectual originality from the mere predictability of short-term outputs. During the post-socialist transition, the ability to reconcile these considerations became part of the broader effort to modernise Czech scientific governance. Grant competition was one influence on mathematical development, but it never replaced the importance of stable research institutions and scholarly communities.

Universities formed the second indispensable pillar of Czech mathematical science. Charles University in Prague maintained a broad disciplinary profile, while Masaryk University in Brno supported research across pure and applied mathematics and mathematical education. Technical universities contributed another perspective by confronting mathematical questions arising from engineering, computation, materials and systems analysis. Regional institutions added distinct areas of expertise and created opportunities for research and advanced training outside the capital. This distribution matters because mathematical creativity cannot be

reduced to the achievements of a single metropolitan centre. Although resources and institutional visibility were unevenly concentrated, the national community developed through cooperation among departments with different strengths. Doctoral education became an increasingly important bridge between those departments and the wider European research environment. Young mathematicians could participate in international seminars and research projects while retaining links with established Czech schools. The university system thus performed two connected functions: preserving mathematical knowledge through teaching and producing new knowledge through research. Its contribution to scientific renewal was ultimately inseparable from the continuity of academic generations.

The internationalisation of mathematics differs in significant respects from the internationalisation of large experimental sciences. A mathematician may conduct important work without access to a large laboratory, yet advanced scholarship still depends heavily on international intellectual exchange. Proofs require careful scrutiny, technical ideas mature through sustained discussion and research problems often emerge from conversations across institutions. Conferences and specialist seminars create places where unpublished ideas are challenged before they become formal publications. Visiting appointments and joint doctoral supervision can bring different mathematical traditions into productive contact. In the 1990s, Czech researchers were increasingly able to participate in these activities as ordinary features of professional life. The availability of electronic communication and digital scholarly resources later intensified this exchange. Nevertheless, international collaboration did not erase the importance of local mathematical schools or shared institutional memory. Rather, it allowed distinctive Czech areas of expertise to interact more systematically with global developments.

European integration provided a further framework for these transformations. The Czech Republic joined the European Union (EU) on 1 May 2004, thereby entering new structures of research cooperation and academic mobility. Participation in European programmes enlarged opportunities for cross-border collaboration and supported the development of research capacity. The European Research Area (ERA) encouraged a more connected environment for scientific institutions, researchers and the circulation of knowledge. Mathematical disciplines could benefit through collaborative networks, fellowships, research projects and connections with European infrastructures. The direct impact varied substantially between pure mathematics and computational or interdisciplinary fields. Some forms of progress remained dependent chiefly upon expert communities and time for concentrated thought rather than expensive technological investment. Other areas acquired new opportunities as shared computing resources and multidisciplinary research centres became increasingly important. European integration should therefore be regarded as a change in the opportunities and conditions of research, not as an automatic explanation for every mathematical achievement. The distinctive contribution of Czech mathematicians continued to depend upon the quality and originality of the questions they pursued.

The notion of a mathematical school is central to this book, although it requires precise interpretation. A school is not merely a department whose members work under one administrative heading. It is better understood as a durable constellation of problems, methods, scholarly practices and relationships through which mathematical knowledge develops. Such

communities may be organised around a particular field, an influential teacher or a sequence of connected research questions. Their vitality depends on the freedom of younger researchers to challenge inherited assumptions rather than repeat them. The Czech experience includes communities associated with mathematical analysis, discrete mathematics, mathematical logic, geometry and mathematical physics. Some have achieved international visibility through highly specialised work that cannot readily be explained by the size of an institution. Others have contributed by sustaining journals, international meetings and rigorous graduate education. The historical task is to identify these different contributions without turning the concept of a scientific school into an uncritical label of prestige.

A particularly revealing example is the development of combinatorics and structural graph theory. Research associated with Jaroslav Nešetřil and international collaborators has contributed to modern understanding of discrete structures and their mathematical organisation. Work on graph homomorphisms and sparsity illustrates how seemingly elementary objects can lead to exceptionally sophisticated theories. Graphs represent relations between objects, but their structural properties also connect with questions in logic, algorithms and computational complexity. The study of sparse graph classes, for example, has generated techniques that cross the conventional boundary between pure mathematics and theoretical computer science. In 2025, Nešetřil and Patrice Ossona de Mendez received the Nerode Prize for influential work on sparsity in graph theory. This recognition does not imply that the entire field originated in the Czech Republic, nor does it diminish the importance of international intellectual collaboration. It demonstrates instead how researchers strongly connected with Czech institutions can help formulate questions and methods with worldwide relevance. Such achievements demand analysis of mathematical substance rather than a mere catalogue of honours. They also show that abstract mathematics can possess significant algorithmic consequences without being created solely for application.

Mathematical logic offers a different illustration of how fundamental ideas connect with new intellectual technologies. Czech research in logic has included investigations of formal systems, many-valued reasoning and the mathematical foundations of uncertainty. The work of Petr Hájek is especially relevant to the history of mathematical fuzzy logic and its rigorous treatment through formal methods. A logical system is not simply a computational procedure, because its significance includes questions of meaning, inference, consistency and provability. Nevertheless, logical investigations can inform the representation of knowledge and the design of reasoning systems. This connection became increasingly important as computer science expanded towards automated reasoning and artificial intelligence. The development of mathematical logic also provides a reminder that innovation may concern an improved conceptual framework rather than a newly manufactured artefact. Its consequences may be understood most clearly through the relationship between proofs, formal languages and methods of computation. An account of contemporary Czech mathematics must therefore examine intellectual foundations alongside visible technical applications.

Mathematical analysis and the theory of differential equations constitute another major domain of continuity and renewal. These fields investigate functions, operators, limiting processes and the behaviour of systems described through mathematical relations. Their theoretical problems

frequently require new existence theorems, regularity arguments, estimates or approaches to non-linear behaviour. Such results may subsequently provide the foundation for models in fluid mechanics, materials research, biology or physical science. Yet the theoretical difficulties are not exhausted by any single application. A method developed for one class of equations may reveal a much broader mathematical structure with independent significance. Research in functional analysis and partial differential equations consequently illustrates the two-way movement between abstract theory and applied modelling. Czech academic institutions have maintained active research environments in these areas, supported by long-standing seminar traditions and international collaboration. The development of these fields is best understood through the mathematical questions posed and the methods used to address them, rather than through broad claims of national excellence. Their history makes clear that mathematical progress is often cumulative, technical and resistant to simple narratives of sudden discovery.

Algebra, geometry and topology further demonstrate the intellectual diversity of Czech mathematical research. These disciplines study different forms of structure, transformation, space and invariance, often at a high level of abstraction. Their questions may initially appear remote from practical technological needs, but they establish concepts that can later cross disciplinary boundaries. In geometry, relationships between local and global properties provide a language for analysing spaces and their transformations. In algebra, the investigation of operations and structural constraints can reveal connections between otherwise distinct classes of mathematical objects. Topological methods address properties preserved under continuous change and contribute to problems in mathematical analysis and discrete mathematics. Research programmes in these fields require specialised knowledge accumulated through sustained training and dialogue. Their importance in a history of mathematical innovation lies precisely in the possibility of building new conceptual languages. Such languages enlarge the range of questions mathematicians can formulate, whether or not their applications are immediately apparent.

The growth of numerical analysis and scientific computing altered the practical reach of mathematical research during the period under consideration. Numerical methods translate mathematical models into computational procedures capable of producing approximate answers. Their reliability depends on questions of stability, convergence, error estimation and computational efficiency. An attractive numerical result is not necessarily trustworthy unless its mathematical and numerical limitations are understood. Researchers working on approximation methods, discretisation and numerical linear algebra therefore contribute to the credibility of computational science itself. More powerful computers increased the scale of problems that could be investigated, but they did not eliminate the need for rigorous mathematical method. On the contrary, larger calculations often introduced new challenges in algorithmic complexity and reproducibility. Czech mathematical and technical institutions increasingly developed research that joined analytical understanding to computational implementation. In this environment, the innovation was frequently an improved method of obtaining reliable knowledge rather than the machine on which the calculation was performed. The distinction is essential for assessing the independent role of mathematics in the digital transformation of science.

The emergence of major computing infrastructures made this interaction more visible beyond traditional mathematical departments. In Ostrava, the IT4Innovations National Supercomputing Center developed within a university-led project supported by European investment and national partners. Its purpose was not confined to mathematics, since high-performance computing serves a wide range of scientific and technological disciplines. Nevertheless, large-scale simulation and advanced data analysis depend fundamentally upon mathematical models and efficient algorithms. The opening of the centre's building in 2014 and the subsequent expansion of its computing systems marked important stages in the national research environment. These developments strengthened opportunities for computationally intensive investigations that could not be undertaken with ordinary departmental resources. They also brought mathematical specialists into closer contact with researchers concerned with physical processes, engineering design and complex datasets. It would be misleading to equate the acquisition of computing hardware with a mathematical discovery in itself. The more important historical question is how expanded computational capacity interacted with original numerical methods and mathematically informed scientific inquiry.

Probability theory and mathematical statistics acquired greater practical visibility as the amount and complexity of scientific information increased. Probability supplies formal methods for representing uncertainty, while statistics investigates what can reasonably be inferred from observations. Their relationship is close but not interchangeable, because probabilistic modelling and inferential decision-making answer different kinds of questions. In a data-intensive scientific environment, the validity of a conclusion depends upon assumptions concerning sampling, measurement, variation and model adequacy. Czech research in statistics and related mathematical fields contributes to a wider ecosystem of empirical investigation and computational modelling. Mathematical reasoning remains important even when automated software makes sophisticated calculations appear effortless. The choice of a model, the interpretation of an estimate and the identification of uncertainty still require theoretical judgement. These challenges became increasingly visible in fields ranging from economics and medicine to environmental research and engineering. The contemporary history of mathematics must therefore investigate not only the creation of statistical tools but also the standards of reasoning that make their use scientifically defensible. This is a central aspect of mathematical innovation in societies increasingly dependent upon evidence expressed as data.

Optimisation and decision theory provide another bridge between mathematical abstraction and practical action. An optimisation problem asks how a defined objective can be improved or achieved subject to explicit constraints. The mathematical form of the problem determines which algorithms are suitable and what can legitimately be inferred from the result. Non-convexity, uncertainty, discontinuity and conflicting objectives often prevent simple solutions. Czech research connected with the Institute of Information Theory and Automation has included mathematical optimisation, variational problems, probabilistic decision models and related methods. Such work illustrates how rigorous analysis can shape decision-support systems without replacing human responsibility for the objectives being pursued. Mathematical methods can clarify trade-offs, establish limitations and identify consequences that would otherwise remain difficult to compare. Yet a technically optimal answer can still be socially inadequate if a

problem has been formulated using inappropriate assumptions. The contribution of mathematics to practical decision-making must therefore be assessed both in terms of methodological innovation and the quality of its problem formulations.

Mathematical physics makes the relationship between theoretical inquiry and the natural sciences especially explicit. Its researchers use advanced mathematical structures to formulate, analyse and sometimes reinterpret physical phenomena. Spectral theory, operator methods, quantum graphs and differential equations are examples of intellectual tools that can be important in this exchange. The movement of ideas is not one-directional, since physical questions frequently stimulate the development of genuinely new mathematics. Czech work associated with mathematical physics belongs to international conversations in which precise analysis is essential to the interpretation of models. Theoretical originality may consist in proving an unexpected property, constructing a new model or establishing conditions under which an approximation is valid. These achievements should not be confused with the experimental discoveries of physics, even when they illuminate the same phenomena. Their distinctive value lies in giving scientific claims a coherent mathematical formulation and explaining the limits of that formulation. The history of mathematical physics thus helps prevent an artificial separation between pure mathematics and its intellectually demanding applications. It also demonstrates that mathematical innovation can arise from problems whose origins lie outside mathematics itself.

The rapid expansion of artificial intelligence during the 2010s and early 2020s intensified public attention to mathematical methods. Many learning procedures rely upon optimisation, linear algebra, probability, numerical analysis and the theory of algorithms. Their performance is also shaped by questions about generalisation, statistical uncertainty, robustness and computational complexity. These issues cannot be resolved through increases in computational power alone. Mathematical research offers tools for explaining why an algorithm succeeds, identifying conditions under which it may fail and designing methods with better-understood properties. Czech institutions concerned with information theory, automation and computation increasingly engaged with this wider transformation. It is important, however, to distinguish mathematical contributions to artificial intelligence from the broader development of software products and commercial systems. The mere deployment of an existing model does not automatically constitute a new mathematical result. A history of innovation should attend to new mathematical formulations, justified algorithms and conceptual advances rather than technological fashion.

The geographical distribution of mathematical science also deserves consideration. Prague possesses an exceptional concentration of universities and Academy institutions, but it does not exhaust the national research system. Brno has developed significant traditions through Masaryk University, Brno University of Technology and their associated scholarly networks. Olomouc, Ostrava, Plzeň, České Budějovice and other centres provide additional academic environments for mathematics, statistics, computation and applied research. The strength of a mathematical department is not necessarily proportional to the size of its city or the visibility of its physical infrastructure. Specialised groups can contribute internationally significant results while remaining comparatively small. At the same time, regional differences in funding, doctoral

recruitment and access to collaboration can influence what research programmes are sustainable. An accurate account must therefore combine recognition of Prague's central position with attention to the diversity of work undertaken elsewhere. This polycentric dimension contributes to the resilience of the Czech mathematical community and the diffusion of mathematical knowledge across society. These regional environments also enable mathematical ideas to circulate between teaching, research and practical scientific needs.

The history of research institutions cannot be separated from the people who sustain them. Mathematics is sometimes described as unusually impersonal because results are judged by proofs rather than by the authority of their authors. This principle is central to mathematical objectivity, but the production of new knowledge remains a profoundly social and educational process. Questions are transmitted through lectures, seminars and conversations, while techniques develop through mentoring, collaboration and constructive disagreement. A successful research school enables younger mathematicians to become intellectually independent rather than simply reproduce the work of their predecessors. International mobility can enrich this process by exposing researchers to competing traditions and unfamiliar approaches. It can also create difficulties for smaller academic systems when talented graduates struggle to find stable research positions at home. The long-term development of Czech mathematics consequently depends upon career structures, doctoral conditions and the ability of institutions to maintain continuity across generations. The achievements of individual scholars are best understood within this broader network of intellectual exchange and institutional support.

The evaluation of mathematical excellence presents particular methodological difficulties. Publication counts and citation measures can offer useful evidence about scientific activity, but their meaning varies across fields and scholarly traditions. In some areas, lengthy collaborative projects and rapid publication cycles are normal, whereas other branches progress through comparatively few highly concentrated works. The impact of a theorem may remain uncertain until subsequent researchers discover connections that were not apparent at publication. A mathematical paper can also be influential because it introduces a technique later adapted in many different settings. Conversely, visibility in an applied project does not by itself establish mathematical originality. This study therefore treats quantitative indicators as context rather than as a complete ranking of intellectual achievement. Where notable researchers or results are discussed, their contributions must be connected to identifiable mathematical questions, publications and scholarly responses. Such an approach is necessary to prevent national scientific history from becoming a series of unverifiable claims of priority or prestige. This methodological caution is especially important when comparing mathematical research across different institutions.

Another central question is how to define innovation within mathematics without reducing the discipline to technological production. In an experimental science, innovation may be embodied in a new instrument, laboratory procedure or material object. Mathematical innovation often takes a different form: a theorem, a proof technique, a novel mathematical structure, an algorithm or a conceptual reformulation of a difficult problem. New methods can transform research even when the objects under investigation remain abstract. A stronger existence

theorem may make a class of equations intelligible, while a more efficient algorithm may render previously inaccessible computations feasible. A probabilistic model can provide a better account of uncertainty, and a new geometric language can reveal relationships overlooked by earlier approaches. These advances may eventually generate technological applications, but their mathematical importance does not depend solely on that outcome. At the same time, application can expose limitations in existing theory and thereby stimulate fundamental mathematical questions. For this reason, the distinction between pure and applied mathematics is analytically valuable but should never be treated as an impermeable boundary.

The endpoint of 2025 provides an opportunity to consider both achievements and unresolved challenges. By that year, Czech mathematics was embedded in European and international research through universities, Academy institutes, specialised conferences and collaborative programmes. Its disciplinary profile included longstanding fundamental areas as well as computational, statistical and algorithmic approaches with growing relevance beyond mathematics. The expansion of digital tools had changed how research was communicated and how complex models could be implemented. Yet the demand for rigorous proof, critical debate and independent reasoning remained essential to mathematical knowledge. Continued progress depended upon stable institutional environments and the ability to recruit and retain researchers with different forms of expertise. It also required a balance between the funding of long-term theoretical work and the support of interdisciplinary programmes with more immediate practical objectives. No single policy instrument or technological infrastructure could resolve these competing demands. The history of the period should consequently be understood as a series of intellectual and organisational adaptations rather than a simple movement towards inevitable success. This distinction also shapes the interpretation of scientific progress across the historical period under review.

This volume develops its argument through three connected perspectives. The first part traces institutional and intellectual change from the establishment of the Czech Republic through European integration to the research environment of 2025. The second examines mathematical fields and achievements, placing particular emphasis on the substance of theoretical results, computational methods and interdisciplinary models. The third investigates the principal institutions and research schools through which these developments were sustained. Special attention is given to distinguishing discoveries originating in Czech research groups from the broader international achievements to which Czech mathematicians contributed. That distinction matters because mathematics is an international language of scholarship and because claims of exclusive national ownership often distort collaborative histories. The book consequently treats institutional affiliation, intellectual contribution and worldwide influence as related but different questions. Its purpose is to explain how Czech mathematical thought evolved within changing scientific structures while preserving the distinctive standards of the discipline. Between 1993 and 2025, Czech mathematics was shaped not by a rupture with its past, but by the continual reinterpretation of inherited strengths in an increasingly interconnected world.

Part One

Historical Evolution of Mathematical Sciences in Contemporary Czechia

1993 as a Watershed: Scientific Continuity and Institutional Reorganisation of Czech Mathematics

The year 1993 occupies a distinctive place in the history of mathematics in the Czech lands because it transformed the political and institutional setting without creating the discipline anew. The independent Czech Republic inherited mathematical communities whose intellectual lineages reached through Czechoslovak universities and academies into much older Central European traditions. Their research embraced analysis, algebra, geometry, logic, probability, numerical mathematics and mathematical physics, although the organisation and visibility of these fields differed considerably. The dissolution of the federation established a new framework for national budgets, institutional accountability and international representation. It did not, however, dissolve the mathematical knowledge accumulated in libraries, seminars, doctoral supervision and long-running research collaborations. A theorem proved before independence remained a contribution to mathematics, while a scientific school formed in the preceding decades could continue under a different state administration. This distinction is essential because historical accounts sometimes confuse a constitutional turning point with a discontinuity in scientific reasoning. Mathematics illustrates particularly clearly how institutional borders may change more quickly than the communities of argument and proof that exist across them. The central question of this chapter is consequently how an inherited mathematical culture acquired new organisational possibilities and constraints after 1993.

A proper interpretation of the watershed begins with the distinction between the political transformation of 1989 and the separation of the Czech and Slovak republics in 1993. The Velvet Revolution opened the prospect of academic self-government, freer international travel and a reconsideration of political controls over research careers. These changes preceded the existence of an independent Czech state and helped establish the conditions in which subsequent institutional reforms became practicable. By contrast, the beginning of 1993 required the allocation of legal competences, budgets and organisational responsibilities to a new national research system. Mathematical research therefore passed through overlapping transitions rather than a single episode of rupture. Scholars who had regained access to international conferences and literature did not need to rediscover that access on the day the federation ended. Nevertheless, they had to adapt to new mechanisms through which their departments and institutes would receive resources and demonstrate scholarly quality. A historically accurate account must recognise both the earlier liberation of intellectual exchange and the later reconfiguration of the institutions supporting it. The interaction between those

processes explains why continuity and innovation appeared simultaneously in Czech mathematics during the early 1990s. Neither process can be reduced to the other.

The deeper genealogy of the Czech mathematical community is inseparable from university education in Prague and other centres of the former Czechoslovakia. Charles University provided a particularly durable setting for mathematical teaching and research, although its faculties and organisational structures changed repeatedly over time. Its Faculty of Mathematics and Physics was established in 1952, thereby concentrating a significant part of mathematics and physics within a specialised academic environment. This institutional date matters because by 1993 the faculty already possessed four decades of experience in shaping doctoral pathways, departments and research seminars. The inheritance was intellectual as much as administrative, consisting of techniques of proof, specialised curricula, scholarly networks and traditions of disciplinary debate. The university environment enabled fundamental fields to coexist with mathematically intensive physics and emerging computer science. Such coexistence encouraged methodological exchange even when research groups pursued questions with no immediately foreseeable technological application. The transition of 1993 therefore encountered an established scholarly ecosystem rather than an undeveloped educational foundation. An assessment of the new period needs to examine how this accumulated capacity was protected and redirected.

The Academy tradition formed a second institutional pillar of mathematical research, complementary to the universities but differently organised. The Mathematical Institute had entered the Czechoslovak Academy of Sciences in 1953 and had therefore acquired a long history within a system of specialised public research institutes. Academy institutes could sustain research programmes whose intellectual horizons extended beyond ordinary teaching cycles and short-term administrative priorities. Mathematical work benefited from this possibility because investigations in analysis or geometry may require sustained conceptual development before results become visible. The Czechoslovak system nevertheless operated under political and economic restrictions that shaped appointments, international contacts and institutional opportunities. After 1989 those restrictions began to loosen, while the academy model itself became a subject of renewed public debate. The challenge was not simply whether specialised mathematical research should survive, but how autonomous academic inquiry could be supported and assessed in a democratic setting. Preserving a research institute could maintain scientific continuity while its governance, recruitment and external evaluation were redesigned. This combination became one of the defining features of the early Czech mathematical landscape. It also distinguished institutional reorganisation from the discontinuation of scientific programmes.

Historical mathematical schools cannot be understood solely as names attached to buildings or administrative units. They arise through relationships among research questions, methods, seminars, teachers, students and the circulation of publications. A mathematical school may persist even when its senior members retire or when its institutional affiliation changes. Conversely, an organisation can retain its original name while losing the intellectual practices that once sustained its reputation. The preservation of Czech mathematical expertise after independence therefore depended upon more than transferring property or approving

organisational statutes. It required continuing doctoral training, access to scholarly literature, opportunities for collaboration and a community capable of scrutinising new proofs. The transmission of tacit knowledge, such as the ability to recognise promising problems or weaknesses in an argument, was particularly important. These practices connected younger mathematicians with disciplinary traditions without obliging them to repeat the research agenda of an earlier generation. Thus continuity should be judged by the reproduction and creative development of mathematical competence rather than by administrative survival alone.

The history of analysis provides a useful example of the depth of the inheritance available in 1993. Functional analysis, differential equations and related areas had developed through university and academy work long before the independent Czech Republic existed. Their problems were often international in formulation, even when investigations took place within a nationally organised research environment. A researcher studying the behaviour of solutions to an equation necessarily engaged with concepts and results beyond state borders. Such work cultivated a culture in which scientific legitimacy depended principally upon the correctness and significance of mathematical reasoning. Yet institutional conditions influenced who could access journals, visit collaborators and participate in developing research conversations. The opening of the early 1990s strengthened the possibility that Czech specialists could enter international discussion more frequently and directly. Independence then placed those researchers within a distinct national framework for academic representation and support. The outcome was a shift in the conditions of participation rather than a sudden replacement of the subject matter of analysis. New organisational choices mattered precisely because the intellectual foundations already existed.

Algebra, geometry and mathematical logic reveal another dimension of the same historical continuity. These disciplines involve the construction of abstract structures whose value cannot always be measured by short-term practical outcomes. Research groups developing such structures depend heavily upon stable networks of criticism, publication and advanced teaching. The mathematical cultures of Prague and Brno preserved a capacity for these activities through changing political circumstances. Following independence, their researchers could re-evaluate professional priorities within a more open European academic environment. International connections were important not because mathematical knowledge became foreign in origin, but because abstract problems advance through conversations conducted across many institutions. The movement of students and visiting academics could transform the choice of questions and methods while preserving the traditions of rigorous argument that supported earlier work. In that sense, inherited scientific schools were resources for intellectual renewal rather than obstacles to modernisation. The early Czech republic thus offered conditions for both maintaining and revising disciplinary identities.

Discrete mathematics and theoretical computer science illustrate how established expertise could intersect with rapidly changing international research agendas. Combinatorics, graph theory and logic supplied methods increasingly important for the mathematical understanding of computation. Their development did not begin with the digital transformation of the 1990s, since abstract structures had been studied independently of immediate applications. Nevertheless, changing computing technologies created new questions concerning algorithms,

complexity and the organisation of information. Czech mathematical researchers entered these discussions with conceptual tools developed through earlier education and scholarship. Greater access to foreign departments, journals and workshops widened opportunities for exchanging results. The resulting interaction strengthened the bridges between pure mathematics and neighbouring computational fields. Such bridges did not erase the distinction between a theorem and a software application, because each requires a different form of validation. They did, however, increase the range of ways in which theoretical mathematics could shape scientific innovation. The post-1993 setting enabled inherited expertise to respond to new intellectual demands.

The geography of mathematical research was already broader than the capital when the Czech Republic was established. Prague concentrated major universities and Academy institutes, yet Brno possessed substantial mathematical traditions connected with Masaryk University and technical education. Other regional universities and specialised departments contributed to the training of mathematicians and the cultivation of applied research. These centres differed in scale, disciplinary emphasis and access to resources, which made the national system uneven rather than uniformly distributed. Regional diversity mattered because the needs of theoretical research and engineering mathematics did not always coincide. Mathematical analysis might be sustained through small groups and seminars, whereas some areas of computational modelling also required access to technical facilities and interdisciplinary teams. The existence of several academic environments increased the opportunities for distinct research directions to emerge. Independence could reinforce regional differentiation through nationally administered higher education and research support. It did not automatically eliminate the concentration of prestige or resources in the largest institutions.

The constitutional and administrative watershed of 1993 was expressed most clearly in the creation of the Czech Academy of Sciences. Act No. 283/1992 Coll. established the Academy and provided the legal basis for its operation in the new state. Under the transitional arrangements, institutes belonging to the Czech part of the former Czechoslovak Academy were transferred into the successor institutional system. This continuity was important for mathematics because it offered a framework within which ongoing programmes and professional communities could remain active. The legislation also connected the Academy to the Czech state budget and recognised international scientific cooperation among its activities. The Academy therefore acquired both national responsibility and a mandate consistent with participation in international research. This legal transition should not be mistaken for the foundation of the individual disciplines pursued within its institutes. Instead, the state created a new structure through which already established research communities could obtain support and exercise scientific responsibility. The distinction between institutional creation and scientific continuity is especially clear in the case of the Mathematical Institute. Its position changed while its accumulated intellectual traditions remained available.

At the organisational level, the Mathematical Institute's incorporation into the Czech Academy preserved a dedicated environment for fundamental mathematical inquiry. The institute's history reaches back before the communist-era Academy and includes successive stages of institutional reorganisation. Its membership in the new Academy in 1993 therefore represented

another stage in an established institutional biography. Researchers could continue to cultivate mathematical analysis, differential equations and other specialised areas while adapting to changed expectations of international visibility. This continuity reduced the risk that administrative separation would interrupt scholarly work requiring extended periods of concentration. Libraries, editorial activities and professional contacts remained important forms of infrastructure even in a field relatively independent of laboratory machinery. The Institute also provided a partner for university researchers whose teaching obligations differed from the research-centred mission of Academy workplaces. Such complementarities helped preserve national capacity without requiring all mathematical work to be concentrated within one institutional sector. The institute's subsequent development demonstrates that continuity can coexist with significant change in research governance.

The broader Academy reform altered the meaning of institutional autonomy for scientists working within publicly funded research. An academy operating in a democratic state faced expectations that its scientific priorities should be formulated through professional mechanisms rather than direct political prescription. At the same time, financial accountability remained necessary because research resources originated largely in public expenditure. These principles did not always fit together without tension, especially when the value of abstract research was difficult to explain through immediate economic results. Mathematics posed a particularly revealing challenge to evaluation models based on rapid deliverables or commercial application. Significant results may emerge after years of unsuccessful attempts, and their eventual consequences may lie far beyond the horizon of a typical grant. A reformed Academy therefore needed to reconcile scholarly independence with procedures capable of assessing research quality. This task required trust in expert judgement while maintaining transparent allocation of public funds. The institutional reform of 1993 made these tensions visible rather than resolving them permanently. Their consequences would continue to influence Czech mathematical research in later decades.

Universities faced parallel questions about autonomy, responsibility and the relationship between teaching and research. The political opening after 1989 allowed academic communities to reconsider governance arrangements inherited from state socialism. Within mathematics, these changes affected the organisational conditions for departments, academic appointments and postgraduate training. Universities were especially important because mathematical schools cannot reproduce themselves without successive generations of students who learn to work independently. Doctoral supervision connects the acquisition of advanced techniques with participation in unresolved research problems. Such work depends upon continuity of staff, opportunities for scholarly debate and sufficient time for students to develop new ideas. The post-1993 university system therefore carried a double responsibility: preserving disciplinary education and enabling internationally competitive investigation. This combination also encouraged exchanges between university departments and Academy institutes. The emerging national research environment depended upon cooperation between these sectors rather than the exclusive dominance of either.

The separation of Czechoslovakia also changed the administrative position of Czech–Slovak scholarly relationships without erasing their intellectual substance. Mathematicians in the two

republics had shared institutions, educational experiences, professional networks and publication venues over decades. The emergence of separate national systems required new arrangements for funding, appointments and official institutional agreements. Yet collaboration across the new border remained possible because common research problems and interpersonal ties did not disappear with federal institutions. Linguistic proximity and longstanding familiarity offered additional continuity in scholarly communication. From the standpoint of mathematics, a joint proof or seminar could retain its significance irrespective of which national budget supported its participants. The new situation nevertheless introduced practical questions concerning travel funding, project eligibility and the recognition of qualifications. Scientific cooperation increasingly had to operate through explicit international or inter-institutional arrangements rather than assumptions derived from membership in a single federation. This episode illustrates the broader difference between the jurisdiction of institutions and the transnational character of mathematical knowledge. Political separation could reorganise research without completely fragmenting the community that produced it.

International reconnection was one of the strongest forces influencing Czech mathematics during the transition. Mathematical research ordinarily requires less expensive physical equipment than many experimental sciences, but it depends intensely upon access to ideas and people. Visiting another department can expose a researcher to an emerging conjecture, a new method or an unfamiliar style of proof. Participation in conferences and specialist schools can be decisive for the development of young scholars whose professional networks are still forming. The freedom to travel and communicate more openly after 1989 therefore altered the opportunity structure of mathematical careers. After 1993, universities and institutes increasingly represented the independent Czech Republic within these exchanges. International publication and collaboration also became more visible components of research assessment and academic reputation. Such connections strengthened the possibility of testing locally developed ideas against an international community of specialists. They did not guarantee equal access for every institution or researcher, since funding and professional inequalities remained substantial.

Scientific publishing formed another essential connection between inherited expertise and the new conditions of international participation. Mathematical results become part of a discipline through precise formulation, rigorous verification and dissemination in venues accessible to other specialists. Established Czech journals and editorial networks represented valuable elements of scholarly infrastructure during institutional transition. Their continuity could preserve disciplinary memory and provide a platform for authors whose work addressed international questions. At the same time, growing opportunities to publish abroad encouraged researchers to place their findings within broader debates. The balance between supporting national scholarly venues and pursuing international visibility was not necessarily simple. Journals need professional editorial standards and reliable financial or institutional backing, while individual careers increasingly depend upon the reception of work beyond national boundaries. A reformed research system had to recognise the scholarly function of publishing without treating every publication outlet as interchangeable. Mathematics thus required not only funds for research activity but also conditions supporting trustworthy communication and

durable access to published results. These questions became integral to the new research culture.

The transition towards competitive funding represented a particularly important practical change after independence. The Czech Science Foundation began supporting standard basic-research projects in 1993 and created a national channel through which investigators could compete for targeted resources. This model differed from reliance on institutional allocations alone because researchers needed to formulate objectives, explain originality and submit proposals for expert assessment. For mathematics, investigator-driven projects were well suited to research areas in which promising questions arise from scholarly work rather than from predetermined industrial demand. Such grants could support personnel, research visits, collaboration and other costs that ordinary departmental budgets might not cover. The competitive mechanism also introduced uncertainty, since not every scientifically serious proposal could receive an award. Researchers consequently needed to balance the long horizon of mathematical discovery against the shorter timetable of project applications and reporting. Funding reform was therefore both an opportunity for scientific initiative and an additional demand on academic labour. Its importance lay less in replacing intellectual judgement with administration than in changing how that judgement entered public decisions.

Contemporary parliamentary documentation makes the early scale of the competitive transition unusually concrete. An official report on the first Czech grant competition in 1993 records 1,822 applications across major disciplinary groupings. It reports 838 awards and an overall allocation of approximately 151.63 million Czech crowns in the first round. The same report identifies 150 million crowns as the initial designated amount, including funds intended for joint projects among Academy institutes, universities and other research organisations. These figures describe the national grant competition as a whole rather than the financing of mathematics alone. They should therefore not be misrepresented as evidence of a particular mathematical success rate or disciplinary budget. Their historical significance lies in demonstrating the speed with which a competitive mechanism became operational in the new republic. They also show that collaborative research across institutional boundaries was recognised as a policy concern from the beginning. The system was already expected to accommodate a diversity of topics, including basic scientific questions without immediate commercial application. Mathematics entered this new environment alongside other disciplines and had to make its intellectual value legible to expert reviewers.

The influence of grant competition upon mathematical research was shaped by the distinctive economics of the discipline. Many mathematical projects require comparatively modest material expenditure, but they depend heavily upon highly trained people and uninterrupted intellectual time. A grant may therefore have substantial scientific value even when it finances few tangible objects. Travel, visiting positions, doctoral assistance and access to publications can alter the productivity of a research group more effectively than equipment procurement. Evaluation procedures must nevertheless recognise that the outcome of research cannot be predicted as precisely as a routine technical deliverable. Promising conjectures sometimes resist proof, while unexpected discoveries may redirect an entire investigation. A proposal can demonstrate mathematical significance and methodological competence without guaranteeing

a predetermined theorem. Competitive funding is most compatible with fundamental mathematics when expert assessment acknowledges this uncertainty rather than interpreting it as deficient planning. The emergence of the Czech grant system consequently raised enduring questions about evidence of quality, reasonable project horizons and scientific risk.

Institutional financing continued to matter alongside the introduction of competitive projects. Universities and Academy institutes needed stable support for academic positions, buildings, libraries and the administrative services that enable research. Competitive grants could supplement these foundations, but they could not realistically substitute for every long-term obligation. The most effective mathematical environments require continuity across years when individual projects begin and end. A seminar series, a departmental library or a doctoral training programme produces benefits that cannot always be assigned to one grant. The tension between stable institutional resources and competitively allocated research funds therefore affected decisions about recruitment and research planning. Smaller groups could be especially vulnerable when they lacked the administrative capacity to compete repeatedly for external support. Larger institutions, meanwhile, could use established networks and expertise to improve their position within the new funding environment. The transition after 1993 was thus not simply an increase in the number of possible funding sources. It was a restructuring of the relationship between institutional stability, researcher initiative and accountability.

The emergence of a more competitive academic culture also altered the criteria by which scientific careers were understood. Membership of a respected institution and participation in an established research tradition continued to matter, but they were increasingly accompanied by evidence of scholarly output and external recognition. In mathematics, such evidence includes published theorems, influential methods, invited lectures, international collaboration and the training of successful younger researchers. None of these indicators can capture the full intellectual importance of an individual's work in isolation. Citation measures may reward fashionable subjects, while important results in specialised fields can remain relatively little cited for years. A rigorous evaluation culture therefore requires disciplinary expertise rather than mechanical comparison across unrelated branches of science. The new environment encouraged Czech mathematicians to communicate their results internationally, but it also exposed them to the limitations of quantitative assessment. Academic careers became more closely connected with international competition and the ability to secure research support. This transformation changed expectations without changing the fundamental criterion of mathematical correctness.

Generational continuity was a further issue in the preservation of Czech mathematical schools. Established researchers possessed specialised knowledge and professional experience accumulated under the previous system, while younger mathematicians encountered a rapidly opening international landscape. Productive renewal required the interaction of these generations rather than the displacement of one by the other. Senior scholars could maintain research programmes, teach advanced techniques and preserve methodological insight. Younger researchers could bring new questions, contacts and approaches learned during periods of mobility abroad. Doctoral programmes offered an institutional space in which these forms of knowledge could meet. Yet recruitment and retention depended upon the

attractiveness of academic careers, the availability of positions and the financial conditions of research. The new republic therefore faced a human-capital problem that could not be solved through constitutional change alone. Preserving a research group in 1993 was only the first step towards ensuring that it would still produce original mathematics many years later. Scientific continuity required active reproduction of expertise rather than passive reliance on distinguished predecessors.

The distinction between fundamental and applied mathematics also acquired new prominence in the changing economic environment. Abstract research pursues relationships whose significance may be conceptual and whose applications cannot be specified in advance. Applied mathematics develops models and techniques for problems arising in engineering, natural sciences, technology and other fields. These orientations often overlap because theoretical advances can later supply the methods needed for practical analysis. Conversely, a difficult applied problem may motivate new questions in differential equations, probability or optimisation. Czech institutions inherited experience on both sides of this relationship, including mathematics cultivated in technical universities and research centres. The transition towards a market economy created changing expectations about the usefulness and funding of academic knowledge. Nevertheless, the reduction of all mathematical work to immediate economic return would have ignored the discipline's distinctive mode of innovation. The post-1993 system had to sustain abstract inquiry while encouraging connections with wider scientific and technological development.

The meaning of scientific independence was itself becoming more complicated as researchers entered increasingly international networks. A nationally funded mathematical institute could pursue questions developed collaboratively with scholars in several countries. The resulting publications might include authors based at foreign universities, while their research drew upon shared methods that had no national ownership. The contribution of Czech mathematics should therefore be assessed through the work of researchers and institutions rather than through exclusive claims to universally applicable concepts. International collaboration can strengthen national research capacity by allowing local groups to engage with problems larger than their immediate resources. At the same time, it can create dependence on external opportunities for mobility, publication and advanced training. The early Czech republic began to negotiate this relationship between domestic institutional responsibility and transnational intellectual activity. Mathematics provides a particularly clear case because its results circulate through proof and communication rather than through physically bounded research facilities alone. National autonomy consequently meant sustaining competent research communities capable of independent judgement within international science. It did not mean conducting mathematics in intellectual isolation.

The legal transition of the Academy, the renewal of university research and the emergence of competitive grants were interconnected rather than separate reforms. Institutional continuity protected the scholarly capacities necessary for researchers to respond to new funding opportunities. Competitive projects, in turn, could support collaboration and travel that helped those inherited capacities develop further. Universities supplied younger researchers and maintained advanced teaching, while Academy institutes offered concentrated environments

for specialised inquiry. International access widened the range of questions and collaborators available to both sectors. The benefits were not automatic, because research quality also depended upon governance, professional trust and adequate long-term support. Nevertheless, by combining preservation with institutional adaptation, the system reduced the danger of a complete break in mathematical expertise. This approach differed from a model in which progress was assumed to require the replacement of established schools. Instead, renewal could emerge through the creative reorganisation of older strengths.

Looking beyond the immediate transition makes the significance of institutional continuity still clearer. The Mathematical Institute later obtained the status of a public research institution in 2007, illustrating that the reorganisation initiated in the early 1990s was not the final stage of governance reform. Its later autonomy and combination of institutional and project resources belonged to a longer history of adapting public research organisations to changing expectations. The reference to this subsequent development is important for interpretation but should not be projected backwards as though the 2007 framework already existed in 1993. Similarly, the mature forms of European cooperation seen in later decades should not be confused with the more tentative process of international reconnection at the start of independence. Historical periods acquire meaning when their distinctive possibilities and limitations remain visible. In 1993, the foremost achievement was the preservation of functioning institutions under a new national legal order. The more demanding task of achieving internationally competitive mathematical research would unfold through repeated reforms, collaboration and generational change. The watershed was therefore a foundation for later transformation rather than a completed model of scientific excellence. Distinguishing foundations from outcomes allows the history of Czech mathematics to be told without retrospective exaggeration.

The first year of the independent republic ultimately demonstrates how durable scientific traditions can operate through a major political transformation. Czech mathematics entered 1993 with established universities, specialised Academy research, experienced scholars and intellectual connections inherited from the Czechoslovak period. The new state reorganised the governance of that inheritance and began to support fundamental research through a competitive national grant mechanism. These changes expanded opportunities for scholarly initiative but also introduced pressures involving evaluation, funding uncertainty and the demonstration of international relevance. Research groups had to adapt without abandoning the long-term forms of inquiry on which significant mathematics depends. The most consequential continuity lay in the survival of expert communities, doctoral traditions and the capacity to formulate and scrutinise original mathematical problems. The most consequential change lay in the institutional environment through which those communities could obtain resources, recruit researchers and cooperate internationally. The year 1993 should therefore be interpreted neither as the birth of Czech mathematical thought nor as a merely administrative footnote. It marked the establishment of a new national framework in which inherited expertise could be preserved, reassessed and connected to a broader European future.

The First Post-Czechoslovak Stage: Mathematical Research, Academic Transformation, and International Reconnection, 1993–2003

The first decade of independent Czech statehood brought mathematics into a new institutional landscape without severing its intellectual connections to earlier Czechoslovak scholarship. By January 1993, researchers had already experienced several years of political opening following the changes of 1989. Freedom of academic travel and communication had widened before the federal state ceased to exist. The new republic nevertheless had to establish durable rules for scientific finance, institutional responsibility and higher education. Mathematics was especially sensitive to these changes because its most valuable resources were often people, libraries, seminars and research relationships rather than expensive laboratories. A newly open environment made it possible to renew contacts with colleagues who had previously been difficult to visit or invite. Yet greater freedom also brought uncertainty about posts, career structures and the distribution of public funds. The years from 1993 to 2003 must therefore be understood as a period of reconstruction through continuity, not a wholesale replacement of existing mathematical traditions. The transformation gradually connected inherited scholarly strengths with an international research culture in which competitive evaluation and sustained collaboration assumed increasing importance.

The institutional inheritance of Czech mathematics was unusually diverse for a relatively small country. Charles University in Prague had long maintained strong traditions in analysis, geometry, algebra and mathematical logic. Mathematical activity also flourished at the universities of Brno and at technical institutions where theoretical methods met problems of engineering and computation. Academy institutes supplied another research setting, allowing specialists to pursue long-term programmes without the full teaching commitments associated with university appointments. Mathematical journals, professional societies and recurring seminars linked these environments into a recognisable scholarly community. Because mathematical problems frequently develop over decades, such networks retained value even when their administrative arrangements changed. Individual mathematicians could preserve research agendas while simultaneously revising the practical conditions under which they worked. Institutional transformation was consequently less visible in new buildings than in the changing responsibilities of departments, funding bodies and research teams. The decade was marked by an effort to make these inherited structures function effectively in an open, internationally comparable system. Understanding this inheritance is essential to distinguishing genuine advances in research from changes in administrative vocabulary alone.

An immediate challenge after 1993 was to stabilise the relationship between universities and the Czech Academy of Sciences. The newly constituted Academy inherited research institutes from its Czechoslovak predecessor and provided institutional continuity for fundamental investigation. Its Institute of Mathematics entered the new organisation with established programmes and a professional community that did not have to be recreated from nothing. The Academy was nevertheless expected to justify the scientific quality and public value of its work more explicitly. University departments faced corresponding expectations to combine teaching with an expanding research mission. Such changes encouraged collaborations between

researchers occupying different formal positions. A mathematician employed at an Academy institute might supervise students with a university partner, contribute to seminars and participate in projects involving several organisations. These arrangements were especially appropriate to mathematics, whose questions often cross the administrative boundaries between analysis, geometry, logic and computation. By the end of the decade, institutional affiliation increasingly shaped opportunities for collaboration rather than defining the limits of intellectual participation.

The establishment of the Czech Science Foundation in 1993 introduced a consequential mechanism for distributing support to basic research. Competitive project funding complemented the institutional resources that continued to sustain universities and Academy institutes. Applicants increasingly needed to articulate research questions, explain their significance and demonstrate that proposed work could be carried out by qualified teams. Such requirements were not incompatible with pure mathematics, but they demanded new habits of planning and communication. Unlike experimental sciences, mathematical research cannot always promise a particular theorem or a predictable sequence of observations. Productive grant assessment therefore had to distinguish serious and original programmes from speculative claims of guaranteed discovery. The new environment could support young groups and emerging areas when proposals were judged on intellectual merit rather than inherited position. It could also privilege researchers with established reputations and greater experience of proposal writing. The resulting balance between opportunity and competition became a defining feature of mathematical careers during the decade. Public grants thus helped change the organisation of research even when the scientific content remained firmly rooted in classical mathematical problems.

The introduction of competition did not eliminate the need for stable institutional funding. Mathematical work often requires sustained concentration over periods that extend beyond the life of a particular project. Libraries, journals, regular workshops and research posts depend on commitments that cannot be renewed from one successful proposal to the next. The Academy and universities therefore remained vital custodians of the continuity on which mathematical progress depends. At the same time, targeted grants made it easier to bring visitors, organise meetings and support work around more sharply defined questions. A productive funding environment combined long-term security with opportunities for scientific initiative. Where resources were scarce, however, researchers had to weigh time devoted to scholarship against growing administrative obligations. This tension was particularly visible for younger mathematicians attempting to establish both an independent research profile and a secure career. During 1993–2003, the central question was not whether competition should replace institutional support, but how both could contribute to resilient scholarly communities.

Changes in academic governance also affected the professional identities of researchers. Under the new conditions, universities and research institutions developed more explicit expectations regarding scholarly output, appointment procedures and external recognition. Departments had to consider how teaching responsibilities could be reconciled with original research. Researchers in small specialisms sometimes depended upon collaboration beyond their home institution to sustain a sufficiently active seminar community. International publications and

invitations gradually became more visible evidence of disciplinary engagement. Yet mathematical reputation could not responsibly be reduced to the sheer volume of articles. A carefully proved theorem of lasting influence might represent more substantive progress than several routine contributions. The transition therefore required evaluation practices sensitive to differences among mathematical fields. A credible research culture also depended on fair opportunities for younger scholars and on the recognition of work whose value becomes apparent only slowly. These concerns accompanied the institutional reforms throughout the decade.

Charles University occupied a central place in the transformation of university-based mathematical research. Its Faculty of Mathematics and Physics connected established pure mathematical fields with a rapidly developing environment of computer science and theoretical computation. Seminar programmes brought students into contact with researchers pursuing questions well beyond the traditional undergraduate curriculum. Increasing international mobility made it easier for lecturers and doctoral students to encounter alternative techniques and approaches abroad. Foreign visitors could contribute directly to graduate training through lectures and collaborative work. At the same time, the faculty maintained the disciplines necessary to preserve rigorous mathematical education. The connection between teaching and research became particularly important in areas whose methods were evolving quickly, including combinatorics and algorithmic mathematics. Faculty life thereby illustrated the broader transition from nationally organised advanced education towards participation in international research conversations. The institution's significance rested not only on its historical prestige but also on its capacity to support new intellectual networks while maintaining demanding foundational training.

One unusually concrete illustration of research organisation in the 1990s is the DIMATIA centre associated with Charles University and collaborating institutions. Its personnel were established in 1996, with Jaroslav Nešetřil as director and Jan Kratochvíl as deputy director. The centre began its regular work in January 1997 and helped bring together mathematicians concerned with discrete structures and related computational questions. Its activities included seminars, workshops and meetings through which researchers and students could interact with international colleagues. In April 1997, the Spring School of Combinatorics at Borová Lada was organised under its auspices. The centre's formal opening programme in May included specialist lectures and a mathematical colloquium. These events were significant because internationalisation was enacted through sustained exchanges, not merely declared in institutional strategies. DIMATIA also exemplified how discrete mathematics could develop in close dialogue with theoretical computer science. Such settings provided an organisational form well suited to an increasingly collaborative discipline. The example shows how local mathematical schools used the freedom of the 1990s to create internationally connected research environments.

The importance of this development becomes clearer when the scholarly character of discrete mathematics is considered. Problems in combinatorics, graph theory and theoretical algorithms often stimulate rapid exchange between different research communities. A single seminar may connect a structural theorem with a computational interpretation that would be less evident

within an isolated department. Prague researchers contributed to such conversations through work on graph structures, homomorphisms, combinatorial reasoning and algorithmic questions. Younger scholars could learn not only results but also the culture of identifying tractable problems and testing conjectures through international discussion. Visits and schools made it possible to form collaborations that endured beyond individual meetings. This process should not be romanticised as an immediate transformation into universal scientific excellence. Institutional resources, language skills and professional contacts remained unevenly distributed across groups. Nevertheless, the development of organised research communities created more favourable conditions for Czech participation in fast-moving international specialisms. By 2003, this networked mode of scholarship was a recognisable component of the Czech mathematical landscape. The implications of this change would become more evident in subsequent decades.

The renewal of mathematical analysis followed a somewhat different rhythm. Researchers studying differential equations, functional spaces and related questions often worked within traditions extending far into the Czechoslovak period. Their progress depended upon technical continuity and sustained engagement with a specialised international literature. New opportunities for travel nevertheless widened access to colleagues using distinct analytical frameworks. The exchange of preprints, participation in workshops and visits to foreign departments encouraged greater familiarity with current research questions. Such activity did not necessarily produce an abrupt change in subject matter. Rather, established questions could be reformulated with modern techniques, while new problems emerged from contact with neighbouring fields. The capacity to maintain stable seminars was essential for advancing detailed arguments and scrutinising proofs. University and Academy researchers thus strengthened the analytical research environment through both continuity and selective intellectual renewal. This combination remains a useful corrective to accounts that identify innovation only with the appearance of entirely new disciplines.

Algebra and geometry likewise benefited from renewed international contact without surrendering their own scholarly identities. Mathematicians in Prague, Brno and other university centres worked in fields whose terminology and methods belonged to worldwide research conversations. International seminars helped reveal connections between abstract algebraic structures, category-based reasoning and geometric questions. These conversations could generate collaborations, but their deeper importance was methodological. Exposure to alternative approaches encouraged researchers to reassess which assumptions were necessary and which techniques could travel between problems. Czech mathematicians also brought to these exchanges longstanding traditions of technical precision and theoretical abstraction. The resulting relationships were reciprocal rather than simply a process of importing mathematical knowledge from abroad. The decade thus involved both the recovery of opportunities previously constrained by politics and the creation of entirely new scholarly contacts. Institutional openness mattered because pure mathematical innovation depends on the unrestricted circulation of ideas as well as on individual ingenuity. Such interactions helped place existing Czech research traditions in a more fully international context. The implications of this change would become more evident in subsequent decades.

Mathematical logic illustrates the relationship between national intellectual inheritance and international scholarly exchange especially well. Work associated with Petr Hájek and other Czech logicians engaged foundational questions that attracted a broad international audience. Developments in formal reasoning and many-valued logics reflected both philosophical depth and increasingly important links with computer science. During the 1990s, these subjects acquired greater visibility as formal methods and knowledge representation became prominent across disciplinary boundaries. Researchers could therefore move between questions of axiomatic foundations and applications in information processing. Such movement did not make abstract logic merely an instrument of technology. Theoretical results retained independent value, even when they later informed computational methods. International publication and specialist meetings were particularly important because foundational logic is sustained by a relatively dispersed global community. Czech mathematical research benefited when its scholars could participate in those communities without the obstacles that had previously constrained scholarly mobility.

Brno contributed a complementary institutional and intellectual environment to Czech mathematics. Masaryk University possessed established traditions in mathematics that could be developed within a transformed higher education system. The city's technical institutions connected applied mathematical methods with engineering and technological education. Research groups across these settings approached questions in algebra, geometry, differential equations, numerical techniques and mathematical modelling. Their specialisations were not identical, and their diversity helped prevent the national research system from becoming exclusively Prague-centred. Cross-institutional teaching and scholarly contacts provided opportunities for cooperation without erasing departmental distinctions. During the transition, regional universities faced the challenge of expanding research while continuing to meet substantial educational needs. Participation in international conferences offered one way of strengthening visibility without requiring the immediate construction of large new institutions. Brno's importance therefore lay in maintaining a substantial second centre for mathematics within an increasingly plural scientific environment. The development of such regional capacity contributed to the long-term resilience of Czech research.

Beyond Prague and Brno, universities in Olomouc, Ostrava, Plzeň and České Budějovice participated in the broader redefinition of mathematical research. Their departments often combined fundamental subjects with teacher education or applications relevant to local technical and scientific programmes. This mixture could be intellectually productive, even when research teams were relatively small. Applied mathematical questions frequently emerged from collaboration with engineers, natural scientists and computer specialists. Regional conferences and seminar series also helped maintain scholarly exchange across institutions. The creation of a genuinely national mathematical research system required such centres to be understood as contributors rather than merely as teaching outposts. Yet differences in staffing and resources shaped the range of specialisations that individual departments could sustain. Internationalisation therefore advanced unevenly, and a successful group might depend heavily on a few particularly active scholars. The important historical development was the gradual

strengthening of opportunities for regional mathematicians to participate in research networks beyond their own institutions.

Higher education reform provided a legal framework for many of these changes. Act No. 111/1998 Coll., the Czech Higher Education Act, clarified the organisation and responsibilities of higher education institutions and their programmes. Its provisions for doctoral study emphasised research and independent creative activity. Doctoral preparation was to proceed through an individual study plan under academic supervision. The original legislation also linked completion to a doctoral examination and defence of a dissertation containing original research results. These requirements formalised the understanding that doctoral education should prepare scholars to contribute new knowledge rather than merely master established material. For mathematics, the distinction was decisive because independent proof and problem formulation are central professional competencies. University departments could organise graduate training around active research groups, provided that supervisory capacity and scholarly standards were sustained. The legislation did not itself create high-quality doctoral education, but it established a structure within which research-centred training could develop. Its long-term significance lay in connecting academic qualification more explicitly with participation in original investigation.

Reform of degree structures also interacted with changes in the everyday practice of mathematical teaching. Theoretical subjects required continuity in proof-based training, but students increasingly needed familiarity with computation and modern scientific communication. Departments could update courses without abandoning rigorous foundations in analysis, algebra, geometry and probability. Research seminars offered another pathway by which undergraduates and postgraduate students encountered unsolved questions. International exchange exposed students to different expectations regarding independent work and research presentation. For doctoral candidates, the ability to read and write mathematical English acquired particular importance because it facilitated participation in international journals and workshops. New methods of electronic communication also made research materials more accessible. None of these developments should be confused with an instantaneous or uniform curricular revolution across every institution. They were cumulative adjustments through which mathematical education became more closely aligned with an open European academic environment.

Academic mobility formed one of the most visible differences between the pre-1989 environment and the decade following independence. Czech mathematicians could visit foreign universities more readily, participate in summer schools and cultivate joint research agendas. Foreign scholars likewise gained more opportunities to spend time in Czech mathematical departments. The benefits of such exchanges were not confined to eventual publications. A period abroad could introduce a researcher to a new proof technique, a different seminar culture or a productive way of formulating problems. Returning scholars often transmitted these experiences through courses, workshops and collaboration with students. International encounters also helped reduce the professional isolation sometimes experienced by small specialist groups. Nevertheless, mobility required funding, institutional support and personal circumstances that were not equally available to all researchers. The early post-Czechoslovak

period was therefore characterised by widening possibilities rather than an entirely frictionless circulation of mathematical talent. It established habits of exchange that would become increasingly normal in subsequent decades.

The practical organisation of scientific travel also changed as universities learned to participate in international schemes and collaborative agreements. Travel funding could come from institutional budgets, competitive grants, hosts abroad or European programmes. Each mechanism had different administrative requirements and often favoured particular types of activity. Short visits were useful for completing work with collaborators, whereas longer placements could reshape a researcher's intellectual development. International conferences offered another essential forum for testing ideas and learning which problems commanded attention. Young mathematicians could establish reputations through presentations well before acquiring extensive publication records. At the same time, attendance alone was not evidence of scientific achievement. The lasting value of mobility depended on whether new contacts produced sustained research, improved training or more demanding methodological standards. By 2003, the ability to organise such interactions had become an important element of institutional competence.

Publication practices underwent a parallel process of international adjustment. Mathematical researchers already valued specialist journals and peer review, but international accessibility now assumed greater practical importance. Departments increasingly treated publication in widely read scholarly outlets as a route to professional visibility and external assessment. English-language presentation enabled findings to circulate across national and linguistic boundaries. This shift was not merely linguistic, because refereeing processes exposed claims to independent scrutiny by specialists from other institutions. For the mathematical sciences, such scrutiny is indispensable even when a proof appears complete to its authors. The expanding availability of electronic journals and preprint circulation further reduced delays in scholarly communication. Researchers could follow developments abroad without relying exclusively on physical access to printed periodicals. Yet different mathematical areas had different publishing rhythms, making simple comparisons of article counts potentially misleading. The decade consequently encouraged international standards while also raising questions about how mathematical quality should be evaluated.

Czech mathematical journals continued to perform important functions during this period. Established periodicals provided continuity for scholarly communication and opportunities for specialised research communities to maintain their own intellectual identity. Their significance did not depend on serving only domestic authors or readers. A reputable national journal can provide an international forum while retaining a recognisable editorial tradition. Researchers and editors faced growing expectations concerning peer review, discoverability and the circulation of articles beyond the immediate region. Libraries simultaneously had to balance the preservation of historical collections with access to current electronic resources. The increasingly digital environment made mathematical literature easier to exchange but also introduced questions of subscription costs and long-term preservation. Publishing infrastructure therefore formed part of the institutional transformation of mathematics, even though it attracted less attention than grants or university reforms. Reliable access to a deep scholarly literature remained essential

because new mathematical results frequently depend on work published many years earlier. The continuity of journals and libraries helped bridge the transition from older scholarly practices to new forms of research communication. The implications of this change would become more evident in subsequent decades.

Computerisation changed the material conditions of mathematical scholarship without displacing its theoretical foundations. Access to electronic correspondence and online bibliographic resources made collaborative work considerably faster. Researchers could circulate manuscript drafts and communicate proofs without waiting for repeated postal exchanges. Symbolic and numerical computing also became more accessible in teaching and selected research areas. These tools enlarged the range of examples and calculations that mathematicians could investigate. However, mathematical validity continued to depend on precise argument, not on the mere production of computational output. In numerical analysis and applied modelling, computation formed part of the research method itself and required careful attention to stability and error. In pure mathematics, digital tools were often most valuable as aids to exploration and communication. The decade therefore saw a practical convergence between traditional mathematical reasoning and increasingly powerful information technologies.

Interdisciplinary collaboration was another characteristic of the transformed research environment. Mathematicians working with physicists, engineers, economists or computer scientists could contribute general methods that travelled across particular applications. Differential equations linked theoretical analysis with mechanics and other problems involving evolving systems. Probability and statistics offered ways to formalise uncertainty in increasingly varied scientific contexts. Optimisation and operations research connected mathematical reasoning with planning and decision-making. Discrete mathematics and logic became particularly relevant to theoretical computing and information systems. Such relationships did not erase the distinction between fundamental and applied investigation. A theorem motivated by an application could retain significance after the original practical problem had been solved. Conversely, abstract mathematical ideas could acquire unforeseen applications many years after publication. The 1990s and early 2000s broadened the institutional settings in which these exchanges could occur.

The changing relationship between mathematics and the economy requires especially careful interpretation. Post-socialist economic restructuring transformed the industrial and technical organisations that had previously commissioned or supported some areas of applied research. Universities and research institutes consequently had to explore new relationships with public agencies and private enterprises. Mathematical modelling could contribute to production, finance, transport or computing, but the demand for such expertise varied by sector. Fundamental mathematics could not reasonably be evaluated only through immediate industrial revenue. Its public value included education, methodological innovation and the long-term intellectual foundations of technological development. Applied groups nevertheless gained opportunities to demonstrate how mathematical methods could address new types of quantitative problem. The resulting system combined multiple forms of relevance rather than

one universal pathway from theorem to product. This balance was important for maintaining a broad national mathematical culture during economic transition.

Younger researchers entered this environment with opportunities that had scarcely existed under the previous political order. International doctoral exchanges and postdoctoral appointments made it possible to encounter research schools abroad at an early career stage. Competitive grants could support emerging groups, although access depended on institutional circumstances and the assessment of proposals. Young mathematicians also faced uncertainties associated with limited permanent positions and the gradual introduction of more explicit performance criteria. Successful career development required research originality, reliable supervision and access to scholarly networks. Mentoring was therefore as important as formal changes in funding. Established mathematicians helped preserve continuity by transmitting research techniques, seminar practices and professional judgement to a new generation. At the same time, younger scholars introduced questions and approaches encountered through mobility and expanding computational fields. The interaction between generations became one of the mechanisms through which Czech mathematics adapted without losing its historical depth. By the end of the decade, this mixture of continuity and renewal was visible across several major research centres.

The growing openness of the system did not automatically resolve inequalities between institutions. Larger universities and established Academy groups often possessed stronger international connections and more administrative capacity than smaller departments. They could provide more specialised doctoral supervision and sustain broader seminar programmes. Limited budgets made it difficult for every institution to support frequent travel, visitors or comprehensive library subscriptions. Some promising researchers might therefore find it necessary to move between Czech institutions or seek positions abroad. This mobility could enrich national science through returning expertise, but it could also place pressure on the retention of skilled mathematicians. The relationship between concentration and regional diversity remained a persistent issue. National excellence required strong specialist centres, while a healthy educational system needed mathematical capability across multiple universities. The early reforms created new possibilities for both aims without eliminating the tension between them.

International cooperation also required a more mature understanding of scholarly contribution. A Czech mathematician publishing with foreign colleagues participated in a shared intellectual achievement that could not always be divided into national shares. Institutional reputation became associated with sustained collaboration and intellectual leadership rather than simple counts of cross-border contacts. Workshops and specialist schools could build capacity even when their immediate outputs were not easily quantified. Academic networks also required reciprocity, including the willingness to host visitors and contribute to communal research activities. The small size of some Czech specialisms made such reciprocity particularly valuable. Researchers often depended on colleagues abroad for complementary expertise and access to wider debates. Yet local traditions mattered because they gave Czech teams distinctive problems, methods and forms of training to contribute. The decade thus involved a movement from reopening international doors towards becoming an active participant in the communities

beyond them. The implications of this change would become more evident in subsequent decades.

By the early 2000s, the Czech mathematical system had acquired several important features of a modern European research environment. The Academy maintained specialised institutions devoted to fundamental investigation, while universities increasingly integrated research into advanced teaching. Competitive funding supported projects alongside continuing institutional resources. Doctoral study had gained a clearer statutory emphasis on original research. Scientific exchange was more international in its participants, language and circulation of publications. New centres and recurring events supplied concrete organisational forms for collaborative mathematics. Traditional subjects such as analysis, algebra, geometry and logic remained active while discrete, computational and interdisciplinary fields gained additional opportunities. These developments were not uniform in pace or scale, and no single reform explains them all. Together they established the conditions for the more systematic European integration that would follow the country's accession to the European Union in 2004.

The historical achievement of 1993–2003 is best understood as an enlargement of the possibilities for mathematical scholarship. Czech mathematics preserved intellectual traditions developed through Czechoslovak universities and Academy institutes while adapting to a new national framework. Research careers became more strongly connected with competition, international evaluation and professional mobility. Departments confronted the demanding task of renewing doctoral education without diminishing the conceptual rigour of mathematical training. Researchers built or revitalised networks that linked Prague, Brno and regional centres with colleagues across Europe and beyond. Journals, libraries, seminars and academic visits remained fundamental, but their operation increasingly reflected global research practices. The new conditions created risks associated with uneven resources, career uncertainty and the pressure to produce measurable outputs. They also allowed talented scholars to formulate more ambitious projects and develop sustained international collaborations. The decade should therefore be seen neither as a period of unqualified success nor as a mere administrative interval before European Union membership. It was the formative stage in which continuity became the foundation for a more open and internationally engaged Czech mathematical science.

European Integration and Scientific Consolidation: Czech Mathematics in the Emerging European Research Area, 2004–2010

The accession of the Czech Republic to the European Union on 1 May 2004 altered the conditions in which mathematical research could be organised and sustained. The change did not suddenly create new mathematical schools, since established communities had already rebuilt many international contacts during the preceding decade. It nevertheless placed those communities within a more comprehensive legal, financial, and institutional framework for European cooperation. Researchers could increasingly approach international collaboration as a normal

element of academic practice rather than as an exceptional opportunity dependent upon personal networks. Mathematics was especially well positioned to benefit from this development because many research questions cross national borders without requiring large experimental laboratories. At the same time, different mathematical fields required different forms of support, ranging from uninterrupted time for theoretical work to substantial computing facilities. The history of Czech mathematics between 2004 and 2010 is consequently a history of consolidation rather than a narrative of instantaneous transformation. Its central achievement was the gradual alignment of inherited intellectual strengths with a European research environment that demanded both openness and institutional reliability. This distinction establishes the interpretative framework for the period as a whole.

The European Research Area was intended to make scientific knowledge, research careers, infrastructures, and cooperation more closely connected across participating countries. For Czech mathematics, the concept supplied an important vocabulary through which universities and research institutes could position themselves within a wider scientific system. National institutions remained responsible for the everyday conditions of employment, teaching, research funding, and academic evaluation. European programmes nevertheless offered additional routes for collaborative investigation, training networks, and movement between research groups. Such routes were valuable because the progress of mathematics frequently depends upon encounters between specialised schools whose technical languages may differ considerably. The value of international integration was therefore intellectual as well as administrative, encouraging the circulation of questions, techniques, and interpretations. A mathematician working in Prague or Brno could participate in an international problem without abandoning a distinct local research tradition. European integration thus created a framework within which national scientific identity and transnational academic practice could reinforce rather than exclude one another.

The period began against the background of the Sixth Framework Programme for research and technological development, which ran from 2002 to 2006. Its collaborative instruments predated Czech accession, and the distinction matters when interpreting the chronology of scientific opening. Czech research teams had already acquired experience of European proposals, partnerships, and reporting before full membership was achieved. Accession nevertheless strengthened their position as participants in the common European institutional order and encouraged more systematic research planning. Universities needed personnel capable of interpreting eligibility conditions, preparing budgets, administering agreements, and coordinating interdisciplinary partnerships. Mathematicians were not always the formal leaders of such consortia, particularly when projects were organised around engineering or information technologies. Their analytical contributions could nonetheless be decisive wherever models, numerical schemes, optimisation procedures, or statistical reasoning were required. This combination of disciplinary expertise and growing administrative capacity became an important feature of Czech participation in European research. The preparation of these projects also encouraged researchers to understand how European scientific institutions worked.

The launch of the Seventh Framework Programme in 2007 introduced a new phase of European research cooperation that continued beyond the period examined here. By 2010 Czech

organisations were adapting to a framework whose ambitions extended from collaborative science to researcher mobility and frontier investigation. The programme created possibilities for mathematics both as an autonomous research discipline and as a methodological contributor to other fields. Fundamental researchers could seek international partners pursuing related theoretical questions, while applied mathematicians could contribute specialised methods to multidisciplinary consortia. Participation was not automatic, however, because successful proposals required research excellence, credible management, and strong international networks. Established academic groups could draw upon reputations built during earlier decades, whereas smaller departments sometimes faced greater coordination burdens. The effects of the programme should therefore be assessed through institutional learning and scientific relationships as well as through grant totals. Its significance lay in the way it made transnational competition and collaboration an increasingly ordinary dimension of Czech mathematical life.

Competitive national funding remained essential during these years and should not be overshadowed by the visibility of European programmes. The Czech Science Foundation continued to provide a principal mechanism for supporting investigator-led basic research through peer-reviewed project competitions. For mathematicians, the ability to define a coherent theoretical problem and demonstrate its scientific importance was especially consequential within such schemes. Research grants could facilitate academic travel, support doctoral researchers, and protect time for work that produced no immediate commercial application. Yet mathematical discovery is often unpredictable, and a valuable line of enquiry may develop more slowly than a conventional project cycle allows. The growing prominence of grant applications consequently introduced new pressures alongside the opportunities of competitive support. Scientific merit had to be communicated in forms intelligible to evaluation panels without reducing difficult theoretical questions to superficial promises. The resulting funding culture reshaped how mathematicians described their work while leaving the underlying standards of proof and intellectual originality intact. Fundamental mathematical inquiry consequently retained its own criteria of excellence within a changing financing system.

Institutional financing and project funding performed different tasks within the research system, and their relationship remained delicate. Long-term institutional support sustained libraries, academic posts, seminars, doctoral supervision, and the continuity of specialised mathematical groups. Competitive grants supplemented these foundations by enabling focused projects and by rewarding successful research initiatives. An institute with an impressive sequence of short grants could still be vulnerable if stable positions and operating resources were insufficient. Conversely, secure institutional structures alone could not guarantee renewal without openness to younger scholars and emerging problems. The years following European accession therefore required a balance between predictability and competition rather than the substitution of one mechanism for another. This was particularly important for fundamental mathematics, where the most influential outcomes may not be foreseeable at the moment funding is allocated. Understanding Czech scientific consolidation requires attention to both forms of financial support and to their different intellectual consequences.

A major organisational development occurred when the legal framework for public research institutions took effect in 2007. Institutes of the Czech Academy of Sciences consequently operated within a reorganised institutional environment intended to clarify their legal position and responsibilities. For mathematics, the significance of this change did not lie in the invention of a new discipline or the wholesale replacement of research personnel. It concerned the conditions under which an existing specialised institute could manage resources, pursue projects, cooperate with universities, and plan its future. Greater administrative clarity could support international agreements and a more professional approach to research management. It could also introduce additional demands for formal reporting and organisational accountability, matters that had previously appeared peripheral to theoretical investigation. The continuity of mathematical groups was especially valuable because scholarly expertise is accumulated through decades of seminars, collaborations, and doctoral training. The reform therefore illustrates how institutional modernisation could coexist with the preservation of an established scientific tradition. The institutional transition was consequently substantial without constituting a rupture in mathematical knowledge.

The Institute of Mathematics of the Czech Academy of Sciences remained a central institution for fundamental research during the period of consolidation. Its activities linked mathematical analysis, differential equations, mathematical physics, numerical approaches, and related theoretical fields within a specialised academic environment. The institute offered an organisational setting in which researchers could devote sustained attention to problems not necessarily aligned with immediate industrial demand. Connections with universities were nevertheless important because doctoral education and the renewal of scholarly generations depended upon regular academic exchange. International visitors, seminars, and collaborative projects helped connect local approaches to active developments elsewhere in Europe. The institute therefore functioned both as a place of independent theoretical enquiry and as a node within a larger research network. The importance of such an organisation should not be measured solely by the amount of experimental infrastructure it controlled. Its accumulated expertise, intellectual continuity, and capacity for long-term research were resources of a different but equally significant kind.

Charles University continued to anchor a broad range of mathematical disciplines within the national university system. The Faculty of Mathematics and Physics brought together research and advanced teaching in fields extending from pure analysis and algebra to discrete mathematics and mathematical modelling. During the years after accession, its international position depended upon the quality of research groups as well as upon the accessibility of European collaboration. Seminars, doctoral programmes, visiting appointments, and conferences offered means by which international exchange could become embedded in ordinary academic life. Theoretical mathematics particularly benefited from environments where disagreement over definitions, proof techniques, and conjectures could be pursued over extended periods. A university contributed to such work by sustaining intellectual communities in which students encountered unresolved problems alongside established knowledge. The interaction between education and research was therefore not an incidental feature of university mathematics but one of its fundamental mechanisms of renewal. The consolidation of the

Prague school reflected this continuing interplay between institutional stability and participation in wider mathematical debates. This made the university simultaneously a research centre and a place where mathematical methods were transmitted.

The Prague tradition in combinatorics and discrete mathematics provides a useful example of international connectedness without loss of local scientific character. Work associated with Jaroslav Nešetřil and collaborating researchers linked structural graph theory, combinatorics, mathematical logic, and theoretical computer science. Such research could be pursued through international conferences and joint papers without requiring expensive physical experimental facilities. Its technical difficulty nevertheless demanded extensive professional networks because important problems often connect several apparently distant areas of mathematics. Research seminars and specialist workshops helped sustain these connections while exposing younger scholars to a wider range of mathematical methods. The importance of the tradition lies in its production of new mathematical questions and conceptual frameworks, not simply in the number of international contacts recorded. Cooperation made it possible to test approaches against the expertise of researchers working in different theoretical communities. The Prague experience illustrates how a national school can remain recognisable while participating fully in a border-crossing field.

Masaryk University in Brno provided another major centre of mathematical consolidation after 2004. Its mathematical research inherited strong traditions in algebra, geometry, analysis, and the relationship between mathematics and informatics. European integration encouraged university groups to deepen international relationships that had already expanded during the 1990s. The resulting collaborations created opportunities for joint seminars, student mobility, specialised conferences, and the participation of Czech researchers in international projects. Mathematical work in Brno also developed within a regional environment shaped by technical universities and growing demand for computational expertise. This proximity encouraged exchanges between abstract mathematical research and application-oriented investigations without making the two identical. The city's mathematical culture therefore combined continuity in established theoretical disciplines with increasing engagement in interdisciplinary questions. Its significance within Czech science demonstrates that post-accession research development was distributed across several academic centres rather than confined to Prague. The process also reinforced the importance of multilingual and internationally connected academic communities.

The relationship between Masaryk University and Brno University of Technology deserves particular attention because their missions were complementary. A research university could sustain investigations of mathematical structures whose practical value was initially uncertain, while a technical university faced problems arising directly from engineering systems. Both settings required sophisticated analytical reasoning, but the timescales and standards of application could differ substantially. Numerical approximation, differential equations, optimisation, and computational geometry offered natural meeting points between these communities. Researchers could exchange methods while maintaining the distinct aims of proof, algorithmic design, and engineering validation. The European research environment increased the incentive to formulate such shared projects in internationally recognisable terms.

Joint research also required effective arrangements for attribution, supervision, access to computing, and the management of interdisciplinary expectations. The broader development was a gradual strengthening of cooperation rather than the dissolution of boundaries between pure and applied mathematics.

Mathematical research in Czechia was not restricted to its two largest university cities. Departments in Olomouc, Ostrava, Plzeň, České Budějovice, and other centres maintained specialised teaching and research traditions with differing disciplinary profiles. Their development after 2004 was influenced by national funding schemes and by the possibilities of international cooperation. Regional research groups often benefited from distinctive concentrations of expertise that could not be inferred merely from institutional size. Small mathematical teams can become internationally relevant when they maintain sustained work on carefully chosen questions and collaborate effectively beyond their home institutions. At the same time, such groups may face disadvantages in recruitment, administration, and access to experienced grant-management personnel. European programmes offered new opportunities but did not remove the structural differences separating major institutions from smaller university departments. The geography of Czech mathematics thus became more interconnected while continuing to exhibit significant inequalities in resources and visibility. These asymmetries shaped the geography of opportunity within the national mathematical system.

The reform of doctoral training formed another crucial dimension of scientific consolidation during the second half of the decade. Doctoral research in mathematics requires mastery of a substantial theoretical tradition before genuinely original problems can be addressed. Access to international seminars, summer schools, visiting researchers, and specialist literature broadened the intellectual environment available to students. European mobility initiatives helped make cross-border study and research an increasingly realistic component of academic careers. Such experiences were valuable not simply for language acquisition but for learning how different mathematical communities formulate and evaluate research questions. Doctoral candidates also required stable supervision and adequate time, especially when their subjects involved technically difficult proofs. The growing emphasis on grants and measurable outputs sometimes created tensions between training and short-term productivity. Successful programmes had to combine international exposure with the patient intellectual formation upon which rigorous mathematics depends.

Research mobility during 2004–2010 should be understood as a two-way process rather than a simple movement of Czech scholars towards Western Europe. Czech mathematicians continued to study, teach, and collaborate at foreign universities and specialist research centres. At the same time, Czech institutions could host visitors whose approaches introduced new techniques and perspectives into local discussions. Short academic visits were often especially productive in mathematics because substantial theoretical progress can emerge from intensive discussion of a single problem. Longer appointments facilitated doctoral supervision, collaborative writing, and the transfer of methods between research traditions. Nevertheless, mobility depended on funding, working conditions, family circumstances, and the ability of institutions to integrate visitors effectively. Internationalisation could therefore

strengthen Czech institutions only if research communities remained sufficiently stable to retain and develop acquired knowledge. The central challenge was to turn temporary circulation into enduring scientific relationships and locally sustained expertise. Continuity at home remained an important condition for productive scientific mobility abroad.

International publication became an increasingly important dimension of evaluation and scholarly recognition during the consolidation period. Mathematical research traditionally circulates through journals, monographs, preprints, conference proceedings, and specialised academic meetings. The significance of any particular result cannot be judged adequately by publication counts alone because papers differ enormously in originality and long-term influence. Nevertheless, visibility in internationally reviewed journals helped Czech researchers participate in common disciplinary debates and reach audiences beyond national institutions. English-language scholarly communication reduced some barriers to collaboration, even as it created additional demands for precision and stylistic competence. Editorial and refereeing practices linked Czech mathematical departments to the critical norms of an international discipline. Participation in these processes required researchers not only to publish results but also to assess the work of colleagues and contribute to shared professional standards. The gradual internationalisation of publication culture therefore changed the organisation of mathematical communication without changing the central authority of valid proof.

Scientific conferences were important instruments of this transformation because they connected formal publication with personal exchange. A conference lecture allowed a researcher to present a new approach before its complete implications had been established in print. Questions from specialists could reveal hidden assumptions, suggest stronger formulations, or identify connections with seemingly unrelated theorems. Czech universities and academic institutes participated in these exchanges through the organisation of meetings and the attendance of researchers abroad. The European setting facilitated shorter cross-border journeys and repeated contact with neighbouring mathematical communities. Specialist workshops were particularly suitable for mathematics because their intellectual value depended more on focused expertise than on the scale of equipment. Yet conference participation was not uniformly accessible, especially to early-career researchers without travel funds or stable institutional support. A mature account of consolidation must therefore consider the infrastructures of discussion and professional inclusion as well as the production of final publications. Conferences also provided a means of sustaining specialised mathematical communities across generations.

One prominent intellectual development was the increasing contact between mathematics and theoretical computer science. Research on graph structures, algorithms, logic, complexity, and combinatorial optimisation operated across a boundary that was institutional rather than strictly conceptual. Computer science posed new questions about effective procedures and computational feasibility, while mathematics supplied tools for formulating and analysing them. Czech research groups participated in these exchanges through university departments and institutes concerned with mathematical and information sciences. A theoretical algorithm could be innovative through its proof of correctness, its complexity bounds, or a new structural insight that made computation possible. Such advances did not always produce a commercial software

package during the same research period. Their relevance therefore needs to be assessed in terms of intellectual contribution as well as subsequent technological use. The years after European accession encouraged this interaction by making informatics a frequent partner in interdisciplinary funding schemes.

Applied mathematics also developed closer relationships with engineering research during the period under consideration. Problems involving fluid movement, heat transfer, elasticity, transport, and industrial processes often required mathematical descriptions that could not be solved analytically in closed form. Numerical analysts contributed by designing approximation methods whose convergence, stability, and accuracy could be investigated mathematically. Engineering researchers supplied physical constraints, experimental context, and criteria for deciding whether computed results were practically meaningful. This division of labour demonstrated that mathematical modelling is not merely the mechanical application of a pre-existing formula. A reliable model requires the selection of assumptions, mathematical formulation, numerical implementation, and critical comparison with the behaviour of the system studied. Cooperation between universities and technical research organisations encouraged the development of these complementary competencies. By 2010 the connection between numerical mathematics and computational engineering was an increasingly important feature of the national research environment. Mathematical methods and questions could circulate across disciplinary boundaries without losing their formal precision.

The relation between mathematical modelling and information technologies became more visible as computational resources improved. Faster computers enlarged the range of models that could be investigated, but greater processing power did not automatically guarantee correct scientific conclusions. Poorly chosen equations, unstable discretisations, inappropriate boundary conditions, and misunderstood uncertainties could invalidate even technically impressive simulations. Mathematical expertise therefore remained essential for constructing reliable numerical experiments and for interpreting their limitations. Czech researchers working in numerical analysis, optimisation, and scientific computing could contribute methods applicable to many disciplinary settings. The growing demand for such skills encouraged collaboration with physicists, engineers, environmental researchers, and specialists in computational information processing. This interaction expanded the social relevance of mathematics while preserving the need for rigorous analytical understanding. Computation became a medium through which mathematical thought reached more complex systems rather than a substitute for mathematical reasoning.

The policy environment changed further with the 2007–2013 programming period of European structural funding. The Operational Programme Research and Development for Innovations was approved by the European Commission in October 2008 and targeted the development of research capacity. Its priorities included centres of excellence, regional research centres, stronger research infrastructure, and conditions linking higher education to scientific activity. Such measures were not designed exclusively for mathematics, and their benefits depended upon the particular institutional and disciplinary context. Nevertheless, they helped establish a more ambitious setting for computational research and interdisciplinary science. Large research infrastructure projects required long preparation periods, including agreements among partners,

technical planning, and institutional commitments. It would therefore be historically inaccurate to describe all infrastructures subsequently built with these resources as fully operational by 2010. The period is better understood as the stage in which policy instruments and project preparations created possibilities realised during the following decade. The scientific value of computation ultimately rested upon the quality of the underlying mathematical formulations.

The emerging plans for large-scale computational research illustrate the distinction between preparation and implementation particularly clearly. Research partners in the Ostrava and Brno regions developed ambitions to combine supercomputing, computer science, and computational mathematics within more substantial institutional frameworks. Their plans reflected the recognition that increasingly complex numerical problems required shared computational resources alongside specialised mathematical expertise. Such initiatives were connected with the funding opportunities that became available through European and national research policy. The IT4Innovations Centre of Excellence project itself began in 2011, which places its implementation outside the chronological boundaries of this section. Treating the project as an achievement already completed in 2010 would obscure the historical sequence of planning, approval, investment, and operation. The importance of the earlier period lies instead in the partnerships and policy priorities that enabled the later infrastructure to emerge. This chronology shows how research consolidation can involve creating conditions for innovations whose concrete results appear several years afterwards.

The distinction between fundamental and applied mathematics became more complex but did not disappear during these developments. Pure mathematics investigates structures and relationships according to questions of consistency, generality, and theoretical significance. Applied mathematics constructs and studies abstractions that may be motivated by phenomena encountered in other sciences or in technological practice. Yet methods created for abstract reasons can acquire applications long after their original discovery, while applied problems can generate entirely new fundamental theories. Institutional funding categories rarely capture the full movement of ideas across this boundary. European programmes frequently emphasised collaboration, innovation, and societal relevance, but national institutions still needed to sustain mathematics without immediate practical deliverables. Czech scientific consolidation depended upon preserving both kinds of work and ensuring that their different criteria of success were understood. The lasting value of mathematical innovation therefore lies in expanding what can be demonstrated, calculated, represented, or explained, not solely in producing readily marketable outcomes. This temporal distinction is essential for attributing scientific developments to the correct historical stage.

The development of mathematical statistics, probability, and optimisation further demonstrated the breadth of this intellectual landscape. Statistical reasoning was increasingly relevant to empirical sciences confronting more complex datasets and demands for reliable inference. Probability theory supplied mathematical languages for systems characterised by uncertainty, randomness, and incomplete information. Optimisation connected abstract questions about feasible solutions and objective functions to decisions in engineering, economics, and logistics. Research environments such as the Institute of Information Theory and Automation provided important connections among these directions. The methodological contributions of

mathematicians could be embedded in larger projects where the principal application belonged to another discipline. Such circumstances make national achievements difficult to describe using an inventory of finished technologies alone. The historical importance of these fields lies partly in their capacity to reorganise how other researchers frame problems and justify conclusions.

European integration also intensified discussion about the assessment of scientific quality and the distribution of research resources. Evaluation systems increasingly considered international publications, competitive grants, research cooperation, and the performance of institutions. These indicators provided information about scientific activity but did not offer a complete measure of mathematical originality. Important mathematical research may result in a single deep theorem rather than a large collection of short publications. Collaborative papers may represent years of concentrated work, while the importance of a conceptual advance may emerge only through subsequent scholarship. Funding authorities consequently faced the difficult task of comparing disciplines with radically different research cultures and material requirements. Czech mathematicians operated within these pressures while continuing to rely on the specialised judgement of peers capable of assessing intricate arguments. The challenge was to make research accountable without confusing administrative measurability with the intellectual value of mathematical knowledge. The influence of such work often appeared indirectly in the quality of research carried out by other disciplines.

The international financial crisis beginning in 2008 introduced an additional context of uncertainty for European research institutions. Economic pressures affected public budgeting and raised questions about the sustainability of ambitious investments and long-term research commitments. Mathematical departments were not necessarily exposed to the same costs as laboratory-intensive disciplines, but they depended upon public salaries, doctoral opportunities, libraries, and stable project funding. A contraction in resources could therefore have consequences even where expensive equipment was not central to research. International projects also required administrative capacity and, in some circumstances, national or institutional contributions that could be difficult to maintain. The crisis did not cancel the larger trajectory of European integration, but it complicated expectations of continuous financial expansion. Long-term mathematical competitiveness depended on institutional resilience under changing economic conditions rather than on the uninterrupted growth of individual programmes. The end of the decade thus revealed the vulnerabilities that accompanied a more ambitious and interconnected scientific system.

By 2010 Czech mathematics occupied a position substantially different from that of the immediate post-Czechoslovak period. Universities and Academy institutes had preserved important mathematical schools while acquiring greater experience of European programmes and international research management. Competitive national grants remained important, and European funding created additional possibilities for networks, mobility, infrastructure planning, and interdisciplinary activity. Research groups in analysis, discrete mathematics, geometry, mathematical logic, probability, optimisation, and computational methods operated within increasingly international professional communities. Technical and information sciences provided expanding areas of application, yet the continuing independence of fundamental

research remained crucial to the intellectual diversity of the system. Major infrastructure investments were moving through planning and early implementation stages, with many of their most visible outcomes still to come after 2010. The stage should therefore be understood as consolidation of institutional capacity, collaborative habits, and methodological possibilities rather than the completion of scientific modernisation. Its legacy was a mathematical research environment better equipped to contribute to European science while retaining the depth of its own accumulated traditions. For mathematics, the preservation of people and research time remained more important than spectacular construction projects.

Internationalisation and Interdisciplinary Expansion: Czech Mathematical Sciences between 2011 and 2020

The years from 2011 to 2020 marked a substantial change in the international positioning of mathematical sciences in the Czech Republic. The institutional reforms of the preceding two decades had created a research environment increasingly familiar with competitive funding, international evaluation and cross-border collaboration. During the 2010s, however, international cooperation moved beyond an aspiration associated with post-socialist recovery and became a routine feature of mathematical work. Research groups in universities and institutes of the Czech Academy of Sciences participated in networks organised around particular problems rather than exclusively around national institutions. Mathematical questions themselves encouraged this development, since proofs, conjectures and abstract methods circulate readily between communities separated by geographical boundaries. At the same time, expensive computational projects introduced infrastructural requirements that pure mathematical research had not traditionally encountered on the same scale. The decade therefore witnessed two partly distinct forms of expansion: the intensification of conceptual collaboration and the construction of shared technological capacity. Neither process replaced the established Czech schools of analysis, algebra, geometry or discrete mathematics, whose intellectual continuity remained an essential resource. Instead, inherited traditions acquired new audiences, additional methodological possibilities and more varied connections with adjacent sciences. Understanding this combination of continuity and widening scientific participation is central to interpreting Czech mathematics during the decade.

It is important not to portray 2011 as an abrupt beginning of international mathematical activity. Czech mathematicians had published internationally and cultivated foreign contacts well before the creation of the independent republic, although the political circumstances of those earlier decades shaped their opportunities. The transformations after 1989 and 1993 made exchanges progressively easier, while accession to the European Union in 2004 enlarged participation in common research programmes. Consequently, the 2010s inherited a scholarly infrastructure of doctoral education, specialised seminars, grant administration and institutional partnerships developed over many years. What changed most visibly was the density and regularity of these

relationships rather than the existence of international mathematics itself. Researchers could remain institutionally based in Prague, Brno or other Czech centres while participating continuously in multinational projects and specialist communities. Joint articles, visiting appointments, international workshops and doctoral supervision became mutually reinforcing components of disciplinary life. These activities also exposed scientific groups to stronger competition for attention and resources within the international research landscape. The decade should therefore be examined as a period of consolidation followed by selective expansion, rather than as a simple narrative of previously isolated scholarship becoming connected.

Charles University in Prague remained one of the principal environments in which the internationalisation of mathematics took an intellectually distinctive form. Its Faculty of Mathematics and Physics maintained research communities spanning mathematical analysis, algebra, geometry, mathematical physics and theoretical computer science. The Department of Applied Mathematics and related groups cultivated particularly visible research on graphs, combinatorial structures, algorithms and discrete geometry. The Centre for Discrete Mathematics, Theoretical Computer Science and Applications, known as DIMATIA, had been established before the decade under review and therefore represented accumulated institutional experience. Its networked character encouraged collaboration among university researchers, academy scientists and colleagues based abroad. The continuity of this arrangement matters because international scientific centres frequently achieve their greatest influence after years of gradual intellectual accumulation. For Czech discrete mathematics, the 2010s brought increasingly systematic attention to the relationship between structural graph theory and computational questions. Theoretical investigations concerned the classification of structures and the identification of properties that allow general problems to be treated efficiently. In this setting, abstract mathematics and theoretical computer science developed not as competing programmes but as deeply interconnected modes of reasoning. Such work demonstrated how an established university school could contribute to international discussions by formulating concepts of broad mathematical relevance.

One of the clearest examples of this conceptual contribution was the theory of sparse graphs and related finite structures. Jaroslav Nešetřil of Charles University and Patrice Ossona de Mendez developed an influential framework for understanding graph classes through structural notions of sparsity. Their monograph Sparsity: Graphs, Structures, and Algorithms, published by Springer in 2012, synthesised definitions, classification methods and algorithmic consequences. The significance of this work lay not in the invention of a single computational procedure but in the articulation of a broad mathematical language. It linked structural properties of graphs with questions in logic, complexity, homomorphisms and algorithm design. In certain graph classes, structural restrictions make it possible to obtain computational results that would not hold in unrestricted settings. The monograph also illustrates a common pattern in contemporary mathematics, whereby abstract theoretical classifications eventually inform methods for practical computation. Although the research was international in authorship, the contribution of a scholar working at Charles University places it within the history of Czech mathematical research. Its emergence shows why national contributions should be assessed

through identifiable intellectual participation rather than treated as discoveries belonging exclusively to one country.

The broadening of graph theory during the decade was not confined to a single research programme. Investigations pursued structural and extremal problems, combinatorial optimisation, graph drawing, discrete geometry and the mathematical properties of computational networks. Some questions were formulated in an entirely abstract language, while others arose from the need to analyse large systems of connected objects. The mathematical structure of a network may represent communications, interactions, dependencies or constraints without depending on the material identity of its vertices. This capacity for abstraction explains why graph theory has become useful across computer science, biology, physics and the social sciences. Nevertheless, the existence of possible applications does not mean that every theorem immediately generates an operational technology. Much of the value of foundational research lies in establishing concepts and results whose practical significance can become apparent only later. Czech research groups benefited from an institutional culture able to sustain both abstract inquiry and interactions with algorithmic disciplines. The resulting relationship between theory and application was therefore productive precisely because it allowed different timescales of scientific progress. In historical perspective, discrete mathematics exemplified how internationalisation could enhance disciplinary depth rather than merely increase the number of partnerships.

Mathematical analysis formed another substantial component of the Czech research landscape between 2011 and 2020. The Institute of Mathematics of the Czech Academy of Sciences and university departments continued investigations connected with function spaces, operators, differential equations and nonlinear phenomena. Such subjects are often less visible outside specialist communities than new computational facilities, but they remain essential to the mathematical description of complex processes. Analytical research asks under what conditions a solution exists, whether it is unique, how regular it may be and how sensitively it depends on initial assumptions. These questions are logically prior to many computational techniques, which can produce numerical approximations without establishing all the properties of the problem being approximated. International collaboration in analysis typically relies on intensive seminars, research visits, shared preprints and detailed work on proofs. The intellectual demands of such collaboration are considerable even when the physical infrastructure required is relatively modest. For Czech mathematicians, participation in these communities allowed local strengths to be situated within broader European debates over analytical methods. The decade thus sustained a form of scientific innovation in which advances were measured principally by conceptual rigour and new mathematical understanding.

The study of partial differential equations occupied a particularly important position at the intersection of pure and applied mathematics. These equations describe change distributed across space and time and arise naturally in fluid mechanics, elasticity, diffusion and many other scientific settings. Research may address existence and uniqueness, regularity, long-term behaviour, singularities or the mathematical consequences of changing physical assumptions. A rigorous treatment of a model requires more than familiarity with the scientific system that inspired it. It also depends on functional analysis, approximation theory, geometric reasoning

and carefully constructed analytical estimates. During the 2010s, Czech groups working on nonlinear problems could engage with international communities concerned with these closely connected mathematical questions. The relevance of the work to mechanics or engineering did not diminish its standing as fundamental research. On the contrary, attempts to understand physical phenomena often exposed mathematical difficulties whose resolution produced methods applicable to different problems. The boundary between pure and applied mathematics therefore appeared increasingly as a working relationship rather than a fixed institutional division. This relationship later became especially consequential when analytical models were implemented through computational simulations.

Geometry and topology retained a parallel importance within Czech mathematical research during the decade. University-based scholars examined geometrical structures, transformations, curvature, discrete configurations and relationships between local and global properties. These investigations continued traditions of abstract reasoning while interacting with analysis, mathematical physics and theoretical computer science. Differential geometry provides tools for understanding smooth spaces, whereas discrete geometry studies configurations that can be described through finite or combinatorial data. Both traditions can contribute to algorithmic methods, although their fundamental questions and preferred techniques often differ substantially. International seminars and collaborative publications offered opportunities to compare approaches formed within distinct national and disciplinary schools. For a mathematical community of limited numerical size, access to such conversations was particularly valuable in specialised subfields. The vitality of geometry was therefore not necessarily reflected in the creation of a separate large research infrastructure. It could instead be observed through the continuation of advanced problems, graduate training and integration into scholarly networks extending beyond Czechia.

Algebra and mathematical logic added further diversity to the research system of the 2010s. Czech mathematical traditions in abstract structures, logical systems and formal reasoning had developed through institutions whose histories preceded the period under review. Their continuing relevance rested on the ability to address contemporary questions in mathematics and theoretical informatics without abandoning disciplinary foundations. Algebra studies relations among operations and structures, while mathematical logic investigates formal languages, inference and the limits of deductive systems. These fields intersect in universal algebra, model theory, categorical approaches and the study of formal representations of knowledge. The growth of computational science made certain questions about formal structures more visible, but did not reduce the disciplines to software engineering. Proof and mathematical explanation remained distinctive forms of scholarly output with standards different from those applied to experimental technologies. International collaboration was important because specialised problems often required conversation among small numbers of experts distributed across many countries. Czech participation in such communities demonstrates that disciplinary strength cannot be judged solely by the scale of institutional equipment. Theoretical research continued to provide intellectual resources from which later computational and interdisciplinary applications could draw.

Probability theory and stochastic methods became increasingly relevant to the mathematical treatment of uncertainty. Randomness enters scientific modelling whenever phenomena cannot be predicted deterministically at the scale or level of detail under investigation. Probability provides a coherent mathematical language for describing such uncertainty, whereas statistics develops methods for learning from observed information. Theoretical work on stochastic processes, dependence, estimation and inference can therefore underpin a wide range of empirical sciences. In Czech institutions, related research was distributed across mathematics departments and organisations concerned with information theory, statistics and automation. This distribution encouraged contact between foundational problems and practical questions arising in data analysis, control and decision-making. Nevertheless, the adoption of probabilistic terminology across disciplines could conceal substantial differences between rigorous mathematical investigation and routine statistical application. The historical significance of the decade lies partly in the increasing circulation of mathematical expertise across those settings. International projects supplied additional opportunities to test modelling methods against problems formulated by partners working in other scientific traditions.

The Institute of Information Theory and Automation of the Czech Academy of Sciences offered an important institutional setting for such interactions. Its research profile connected mathematical statistics, optimisation, decision systems, information processing and the theoretical study of automation. These domains share an interest in how information can be represented, interpreted and used when decisions must be made under constraints. Mathematical methods are indispensable because uncertainty and optimisation require assumptions precise enough to support reproducible conclusions. Scientific questions may concern the best allocation of resources, the robustness of an estimator or the response of a controlled system to changing conditions. Addressing them can involve probabilistic models, numerical algorithms, variational techniques and computational experiments. The institute consequently exemplified a broader trend in Czech science towards collaborative work that crossed older boundaries between mathematics and engineering. Its significance should not be measured only in terms of commercial products, because methodological advances may become useful through later adoption by other researchers. The decade reinforced the importance of institutions capable of connecting theoretical competence with diverse problem domains. This intermediary role was especially relevant as data-intensive scientific practice expanded across universities and research centres.

Optimisation was one of the mathematical fields most clearly positioned between abstract theory and practical decision problems. The central task is to identify a preferable or optimal solution within a set of constraints, which may be simple in appearance yet mathematically demanding. Different forms of optimisation arise in logistics, telecommunications, finance, scheduling, engineering design and statistical learning. Their tractability depends on properties such as convexity, differentiability, discrete structure and the dimensions of the solution space. Research therefore ranges from foundational theorems about existence and duality to the design of efficient numerical methods. Czech mathematicians working across applied mathematics, informatics and technical universities participated in this broad intellectual landscape. Internationalisation helped circulate algorithmic techniques and opened questions arising from

scientific and industrial partners outside the national system. A useful historical distinction separates the development of a mathematical method from its subsequent implementation in a particular industrial context. Both may qualify as innovation, but they demand different forms of evidence and should not be conflated in evaluating scientific achievement.

The most visible infrastructural development affecting computational mathematics occurred in Ostrava. The IT4Innovations National Supercomputing Center emerged as a strategic initiative associated with VSB–Technical University of Ostrava and investments in research capacity. Its Anselm supercomputer entered service in 2013, providing a significant domestic resource for scientific computation. The purpose of such a facility was not simply to accelerate existing calculations, although performance was an important practical consideration. Access to large-scale computing also changed which mathematical models and numerical experiments researchers could realistically investigate. Some problems require fine spatial resolution, long simulation periods or repeated computations across extensive sets of parameters. Without adequate resources, the underlying mathematical formulation may be sound while the associated numerical investigation remains impracticable. The availability of national computing capacity therefore strengthened connections among numerical analysis, scientific software, engineering and data-intensive science. Unlike a theorem, a supercomputer is not itself a mathematical result, but it can reshape the conditions in which mathematical methods are developed and tested. The Ostrava initiative thus represents an institutional and technological form of innovation complementary to advances in abstract mathematics.

The expansion of IT4Innovations continued through the middle and later years of the decade. In 2015 the centre launched Salomon, a machine with a theoretical peak performance of approximately two petaflops that entered the TOP500 ranking at fortieth place. The arrival of a substantially more powerful system increased the range of calculations available to users from research institutions and other organisations. Computational work in materials research, engineering, physical modelling and data analysis can all depend on mathematical algorithms suited to parallel machines. Performance improvements, however, create new mathematical challenges involving convergence, stability, scalability and efficient allocation of computational tasks. A faster processor does not automatically make a poorly formulated model scientifically valid or an unstable numerical method reliable. Indeed, greater computational power can make it easier to generate impressive-looking results whose assumptions have not been adequately examined. For this reason, the expansion of national hardware capacity also increased the importance of skilled mathematicians able to evaluate methods critically. The decade illustrates how digital research infrastructure and rigorous mathematical reasoning became increasingly interdependent.

Further developments near the end of the period reflected the growing importance of specialised computational architectures. In 2019 IT4Innovations introduced the Barbora supercomputer as an extension of its existing resources. The centre also installed an NVIDIA DGX-2 system in the same year for work involving artificial intelligence calculations. These developments indicated that scientific computing was becoming more heterogeneous, with hardware designed for different computational workloads. Such diversity matters because mathematical algorithms differ in their needs for memory, communication, precision and parallel processing. Methods

suitable for a traditional simulation may not be optimal for the matrix operations characteristic of certain data-driven approaches. The growing importance of specialised hardware encouraged collaboration among mathematicians, computer scientists and researchers familiar with specific applications. Nevertheless, retrospective accounts must avoid attributing systems installed after 2020 to this earlier decade. For example, subsequent supercomputing developments belong to the following historical stage even when their planning commenced before it. The chronology is important because infrastructure decisions, construction and scientific use do not necessarily occur in the same year.

Scientific computing also depended on communications networks, storage and accessible services rather than processors alone. The Czech academic networking organisation CESNET supported a broader research e-infrastructure, while Masaryk University contributed computing and cloud services through CERIT Scientific Cloud. During the 2010s these resources became increasingly relevant to collaborations requiring the transfer and processing of substantial datasets. In 2019 the national infrastructure roadmap grouped CESNET, CERIT Scientific Cloud and IT4Innovations within the shared e-INFRA CZ framework. The integration reflected a recognition that contemporary computational research requires coordinated technical services and user support. Distributed scientific teams need reliable means of exchanging data, accessing remote resources and preserving research outputs. For mathematical disciplines, such services are most directly valuable when theoretical work is accompanied by computation or large-scale data processing. They do not imply that all branches of mathematics became dependent on digital infrastructure to the same extent. Rather, the development widened the technological possibilities available to the national research system while preserving the methodological diversity of mathematics.

The construction of computing capacity contributed to the increasing visibility of numerical modelling in engineering and physical sciences. Mathematical models translate selected features of a real process into variables, relationships and assumptions that can be investigated systematically. Numerical procedures then approximate solutions when an exact expression is unavailable or insufficient for practical purposes. In fields such as fluid mechanics, materials modelling and structural engineering, simulation may reveal patterns that would be difficult or expensive to observe experimentally. Yet a model remains a deliberate simplification, and its reliability depends upon the adequacy of the underlying assumptions. Validation against empirical knowledge is therefore necessary when computational output is used to make claims about physical systems. Czech collaborations during the decade increasingly brought together researchers familiar with mathematical theory, software implementation and domain-specific constraints. This arrangement was a productive response to scientific problems whose complexity exceeded the expertise of a single discipline. The historical significance of modelling consequently rests both on improvements in tools and on the organisation of knowledge across research communities. Mathematics became more widely used while remaining responsible for assessing the logical and numerical foundations of those uses.

The relationship between mathematics and computer science deepened beyond the immediate need for supercomputing resources. Algorithmic research examined how computational tasks could be solved efficiently and what limits could be established for particular classes of

problems. Discrete structures, optimisation, logic and numerical methods all contributed techniques relevant to this work, although they developed different criteria of success. Computational complexity addressed resources required to solve a task, while numerical analysis considered how accurately a solution could be approximated. Formal methods offered means of describing the logical properties of computational systems, and statistics supplied tools for reasoning about uncertain data. These connections produced a research environment in which mathematical abstraction was regularly placed in dialogue with practical computation. Charles University's theoretical computer science groups provided one important example, alongside mathematical work at technical universities and Academy institutes. International collaborations made it possible to compare algorithms and theoretical approaches against a wider range of problems and scientific traditions. The decade therefore strengthened mathematics as a foundation of the digital sciences without making it identical to those disciplines.

Research in data analysis expanded as scientific fields generated larger and more complex bodies of information. Biology, physics, environmental science and engineering all increasingly relied on measurements whose interpretation required statistical and computational expertise. The mathematical challenge was not merely to process greater quantities of data but to identify meaningful patterns while controlling uncertainty and error. Methods that perform well on a particular dataset can fail when underlying conditions change or important assumptions are violated. This difficulty underscores the importance of probability theory, statistical inference, optimisation and numerical linear algebra within data-driven science. During the later 2010s, machine-learning techniques drew increasing attention to mathematical questions about estimation, generalisation and algorithmic performance. The Czech research environment participated in this broader transformation through collaborations spanning universities, applied institutes and computing centres. Care is nevertheless required in historical analysis because the rapid growth of applications should not be confused with the invention of their mathematical foundations. Many contemporary techniques rely on concepts developed much earlier and adapted to new computational contexts. The history of the decade is therefore one of recombination as well as genuine methodological innovation.

The expansion of international funding programmes also influenced the organisation of Czech mathematical research. Horizon 2020, the European Union's research and innovation framework for 2014–2020, supported international projects, scientific mobility and collaborative knowledge production. Its structures built upon earlier European programmes but operated within a research environment increasingly oriented towards multinational consortia. Mathematicians could participate directly in theoretically focused projects or contribute methods to interdisciplinary research addressing broader scientific questions. Such participation demanded the ability to articulate research aims for evaluators and partners whose expertise was not always confined to the same subfield. Proposal preparation, reporting and international coordination consequently became more prominent aspects of academic work. These administrative requirements could improve cooperation while also placing additional demands on smaller research groups. The effects were therefore mixed, and successful participation should not be equated with the absence of institutional difficulties. European research funding

helped create opportunities, but the intellectual direction of a project still depended on the questions and expertise of participating scientists.

National grant mechanisms remained equally important despite the growing influence of European programmes. The Czech Science Foundation continued supporting fundamental investigations, including mathematical research not immediately linked to technological applications. Competitive grants enabled researchers to organise teams, support doctoral and postdoctoral participation, and pursue defined programmes over several years. For theoretical mathematics, funding frequently facilitates time for sustained research, travel and scientific exchange rather than large purchases of laboratory equipment. For computational mathematics, projects may also require software development, technical personnel or access to specialised resources. The differing cost structures illustrate why uniform measures of scientific performance are not equally informative across all mathematical fields. A mathematically important proof may be produced by a small team, whereas a numerical simulation can require an extensive technical environment. The 2010s encouraged closer attention to international visibility and external evaluation, but these mechanisms could not fully capture the diversity of mathematical achievement. Maintaining support for different research models was therefore a condition of institutional balance rather than a minor administrative issue. The scientific system's resilience depended on preserving pathways for both long-term theory and application-oriented investigation.

International mobility had particular importance for the reproduction of mathematical expertise across generations. Doctoral candidates and early-career scholars increasingly encountered research environments in which foreign visits and collaborative publications formed a normal part of academic development. Working abroad could provide access to specialised seminars, new mathematical techniques and researchers addressing similar problems from different perspectives. Visiting scholars also contributed to Czech institutions by bringing alternative research approaches and establishing durable professional relationships. Mobility was therefore not simply a movement of individuals away from the national system but a circulation of knowledge between interconnected centres. Nevertheless, prolonged international careers could complicate the retention of researchers within universities facing resource and salary constraints. Mathematical disciplines are especially sensitive to this tension because international reputation and intellectual community may matter more than access to local equipment. Institutions consequently needed to support credible research conditions and attractive career prospects alongside opportunities for travel. The 2010s brought more extensive international engagement, but the distribution of its benefits remained dependent on disciplinary and institutional circumstances.

International conferences and specialist workshops continued to function as essential mechanisms of mathematical communication. Unlike disciplines whose progress is frequently organised around shared instruments, mathematics often advances through sustained discussion of propositions, examples and possible proofs. A seminar can expose a weakness in an argument, suggest a new interpretation of a problem or reveal an unexpected connection between methods. Research visits may have equally lasting consequences when scholars develop collaborative work across several years. The activities associated with DIMATIA

illustrate how a Czech centre could foster such exchanges in discrete mathematics and theoretical computer science. Other institutions maintained seminars and conference networks appropriate to their respective areas of expertise. International publication remained important, but the finished article often represented the outcome of a much longer intellectual process. The decade's increased connectivity should therefore be understood in terms of research practice as well as bibliometric visibility. The ability to maintain recurring scholarly conversations constituted a significant form of institutional capacity. Such developments must be assessed through documented scientific practices rather than through general claims about inevitable progress.

Publication culture also underwent further international consolidation during the period. Researchers working in Czech mathematics published articles and monographs for specialist readerships whose members were distributed across Europe and the wider world. English-language dissemination enabled results to enter the international literature more readily, although publication language alone does not determine scientific quality. Different mathematical fields retained distinct publication rhythms, authorship conventions and expectations regarding the length of research papers. A complex theorem may require a lengthy proof, while another important contribution can be communicated through a concise article. Books such as the 2012 monograph on sparsity demonstrate the importance of extended synthesis in consolidating emerging research directions. Research assessment therefore needs to recognise different forms of scholarly contribution rather than rely exclusively on uniform numerical indicators. International journal participation may indicate the visibility of a national scientific community, but individual intellectual achievements must be examined through their content. Czech mathematics in the 2010s developed within this international publishing environment while retaining differences among its constituent schools.

Regional differentiation remained important despite the international prominence of institutions based in Prague. Masaryk University in Brno sustained mathematical investigations across several foundational and applied fields, while Brno University of Technology connected mathematical expertise with engineering and computation. Research groups in Olomouc, Ostrava, Plzeň and other centres contributed additional disciplinary strengths and educational capacity. These institutions should not be viewed simply as smaller replicas of the country's most prominent university departments. Their scholarly profiles reflected local histories, academic staffing, technical specialisations and opportunities for collaboration. The emergence of substantial computational infrastructure in Ostrava provided a particularly visible example of how regional investment could influence national scientific geography. Yet the significance of a regional mathematical community also rests in its graduate training, publications and sustained research relationships. Geographical diversity can create resilience when institutions develop complementary areas of expertise rather than competing to duplicate every programme. During the 2010s, internationalisation consequently combined with an increasingly differentiated national research landscape. Such developments must be assessed through documented scientific practices rather than through general claims about inevitable progress.

The growth of interdisciplinary projects raised important questions about the identity of mathematics as a scientific discipline. Mathematicians collaborating with biologists, physicists

or engineers must often translate disciplinary problems into statements that can be analysed with precision. This translation can reveal missing assumptions or suggest that several apparently unrelated problems share an underlying mathematical structure. Conversely, practical scientific questions may expose limitations of established mathematical tools and stimulate new theoretical investigations. The resulting exchange is most productive when both the mathematical and the domain-specific aspects of the research are treated seriously. An elegant formal model may be scientifically weak if it omits essential features of the phenomenon it is intended to describe. An empirically promising computational technique may likewise require further mathematical work to clarify its reliability or limits. Czech collaborations in the decade reflected the international movement towards such reciprocal relationships between theory and application. The relevant innovation frequently lay in the interaction among concepts, models and methods rather than in a single isolated outcome.

The increasing computational capacity of the decade introduced additional methodological responsibilities. Results from numerical experiments must be assessed in relation to discretisation choices, rounding errors, convergence behaviour and sensitivity to uncertain parameters. Larger calculations can improve approximation but may also magnify the difficulty of identifying mistakes embedded in software or model design. Parallel computation adds further concerns, including how algorithms communicate across processors and how efficiently they use available memory. Such problems belong simultaneously to numerical mathematics, computer science and the scientific discipline motivating the calculation. The development of high-performance computing therefore required researchers capable of integrating mathematical analysis with technical knowledge. Infrastructure investment was most valuable where institutions also maintained expertise in algorithms, scientific programming and methodological verification. The expansion of resources at IT4Innovations illustrates this broader transformation in the conditions of computational inquiry. By 2020, computational capability had become a strategic component of the national scientific system rather than a peripheral service for specialised users. Such developments must be assessed through documented scientific practices rather than through general claims about inevitable progress.

The final year of the decade introduced an unexpected test of international research routines. The coronavirus pandemic in 2020 disrupted travel, face-to-face conferences and customary forms of academic exchange across Europe. Mathematical collaboration could continue remotely in many cases because numerous research activities depend primarily on communication and access to scholarly materials. Nevertheless, informal seminars, visiting appointments and extended conversations remained difficult to reproduce completely through digital platforms. Teaching and doctoral supervision also required rapid adaptation as universities shifted substantial activities away from physical classrooms. Computational projects faced their own challenges where access to facilities, technical maintenance or coordination depended on changed working conditions. The disruption demonstrated both the flexibility of mathematical work and the importance of institutional relationships established before the crisis. Teams with mature collaborative networks were better positioned to preserve intellectual continuity despite limits on physical mobility. For historical purposes, 2020 should

therefore be understood both as the end of a decade of expansion and as the beginning of a new phase of scientific adaptation.

A balanced account of the 2010s must recognise the limitations accompanying increased research opportunities. International competition placed pressure on universities to recruit and retain highly qualified mathematicians while maintaining stable conditions for teaching and research. Competitive funding created openings for ambitious projects but also exposed smaller groups to uncertainty and administrative workload. The development of major computational facilities altered the possibilities for applied research without distributing every technological benefit equally across disciplines. Highly visible projects could attract greater attention than less conspicuous foundational work whose results required many years to mature. Research assessment risked conflating publication volume, grant income, intellectual originality and practical utility, although these are distinct dimensions of scientific activity. Regional inequalities and differences in institutional resources also shaped access to international networks and advanced infrastructure. These constraints do not negate the progress of Czech mathematics, but they make its development more historically credible and analytically precise. A mature interpretation should therefore distinguish institutional success from proof of universal improvement throughout the scientific system. The decade's achievements emerged within persistent tensions between concentration and diversity, competition and continuity, and application and theory.

By 2020, Czech mathematical sciences had acquired a research environment substantially more internationally integrated and technologically diversified than that of the early post-independence years. Theoretical schools continued contributing to analysis, geometry, logic, combinatorics and related mathematical fields through international exchange and sustained academic institutions. Discrete mathematics offered visible examples of conceptual innovation, particularly through research on graph structures and their algorithmic consequences. Probability, statistics and optimisation supplied foundations for increasingly data-intensive investigations and interdisciplinary decision problems. The development of IT4Innovations and coordinated e-infrastructure strengthened the practical conditions for numerical modelling and high-performance scientific computing. Universities and Academy institutes remained central because equipment and projects could not substitute for trained researchers, specialised schools or rigorous scholarship. The decade also revealed that scientific innovation could take the form of a theorem, classification, numerical method, research network or technological platform. These categories are connected, but each requires its own criteria for assessing scientific significance. The historical movement from institutional consolidation towards wider intellectual and computational participation prepared the conditions for developments after 2020.

Contemporary Stage: Mathematical Innovation, Computational Intelligence, and Emerging Research Directions, 2021–2025

The years from 2021 to 2025 marked a distinctive stage in the development of mathematical sciences in the Czech Republic. Their significance did not arise from a sudden replacement of established disciplines by fashionable technologies. Instead, mathematical research operated within a scientific environment transformed by digital infrastructure, international cooperation and renewed interest in the mathematical foundations of artificial intelligence. The coronavirus pandemic had already demonstrated the importance of mathematical reasoning for understanding uncertainty and evaluating possible interventions. At the same time, advances in computing made previously impractical simulation and optimisation problems increasingly accessible. Czech mathematical institutions entered this period with research traditions extending across analysis, geometry, algebra, probability and discrete mathematics. Their strength depended upon the continuity of these traditions as well as their responsiveness to emerging questions. The contemporary stage should therefore be studied as a convergence of fundamental inquiry, computational capability and institutional adaptation. A historical account must distinguish genuinely new results from the wider international trends within which Czech researchers participated.

Mathematics occupied a singular position within this changing research landscape because its intellectual products were often independent of specialised physical equipment. A theorem, proof or mathematical construction could alter an entire field without requiring the construction of a laboratory. Nevertheless, increasingly complex mathematical applications depended on reliable computational infrastructure and accessible research data. The interaction of conceptual autonomy and technological dependence became particularly visible after 2021. Researchers could formulate a model using purely mathematical assumptions and then test its computational behaviour on powerful machines. Conversely, computational experiments frequently suggested conjectures that demanded rigorous theoretical explanation. In this respect, intellectual continuity and methodological renewal reinforced one another. Such exchanges did not diminish the traditional importance of proof, which remained the decisive standard for mathematical validity. They instead enlarged the range of questions that researchers could explore and the contexts in which mathematical knowledge could be applied. Czech scientific development during these years therefore illustrates how mathematical innovation can be both abstract and operational.

The institutional foundations of Czech mathematics continued to rest on universities and the Czech Academy of Sciences (CAS). The Faculty of Mathematics and Physics at Charles University sustained substantial research in both pure and applied mathematics. The Institute of Mathematics of the CAS remained a prominent site for work in mathematical analysis, differential equations and related disciplines. Mathematical activity also extended through Masaryk University, the Czech Technical University in Prague and other university departments. These institutions differed in their research profiles, teaching responsibilities and proximity to technological applications. Their diversity allowed mathematical ideas to move between theoretical seminars, doctoral programmes and interdisciplinary projects. The relationship

between academy institutes and universities was particularly important for the preparation of young researchers. A mathematical research community requires both specialist continuity and a regular renewal of expertise through education. Institutional strength consequently depended less on a single spectacular development than on the maintenance of an interconnected scholarly environment.

The final stage of the period covered by this book also witnessed an increasingly close relationship between national and European research policy. European programmes supported international projects that demanded shared methodologies, comparable standards and coordinated research plans. For mathematics, these frameworks offered opportunities ranging from individual mobility to multidisciplinary partnerships involving engineers and computer scientists. International collaboration nevertheless remained scientifically meaningful only where the participating groups possessed their own substantive expertise. Czech researchers did not simply adopt external research agendas, but contributed distinctive mathematical approaches to shared questions. European integration also brought heightened expectations regarding publication quality, research transparency and evaluation. Institutional diversity also enabled individual disciplines to respond differently to the same policy changes. These expectations could encourage scholarly excellence while simultaneously increasing administrative burdens on small research groups. The experience of the early 2020s consequently combined opportunities for wider influence with pressures on available time and institutional resources. A balanced assessment must recognise both dimensions rather than interpreting internationalisation as an unqualified measure of progress.

Research in mathematical analysis remained central to the Czech mathematical landscape during 2021–2025. Analysts continued to examine functions, operators, functional spaces and the mathematical structures underlying differential equations. These investigations addressed fundamental questions concerning the existence, uniqueness and qualitative behaviour of solutions. Many such questions are formulated independently of immediate applications, yet their answers can shape the reliability of physical and engineering models. The analytical tradition also provides mathematical language for understanding stability, regularity and approximation. Contemporary computational methods increased the visibility of these connections without replacing the need for theoretical argument. Researchers working with complex models still had to understand when numerical procedures converged and when apparent solutions were misleading. This relationship between rigorous analysis and practical computation remained one of the defining features of contemporary mathematical work. It linked established Czech scientific strengths with the methodological demands of increasingly data-intensive research.

Differential equations offered another prominent bridge between inherited mathematical expertise and emerging scientific questions. Partial differential equations describe phenomena involving the interaction of spatial variation, time and physical constraints. Their mathematical study requires careful attention to boundary conditions, nonlinearity and the properties of admissible solutions. Applications may involve fluid motion, transport, diffusion, materials or biological systems, but the governing mathematics cannot be reduced to any one application. Progress in the theoretical investigation of such equations can therefore influence several

scientific disciplines at once. During the early 2020s, computational resources made more elaborate numerical treatments of coupled systems possible. Questions of regularity and approximation accordingly connected fundamental theory with numerical practice. These opportunities also exposed difficulties involving uncertainty, multiscale behaviour and the numerical approximation of unstable dynamics. The practical value of mathematical results depended on understanding the assumptions under which a model remained valid. Czech mathematical research occupied this important intellectual territory between abstract structural questions and scientifically motivated computation.

Geometric research likewise retained relevance in a period dominated by discussions of data and algorithms. Modern geometry studies structures and transformations whose importance extends beyond visual descriptions of shapes. Geometric methods inform the theory of manifolds, differential systems and mathematical models of physical interactions. They also interact with optimisation and mathematical descriptions of high-dimensional information. Topology supplies concepts of continuity and structural invariance that often clarify relationships obscured by direct calculation. The continuation of algebraic and geometric research demonstrated that digitalisation did not eliminate the significance of foundational mathematics. Rather, new computational problems frequently generated demand for deeper structural insights. Czech university departments provided educational and research environments in which such connections could be pursued. The persistence of geometry and topology thus illustrated continuity within a research system increasingly attentive to interdisciplinary applications.

Algebra and abstract mathematical structures offered a complementary perspective on the contemporary research environment. Their objects may be defined by operations, relations and axioms rather than by direct measurements of the physical world. Yet the resulting theories can illuminate formal languages, computation, logical systems and symmetries. Czech mathematical scholarship retained important traditions in structural approaches to mathematics. In this context, innovation could mean discovering a new property of an established class of structures or defining a productive relationship between previously separate theories. Such contributions often require sustained specialist engagement and cannot be judged by their immediate commercial usefulness. The generality of algebraic language could reveal similarities between otherwise distant problems. Their longer-term influence may emerge only when other researchers identify unexpected connections. International seminars, publications and doctoral training remained essential mechanisms through which these ideas circulated. The development of algebra therefore belonged to the same historical narrative as advances in computation, although its modes of progress were often less publicly visible.

Discrete mathematics became especially visible in the international position of Czech research during these years. The field investigates finite or countable structures and develops methods relevant to graphs, networks and algorithms. Its importance increased as large computational systems made questions of structural complexity more consequential. The school associated with Jaroslav Nešetřil contributed to the development of structural approaches to graph theory. Work with Patrice Ossona de Mendez on sparse structures offered a conceptual framework for understanding broad families of graphs. This programme connected mathematical classification

with algorithmic questions and logical descriptions of structures. Its influence illustrates how abstract mathematical ideas can reshape research agendas extending beyond their original formulation. The significance of the Czech contribution arose from sustained theoretical work rather than from the availability of any particular computing facility. The early 2020s provided a particularly visible moment in the international recognition of this longer intellectual development.

An important recognition of this research appeared in 2025, when Jaroslav Nešetřil and Patrice Ossona de Mendez received the Nerode Prize. The award acknowledged pioneering contributions to the theory of sparsity in graphs and its consequences for theoretical computer science. The underlying programme had developed across many years, making the award a recognition of cumulative achievement rather than a discovery confined to 2025. Sparse graph classes are significant because structural restrictions can reveal patterns that remain hidden in unrestricted graphs. Such restrictions can affect what kinds of algorithmic procedures or logical characterisations are possible. The relationship between graph structure and computational complexity gives the research a wider disciplinary significance. Its applications demonstrate that rigorous classification can have consequences beyond pure theory. Charles University thus provided a notable example of Czech participation in an internationally influential theoretical programme. The award also demonstrated that mathematical leadership may be recognised through general principles whose applications continue to evolve. Its historical importance should be interpreted through the development of the research field, not merely through the prestige of the prize.

Theoretical computer science formed an increasingly important point of contact between mathematics and digital technologies. Algorithmic investigations require precise distinctions between computability, computational complexity and practical performance. Mathematical work can establish limits on what algorithms can achieve as well as identify conditions under which efficient methods exist. Research involving graphs, combinatorial structures and formal systems is especially relevant to these questions. The growth of machine learning did not displace classical algorithmic theory, because reliable computational methods still depend on well-defined assumptions. Indeed, the opacity of some data-driven systems increased interest in mathematically interpretable procedures. Czech mathematical and computer-science groups contributed to the international discussion through their established theoretical specialisations. The contemporary challenge was to preserve conceptual rigour while confronting the demands of increasingly complex applications. This challenge connected traditional discrete mathematics with the emerging intellectual environment of artificial intelligence.

Probability theory acquired renewed public and scientific visibility during the pandemic and its aftermath. The interpretation of uncertain observations depends on probability distributions, conditional reasoning and assumptions about underlying processes. Stochastic models can describe random variation in epidemiology, finance, engineering and many other domains. Their value, however, depends on whether the chosen assumptions correspond adequately to the phenomena being studied. The widespread use of numerical simulations did not eliminate this problem of model validity. On the contrary, sophisticated computation could conceal

conceptual weaknesses beneath apparently precise quantitative results. The quality of a forecast depends as much on the interpretation of uncertainty as on the complexity of its implementation. Czech researchers working in probability and related areas belonged to a wider international community confronting these methodological questions. The early 2020s emphasised the distinction between mathematically well-defined uncertainty and uncertainty arising from incomplete knowledge. That distinction became increasingly important for responsible interpretation of quantitative scientific evidence.

Mathematical statistics became more closely connected with large-scale data analysis during the period under examination. Increasing quantities of digital information generated demand for methods capable of extracting reliable conclusions from heterogeneous datasets. Statistical inference nevertheless remained governed by questions of sampling, bias, uncertainty and the interpretation of evidence. Larger datasets did not automatically guarantee more accurate or more meaningful conclusions. Researchers therefore needed to combine computational tools with careful mathematical assessment of modelling assumptions. Connections between statistics, optimisation and machine learning became particularly prominent. These connections could support new forms of analysis while also creating difficulties concerning reproducibility and interpretability. Czech institutions active in statistics and information processing participated in an international methodological transformation of this kind. The contemporary significance of statistical mathematics thus rested on its capacity to distinguish credible inference from computationally impressive but inadequately justified results.

Optimisation remained a foundational mathematical instrument across many emerging research domains. It concerns the identification of admissible solutions that minimise or maximise specified objectives subject to constraints. Classical mathematical questions regarding convexity, existence and optimality continued to influence modern numerical algorithms. Machine learning made optimisation unusually visible because training computational models often requires repeated adjustment of large collections of parameters. Yet not every optimisation problem has the same mathematical properties or admits the same guarantees. The distinction between global solutions, local solutions and computational approximations remained essential. The distinction between correlation, inference and explanation remained a recurring scholarly concern. Czech research groups working in optimisation and applied analysis consequently possessed expertise relevant to both traditional engineering and contemporary data-driven science. The expansion of applications could stimulate mathematical innovation, but did not remove the value of rigorous theoretical investigation. Optimisation illustrated the productive interaction between established mathematical knowledge and newly prominent computational tasks.

The development of artificial intelligence (AI) gave new urgency to discussions of mathematical foundations. Although contemporary AI is often presented as a technological field, its models depend on linear algebra, probability, optimisation and approximation theory. Machine-learning procedures use mathematical representations to connect observed data with predictive or generative behaviour. Their performance depends on modelling choices and the statistical properties of training information. Mathematicians can therefore contribute by analysing generalisation, stability, uncertainty and the sensitivity of models to perturbations. Such work is

different from merely applying an available software package to a dataset. It asks why a method should work, when it can be trusted and under what conditions it may fail. The Czech research environment offered numerous opportunities for interaction between mathematically trained specialists and computer scientists. The period 2021–2025 accordingly marked a further convergence of mathematical reasoning and intelligent computation, rather than a simple transfer of mathematics into engineering.

Interest in interpretable and trustworthy computational methods also increased across European scientific policy. Complex machine-learning systems may achieve impressive performance while remaining difficult to explain in conventional causal or mathematical terms. Problems of interpretability generate questions about identifiable assumptions, uncertainty estimates and sensitivity to changes in data. Mathematical analysis can help distinguish empirical regularities from properties that hold under specified theoretical conditions. Formal reasoning also provides tools for examining the robustness and limitations of computational procedures. These questions are particularly consequential when algorithms inform decisions affecting people or public services. It also required collaboration among specialists whose disciplinary vocabularies were not always identical. No single mathematical technique can eliminate every source of uncertainty or social risk. Nevertheless, rigorous modelling can clarify what a system has actually demonstrated and what remains unverified. Czech mathematical and computational research participated in this broader effort to make technically powerful methods intellectually accountable.

High-performance computing became one of the clearest material expressions of the changing scientific environment. The IT4Innovations National Supercomputing Center in Ostrava provided researchers with access to resources far beyond ordinary departmental computing facilities. In 2021, the Karolina supercomputer entered operation as part of cooperation with the European High Performance Computing Joint Undertaking. The system combined substantial computing capacity with infrastructure intended for numerical simulations and data-intensive workloads. It was therefore relevant to applied mathematics, engineering, materials research and computational approaches to AI. Its significance should not be interpreted as an independent mathematical discovery. Rather, such infrastructure increased the practical scale on which mathematical models and algorithms could be investigated. The distinction matters because computational capacity and intellectual originality are related but not identical forms of scientific development. Karolina marked a concrete extension of Czech research capability within an increasingly interconnected European computational landscape.

Karolina also exemplified the shift from national infrastructure towards jointly governed European access. Its design and procurement were connected with a broader effort to coordinate advanced computational resources across Europe. This model helped avoid the expectation that every country should independently build every specialised computing capability. Research communities could instead contribute to shared facilities while retaining nationally based expertise. For mathematical researchers, such access created opportunities to explore larger problems and compare computational techniques on demanding workloads. The operation of powerful systems nevertheless requires specialist knowledge of software, parallel computation and numerical performance. The broader research value of the system therefore

depended on well-trained communities capable of using it. An algorithm that works on a small computer may behave differently when deployed across many processing units. The challenge therefore included not only access to hardware but also the mathematical design of scalable methods. High-performance computing became a catalyst for methodological exchange between mathematicians, software specialists and domain scientists.

Scientific computing during these years involved more than the use of ever faster processors. Numerical methods have to control approximation errors, preserve relevant mathematical properties and operate efficiently on available hardware. Parallel algorithms introduce additional questions about communication costs, stability and the distribution of computational work. Problems involving multiple physical scales may require adaptive strategies rather than uniformly increasing computational resolution. Similarly, modelling uncertain systems can demand repeated simulations whose computational costs grow rapidly. Such challenges create opportunities for collaboration between numerical analysts and specialists in physical or biological applications. The contribution of mathematical research lies in making computation both more efficient and more reliable. Hardware improvements may accelerate a flawed calculation without improving its scientific meaning. For this reason, the contemporary growth of computational infrastructure strengthened rather than weakened the case for rigorous numerical mathematics.

Mathematical modelling in the natural sciences remained a major point of interdisciplinary exchange. In physical sciences, mathematical equations connect measurable quantities with hypotheses about underlying mechanisms. In biological research, models may represent population dynamics, molecular processes or interactions across complex systems. Economic applications often require formal treatment of incentives, constraints and uncertainty. Engineering problems may involve coupled systems in which mechanical, thermal and electrical processes interact. Across these different contexts, the mathematician must decide which relationships are essential and which features may be simplified. Its services were most valuable when embedded in well-formulated scientific projects. A successful model can illuminate mechanisms, support predictions or reveal consequences that were not intuitively obvious. Its usefulness depends on both internal mathematical consistency and empirical relevance. Czech researchers working across university and academy boundaries continued to contribute to this form of interdisciplinary problem-solving.

The early 2020s also reinforced the importance of uncertainty quantification in computational science. A numerical result can appear precise even when it depends on imperfect measurements or uncertain assumptions. Researchers therefore need to distinguish numerical error from uncertainty in input parameters and uncertainty associated with model structure. These forms of uncertainty may interact in ways that complicate conventional interpretation. Probabilistic methods, sensitivity analysis and robust optimisation offer different tools for investigating such problems. No method is universally appropriate, and methodological choices require justification in relation to the research question. Mathematical reasoning helps make these choices explicit rather than leaving them hidden within software implementation. The growing importance of reproducibility further encouraged transparent reporting of algorithms and computational assumptions. The development of uncertainty-aware modelling

consequently represented an important methodological theme across Czech and international computational research.

The social organisation of mathematical research also evolved during the period under consideration. International conferences, joint publications and research visits had already become established features of Czech academic life before 2021. Pandemic restrictions temporarily disrupted many forms of physical mobility and accelerated the use of online communication. Remote seminars created new opportunities for participation but did not reproduce every benefit of direct scholarly contact. Mathematical research relies heavily on discussion, informal exchange and the careful presentation of developing arguments. International collaboration may begin through formal projects but often matures through sustained relationships between individuals. The translation between mathematical description and empirical evidence remained a central intellectual task. As travel and in-person meetings resumed, research groups combined inherited practices with additional digital forms of communication. This transformation affected academic culture without necessarily altering the mathematical validity of the results produced. The contemporary Czech mathematical community therefore experienced institutional and communicative change alongside scientific development.

Research funding constituted another important structural condition of mathematical development. Fundamental mathematical projects can require relatively modest equipment budgets but substantial continuity of researchers' time. Applied and computational projects may additionally depend on software, data services and powerful shared infrastructure. Competitive grants can encourage originality and international peer evaluation, yet short project cycles may conflict with long-term theoretical work. A difficult mathematical problem does not necessarily yield a result within the timeframe anticipated by a funding application. Research systems therefore need to accommodate uncertainty in discovery without abandoning standards of accountability. European funding opportunities expanded the possible scale of collaboration but also increased the complexity of project administration. The sustainability of mathematical schools depended on balancing competitive opportunities with reliable institutional support. The period 2021–2025 illustrates why mathematical innovation cannot be reduced to a simple count of funded technological projects.

International recognition offers a useful but incomplete measure of national mathematical achievement. Awards, prominent journal publications and major invitations can reveal the influence of particular researchers or scientific schools. Such indicators should nevertheless be interpreted in relation to the collective and cumulative nature of mathematical discovery. Many important results emerge from cooperation between scientists working in several countries. A Czech institutional affiliation does not by itself establish that an entire result originated within a single national community. Conversely, participation in international research can represent substantial intellectual leadership rather than secondary involvement. These changes also affected how researchers entered and sustained international collaborations. The recognition of work on sparse graph structures illustrates the importance of tracing a result's actual scholarly development. Similar caution is required when assessing computational infrastructures and interdisciplinary projects. Historical analysis should identify the nature of each Czech

contribution instead of treating every international achievement as an exclusively national accomplishment.

The close relationship between mathematical research and public expectations created further tensions during this period. Digital transformation encouraged institutions to emphasise measurable applications and practical social benefits. Yet fundamental mathematics often develops without a clear timetable for application. The history of science repeatedly shows that abstract ideas can acquire practical relevance through connections not anticipated by their authors. This possibility does not justify indiscriminate claims that every theoretical result will eventually become a commercial technology. Rather, it supports a diverse research environment in which fundamental and applied mathematics are valued for their different contributions. Public communication should make these distinctions intelligible to audiences unfamiliar with mathematical research. The Czech experience shows why a mature innovation system needs both conceptual freedom and responsible engagement with practical challenges. The meaning of mathematical innovation is therefore broader than technological transfer, while remaining open to its possibilities.

By 2025, Czech mathematical sciences displayed a combination of disciplinary continuity and expanding computational integration. Established research in analysis, geometry, algebra, probability and discrete structures remained integral to the national scientific landscape. Theoretical results continued to enter international mathematical discussions through scholarly publication and cooperation. Computational resources in Ostrava and elsewhere broadened the practical reach of numerical modelling and data analysis. The increasing prominence of AI created additional questions for optimisation, statistics, logic and the theoretical study of algorithms. These developments did not proceed uniformly across every institution or research field. Long-term intellectual developments could be obscured by metrics focused on short-term visibility. Differences in funding, personnel, infrastructure and international visibility remained important features of the national system. Nevertheless, the period demonstrated the capacity of Czech mathematical communities to participate in contemporary scientific transformations without abandoning their intellectual inheritance. The resulting landscape was more interconnected and computationally capable than that of the early post-independence years.

The years 2021–2025 thus complete the historical progression examined in the first part of this book. The initial phase after 1993 involved institutional adaptation and the preservation of inherited mathematical traditions. European integration subsequently expanded funding opportunities, scientific mobility and participation in international projects. The 2010s strengthened interdisciplinary links and contributed to a major improvement in computational resources. During the early 2020s, these earlier developments converged around questions involving complex systems, data, algorithms and mathematical foundations. The distinctive achievement of the period was not the replacement of traditional mathematical science by computational intelligence. It was the establishment of more varied pathways through which rigorous mathematical knowledge could influence international research and practical inquiry. The following part therefore turns from historical transformation to the principal mathematical fields and achievements that give this development its intellectual substance. Understanding

those achievements requires sustained attention to mathematical ideas themselves, the researchers who developed them and the institutions that made their work possible.

Part Two

Major Achievements and Research Directions of Czech Mathematical Sciences, 1993–2025

Mathematical Analysis and Functional Analysis: The Development of Modern Analytical Methods

Mathematical analysis occupies a distinctive position in the history of Czech science because it supplies both a language for abstract reasoning and a foundation for models of observable phenomena. Between 1993 and 2025, researchers working in the Czech Republic investigated functions, limiting processes, inequalities, operators and infinite-dimensional spaces within increasingly international mathematical communities. Their work was not confined to reproducing established European theories, even though those theories provided indispensable intellectual foundations. It also addressed problems concerning the sharpness of estimates, the geometry of functional spaces and the precise circumstances under which analytical constructions remain valid. A theorem establishing the best possible setting for an embedding may have no immediate industrial product, yet it changes what mathematicians can prove thereafter. Such achievements deserve to be recognised as mathematical innovation in their own right. The Czech experience is particularly revealing because inherited mathematical traditions survived the institutional changes of the early 1990s while research agendas became more outward-looking. Understanding this development requires attention to specific research questions rather than an undifferentiated account of national scientific progress. The period therefore illustrates how intellectual continuity can coexist with a genuine transformation of methods and collaborations.

The intellectual inheritance of Czech analysis preceded the establishment of the independent Czech Republic by many decades. University mathematics in Prague, Brno and other centres had cultivated rigorous traditions in differential equations, real analysis, approximation, mathematical physics and related fields. The Mathematical Institute of the Academy represented another institutional environment in which specialised research could be pursued over extended periods. These traditions supplied expertise that remained valuable when funding, evaluation and international mobility changed after 1989 and 1993. At the same time, disciplinary continuity must not be mistaken for intellectual immobility. Problems of global interest were evolving rapidly as functional analysis became more closely connected to partial differential equations, nonlinear phenomena and sophisticated function-space techniques. Czech researchers consequently participated in a transformation shared with analytical schools elsewhere in Europe. The most meaningful historical question is how established expertise was redirected towards new forms of mathematical investigation. An account organised around individual results and research networks offers a firmer basis for answering that question than

institutional chronology alone. It also allows domestic contributions to be distinguished from developments imported through international cooperation.

One enduring line of research concerned inequalities that compare the size of a function with the size of its derivatives. Such inequalities form a bridge between local information and global behaviour, which makes them fundamental to modern analysis. Sobolev-type inequalities are especially important because they specify when control of derivatives yields integrability, continuity or other regularity properties. Czech research groups pursued weighted, limiting and nonstandard variants of these questions, frequently in collaboration with colleagues abroad. Their task was often to identify assumptions that are not merely sufficient but mathematically optimal. The distinction matters because a convenient estimate may conceal a substantially sharper theorem. Function spaces adapted to these problems can record behaviour that classical power-integrability scales fail to detect. The resulting theory supports both abstract understanding and subsequent investigations of differential equations. In this setting, advances emerge from clarifying the exact boundary between what can and cannot be established.

A particularly well documented setting for these investigations was the Prague seminar on function spaces. Its historical archive records discussions of weighted inequalities, Hardy operators, Orlicz spaces and Sobolev embeddings from the years surrounding Czech independence onwards. The archive also reveals repeated visits by foreign analysts and sustained conversations between Academy and university researchers. This evidence is valuable because mathematical schools develop through regular criticism, shared problems and the circulation of preliminary ideas as well as through finished journal articles. Seminars provided opportunities to test proposed generalisations before they became settled results. They also brought young researchers into contact with problems whose technical details could rarely be learnt from standard textbooks alone. The continuity of the seminar across political and institutional change illustrates the durability of a research culture organised around difficult analytical questions. Its international participation demonstrates that Czech mathematics was embedded in a larger community rather than enclosed within national borders. A history of scientific achievement must therefore examine the social structures through which abstract arguments are refined. Those structures can be decisive even when they do not appear in the title of a published theorem.

The theory of rearrangement-invariant function spaces became an important framework for sharpening classical embedding results. Such spaces measure the magnitude distribution of functions without depending on the particular location of their values. This property makes them particularly useful when analytical estimates can be reduced to inequalities involving decreasing rearrangements. In a paper published in the Journal of Functional Analysis in 2000, David Edmunds, Ron Kerman and Luboš Pick studied optimal Sobolev embeddings formulated through rearrangement-invariant quasinorms. Their investigation asked how domain and target quasinorms could be selected without unnecessarily weakening the corresponding inequality. It thereby addressed the strength of the functional setting rather than merely the existence of some embedding. The work exemplifies a form of Czech participation in international research that is identifiable through a concrete co-authored result. Its significance lies in the precision with which a familiar analytical principle could be reformulated. The word optimal carries a technical

burden here: it concerns mathematically demonstrable limits on improving the chosen functional scales.

Luboš Pick's research provides a useful example of the movement between institutions and the deepening of a specialised programme. Work associated with the Prague analytical community explored Hardy-type operators, Lorentz and Orlicz constructions, and the behaviour of embeddings at critical thresholds. These subjects are related, but they do not collapse into a single general technique. An operator inequality asks whether a transformation acts boundedly between specified spaces, whereas an embedding theorem asks whether functions satisfying one kind of control automatically possess another. Critical situations are especially delicate because standard exponents may no longer give an adequate description of endpoint behaviour. Logarithmic refinements and more flexible function scales can then become necessary. The intellectual contribution of such investigations is to make analytical distinctions visible that coarser categories would overlook. These methods also make it possible to compare apparently separate problems through common functional principles. The Prague school thus contributed not just isolated inequalities but a systematic vocabulary for analysing their exact range of validity. That vocabulary travelled through publications, doctoral research and international seminars.

Research on Hardy-type inequalities represents another thread connecting Czech functional analysis to wider mathematical developments. In their simplest forms, these inequalities control averages or integral transforms in terms of suitable norms of the original function. More sophisticated versions incorporate weights, monotonicity, nonclassical integrability or higher-order differentiation. The historical Prague seminar lists work by Alois Kufner, Bohumír Opic, Miroslav Krbec, Petr Gurka and other researchers on such questions. These records establish the continuing presence of an analytical research programme without requiring every seminar presentation to be treated as a major breakthrough. They also show that collaboration linked the Mathematical Institute, universities and international partners. The attraction of Hardy-type methods is that they often convert difficult multidimensional estimates into more manageable integral inequalities. Such reductions are valuable only when the resulting equivalences and assumptions are established rigorously. For that reason, advances in integral inequalities can influence entire families of otherwise unrelated analytical problems.

Orlicz spaces and related generalised integrability classes offered a particularly flexible language for these investigations. Classical Lebesgue spaces describe integrability through power functions, but some problems exhibit growth that does not fit a single fixed exponent. Orlicz-type structures permit more general growth functions, allowing analytical estimates to reflect that additional complexity. Weighted and logarithmic variants further enlarge the range of behaviours that can be examined. Czech researchers used such settings to study embedding, approximation and operator boundedness questions. The challenge is not merely to introduce a larger catalogue of spaces, since arbitrary generalisation can make a theory less informative. A successful result identifies structural conditions that are both verifiable and relevant to a precise theorem. It then explains why those conditions allow an argument unavailable in a narrower framework. The value of this work lies in making difficult boundary cases mathematically

tractable. Its broader lesson is that innovation may consist in choosing the right language for an existing problem.

Another family of results concerns the relationship between analytical embeddings and geometrical properties of domains. The behaviour of functions is affected not only by differential expressions but also by the structure of the region on which those functions are defined. Irregular boundaries and geometrical constraints can require substantially different estimates from those used on smooth domains. Research connecting Sobolev spaces with Morrey, Campanato and Hölder-type settings explores how integrability and differentiability control local oscillation or continuity. Andrea Cianchi and Luboš Pick published a 2003 study specifying conditions for embeddings into these classes and investigating optimal source spaces. That collaboration illustrates how Prague-based expertise contributed to an international programme linking abstract function spaces with geometrical regularity. Such work is relevant to differential equations, although a direct application cannot be assumed for every theorem. Mathematical significance depends first on the exact statement and proof of the embedding result. The passage from norm estimates to qualitative information about functions is one of the most productive features of modern analysis.

The notion of compactness occupies a central place in functional analysis because it can turn an infinite collection of possible behaviours into a manageable convergent subsequence. In finite dimensions, bounded sets display compactness properties that often fail in infinite-dimensional settings. Researchers must therefore identify additional hypotheses that recover sufficiently strong forms of convergence. Compact embeddings, compact operators and measures of noncompactness provide different tools for approaching this difficulty. Czech analytical work on embeddings and operators belongs to this wider conceptual landscape. When an embedding is compact, sequences controlled in one function space may converge more strongly in another. Such a conclusion can be decisive when proving the existence of solutions to nonlinear equations or variational problems. Yet compactness is never merely an informal synonym for boundedness, and the distinction must be maintained carefully. The search for precise compactness criteria therefore represents a substantive part of the theoretical achievement of analysis. It also illustrates the dependence of applications on seemingly abstract structural theorems.

The geometry of Banach spaces constituted a second major domain of Czech functional-analytic research. A Banach space is a complete normed vector space, providing a natural setting for limits, operators and infinite-dimensional approximation. Its geometry includes questions concerning dual spaces, weak topologies, convexity and the existence of useful systems of coordinates. These are not only formal properties, because they determine what kinds of convergence and representation can be expected. At Charles University, research associated with Ondřej Kalenda addressed compact spaces and Banach-space structures, including Valdivia compacta and spaces of continuous functions. His publication record and habilitation work document a sustained programme linking topological constructions with functional analysis. The study of such objects demonstrates that topological classification can carry substantial consequences for the organisation of infinite-dimensional spaces. It also shows how internationally significant mathematical questions may arise from highly specialised theoretical

traditions. The resulting research cannot be adequately described as a subsidiary application of calculus.

Spaces of continuous functions provide a particularly fruitful interface between topology and functional analysis. Such spaces encode the structure of a compact topological domain within a Banach space of functions defined upon it. Properties of the underlying compact space may influence the existence of projections, bases and other forms of linear organisation. Conversely, the geometry of the Banach space can reveal information about the topological object. Valdivia compact spaces belong to an important family studied through this interaction. Czech work on these constructions contributed to a conversation about what infinite-dimensional spaces can look like when familiar finite-dimensional intuitions no longer apply. Questions concerning weak compactness or descriptive complexity may determine whether a representation theorem is possible. Their technical nature should not obscure the wider goal of understanding the architecture of infinite-dimensional mathematical objects. This field supplies an example of theoretical innovation whose value is measured through structural understanding rather than immediate computation. It also reveals an enduring dialogue between mathematical disciplines.

Descriptive aspects of functional analysis formed a closely related research direction. Some mathematical sets can be classified through the complexity of their construction from open and closed sets. Such classifications become relevant when studying measurability, continuity and the structure of spaces of functions. Petr Holický's work on generalised analytic spaces, completeness and fragmentability offers a documented Czech example of this interaction. His article appeared in the Czechoslovak Mathematical Journal in 2001 and can be located through the Charles University preprint archive. Fragmentability investigates whether complicated spaces can be decomposed into pieces with controlled local behaviour. This concept can inform questions in topology and the geometry of Banach spaces without making those subjects identical. The research demonstrates how descriptive methods can help identify distinctions that purely metric arguments would overlook. Its place within modern Czech mathematics is significant precisely because it broadens the usual understanding of what analytical research encompasses.

Operator theory studies transformations between functional spaces and is therefore inseparable from the language developed by Banach-space analysis. Fundamental questions concern boundedness, compactness, spectral properties and the response of an operator to changes in its domain. Integral operators provide concrete examples because they transform functions through integration against a kernel. Differential operators provide another class, arising when mathematical laws involve derivatives and boundary conditions. Czech researchers approached these problems through function-space estimates, differential equations and mathematical physics. An operator may behave well on one integrability scale and fail dramatically on another. Determining the exact threshold often requires estimates more refined than a simple application of classical inequalities. This is why studies of operators and studies of functional spaces frequently develop together. The connection turns function-space theory into a means of investigating transformations rather than a catalogue of definitions. It also links abstract analysis to evolution equations and spectral problems.

Spectral ideas make the relationship between abstract operator theory and physical models especially clear. In elementary linear algebra, eigenvalues describe transformations on finite-dimensional spaces, but infinite-dimensional operators introduce new complications. Their spectra may include continuous components or depend sensitively upon boundary conditions and functional domains. Mathematical physics therefore requires careful specification of the operator before any physical interpretation is justified. Czech research in spectral theory developed within networks spanning universities and Academy institutions. The work associated with Pavel Exner and collaborators illustrates one prominent strand, although its detailed achievements belong more fully to the separate section on mathematical physics. Here its significance is methodological: functional-analytic language supplies the framework in which spectral questions become rigorous. This relationship exemplifies the ability of abstract mathematics to shape the investigation of physical systems. A useful historical analysis must nevertheless distinguish general international methods from results established by particular Czech research groups.

Infinite-dimensional analysis also provides essential tools for understanding partial differential equations. A solution may not possess classical derivatives everywhere, making the traditional pointwise formulation inadequate. Weak formulations instead permit equations to be interpreted through integrals and test functions within suitable spaces. The success of this approach depends upon properties such as completeness, continuity of linear functionals and appropriate compactness. Research on Sobolev spaces, operator bounds and embeddings contributes directly to this theoretical framework. Czech mathematicians participated in these areas through several interconnected schools rather than a single centralised programme. Distinct methods were required for elliptic, parabolic, evolutionary and nonlinear settings. Consequently, the analytical foundations of equation theory cannot be reduced to one universal technique. The importance of modern functional analysis is that it makes this diversity of solution concepts precise. It provides a disciplined way of deciding what it means for a mathematical model to possess a solution.

Nonlinear analytical problems pose challenges that do not arise in the same form for linear operators. Superposition may fail, solutions may not be unique and their regularity can depend strongly on growth assumptions. Variational methods offer one route by seeking critical points or minimisers of functionals defined on appropriate spaces. The existence of those minimisers often relies on weak convergence, lower semicontinuity and coercivity. Each of these properties has an explicitly functional-analytic meaning. Czech investigations of nonlinear differential equations and variational problems therefore shared conceptual foundations with research on Banach spaces and embeddings. The exchange was not merely one-directional, since demanding problems in nonlinear analysis can motivate new function-space constructions. A productive analytical school develops by moving repeatedly between abstract questions and the specific obstacles encountered in applications. This iterative relationship helps explain why the boundaries between analysis and differential equations became increasingly permeable after 1993.

A notable feature of Czech mathematics during the period was the coexistence of university-based and Academy-based analytical research. Charles University's Faculty of Mathematics and

Physics sustained seminar activity, postgraduate supervision and publication in international journals. The Mathematical Institute of the Czech Academy of Sciences contributed another environment for fundamental work on inequalities, operators and related subjects. Researchers also held appointments or maintained collaborations across institutions, demonstrating that the intellectual map was not identical to the administrative one. This arrangement helped preserve specialist traditions while introducing younger mathematicians to international networks. It did not eliminate pressures arising from competition for grants and permanent positions. Nevertheless, it enabled different research groups to share difficult problems without creating a single institution responsible for every branch of analysis. The most convincing evidence of this interaction is found in co-authorship, seminars and research archives rather than in promotional descriptions. Institutional diversity should therefore be understood as a source of complementary expertise, not proof of uniform national performance. Actual achievements require examination at the level of publications and established results.

International collaboration was particularly important in areas where a theorem required several complementary forms of expertise. The research of Edmunds, Kerman and Pick on optimal embeddings illustrates a collaboration between mathematicians working in different national environments. Similarly, work involving Cianchi and Pick connected the Prague function-space tradition with major developments in European analysis. Such collaboration should not be interpreted as a loss of national scientific identity. Mathematical arguments are commonly formulated in a transnational setting, while research institutions still provide the continuity needed to sustain expertise. Shared authorship also demands care in assigning credit, because a collective theorem cannot be divided automatically into separate national achievements. The accurate historical claim is that researchers associated with Czech institutions made identifiable contributions within international teams. This distinction makes the account more credible than an indiscriminate celebration of every collaborative publication. It also reflects the actual social organisation of mathematical discovery in the late twentieth and early twenty-first centuries.

The development of research journals and preprint channels altered the circulation of Czech analytical work after independence. A mathematical result becomes durable through a proof that other specialists can scrutinise and reproduce. International journals make that scrutiny visible, although the significance of a paper still depends upon its content rather than the journal title alone. The Charles University preprint archive preserves examples of work that subsequently appeared in established journals of analysis and topology. It permits the relationship between preliminary circulation and formal publication to be traced with unusual clarity. Such resources also reveal the range of problems pursued within a department at a given time. Digitisation made older studies easier to locate and encouraged scholars to connect related arguments across different publication venues. The availability of these records is particularly useful when reconstructing scientific achievements over a period as long as 1993–2025. A sound history must still distinguish dated records of discussion from independently validated scholarly results. Archival evidence and peer-reviewed articles play different but complementary roles.

Training new analysts required more than the transmission of standard theorems in undergraduate lectures. Doctoral students had to learn how to recognise a genuinely open

problem and distinguish it from a question already settled in another terminology. Seminar presentations trained them to expose arguments to demanding questioning, often before a proof was complete. Research visits introduced unfamiliar techniques and helped identify potential collaborators. The mathematical vocabulary of function spaces is especially intricate, so sustained mentoring is important for avoiding errors in assumptions and endpoint cases. Czech doctoral programmes benefited from opportunities for contact with foreign institutions during the post-1993 opening. The research records of Prague also demonstrate how students and younger scholars could participate in specialised discussions alongside established mathematicians. These mechanisms contribute to the continuity of analytical schools even when their members subsequently work in other countries. Scientific mobility should therefore be evaluated not only through departures and arrivals but through the knowledge circulated between generations.

The period after 2004 brought further opportunities for Czech analysts to participate in European research projects and specialist networks. Membership of the European Union expanded the framework within which institutions could cooperate and seek competitive support. Yet analytical research often requires modest equipment compared with experimental sciences, so its needs cannot be measured solely through laboratory construction. Access to literature, research time, doctoral support and travel funding can be more decisive for theoretical progress. Some branches also benefit from powerful computation, but proofs concerning general functional spaces cannot simply be replaced by numerical experiments. Funding systems therefore need to accommodate results whose relevance may be apparent only after years of subsequent mathematical development. Czech analytical groups operated within this broader tension between short project cycles and long intellectual horizons. The resilience of a school depends in part on whether it can preserve specialised knowledge between funding rounds.

The distinction between pure and applied mathematics becomes less rigid when analysis is examined closely. A sharp embedding theorem may arise from an abstract problem about rearrangements, yet later become useful in controlling solutions of differential equations. Conversely, a physical model may motivate a new estimate whose formulation is valuable far beyond the originating application. This movement does not mean that every mathematical theorem should be justified by a promised practical outcome. Its primary achievement remains the discovery and verification of a new mathematical relation. The possibility of application reflects the generality and precision of that relation rather than a guarantee of immediate commercial benefit. Czech analytical research offers numerous examples of concepts that serve both theoretical and methodological purposes. Operator theory and infinite-dimensional spaces supply a common language across these contexts. A balanced account therefore values autonomous theoretical inquiry while tracing genuine connections to other fields. Neither side of the relationship should be exaggerated.

The rise of computational mathematics during the 2010s and 2020s changed the environment in which analytical ideas were used. Numerical approximation can explore complex models, test conjectures and reveal behaviour inaccessible to elementary calculations. However, mathematical analysis remains necessary to establish convergence, stability and the consistency of numerical schemes. A computation on a finite grid cannot by itself prove that an

infinite-dimensional problem is well posed. Function-space regularity and operator estimates can specify the assumptions under which approximation is legitimate. Czech investments in computing infrastructure therefore complemented, rather than superseded, the older analytical traditions. Their significance should be understood as part of a broader methodological ecosystem. Abstract proofs and computational experiments answer different questions, even when they concern the same equations. The boundary between them is most productive when each approach recognises the other's limitations.

The growing influence of data-driven methods has similarly renewed interest in fundamental analytical questions. Machine learning models frequently involve optimisation over high-dimensional parameter spaces and approximation by families of functions. Their practical performance does not automatically resolve questions about convergence, stability or generalisation. Functional-analytic concepts can help formulate these issues, although the relevance of any specific classical theorem must be demonstrated rather than assumed. Czech mathematics entered this field through the intersection of analysis, probability, optimisation and computing. The shift offered opportunities to reinterpret established theories while also posing problems that older frameworks do not yet solve. It would be misleading to attribute modern artificial intelligence wholesale to functional analysis or to any one national school. The more defensible conclusion is that analytical reasoning forms part of the conceptual resources needed to assess increasingly complex computational methods. This continuing relevance explains why abstract research remains significant within an apparently technology-led period. Its questions evolve even as its commitment to proof remains constant.

Assessing the international contribution of Czech analytical schools requires a careful standard of evidence. A university listing a research topic establishes institutional activity but does not, by itself, demonstrate a breakthrough. A seminar record confirms that an argument was discussed, yet discussion is not equivalent to an accepted theorem. A peer-reviewed article with an identifiable mathematical result provides stronger evidence of an intellectual contribution. Subsequent developments may reveal whether the result opened a sustained programme or remained a specialised refinement. The examples of optimal Sobolev embeddings and research on compact and Banach-space structures satisfy the need for more specific documentation. Even here, their precise mathematical scope matters more than expansive claims about national leadership. A rigorous monograph should describe authorship, the problem addressed and the nature of the result without inflating its significance. Such methodological restraint is especially important for a field where innovation is sometimes invisible to readers outside mathematics.

The geography of Czech analytical research also requires a nuanced interpretation. Prague was a major concentration of specialist activity because of its universities and Academy institutions. Brno and other academic centres maintained mathematical traditions and contributed to the broader circulation of analytical knowledge. Nevertheless, regional presence in a field is not sufficient evidence for treating every centre as an equivalent international school. The importance of an institution depends on its actual research record, personnel and scholarly networks during the period concerned. Mathematical communities are often small enough that a few influential individuals can shape a research profile. Their movement between positions can also alter the geography of expertise without changing the underlying scientific problems. For

these reasons, national history should follow researchers and collaborations as well as the addresses of departments. This approach better captures the flexible organisation of fundamental mathematical science. It avoids confusing administrative size with analytical influence.

The transformation from 1993 to 2025 was therefore cumulative rather than a succession of disconnected intellectual fashions. Hardy and Sobolev inequalities continued to matter as researchers pursued sharper and more general formulations. Function-space theory developed increasingly refined tools for critical embeddings and nonclassical growth. Banach-space geometry and topology sustained a parallel programme concerned with the structure of infinite-dimensional objects. Operator methods linked these theories to spectral questions and differential equations. International co-authorship and specialised seminars allowed arguments to circulate beyond national boundaries. Universities and Academy institutions maintained the settings in which such long-term research could continue. New computational and data-oriented problems created further reasons to revisit analytical foundations. The enduring characteristic of Czech analysis was thus not attachment to an unchanging catalogue of results but the capacity to extend rigorous traditions into new intellectual settings.

Seen in the perspective of the whole book, mathematical analysis provides a model for understanding innovation without reducing it to technological invention. Its progress often takes the form of an improved estimate, a sharper classification or a more powerful theorem about operators and spaces. These results can influence neighbouring disciplines, but their original mathematical worth does not depend upon a predictable practical application. Czech researchers participated in this process through concrete international publications and the sustained activity of recognisable analytical communities. The history also reveals why research infrastructures for mathematics include seminars, libraries, doctoral supervision and durable collaborative networks. Functional spaces and infinite-dimensional systems require patience because their conceptual difficulties cannot be overcome merely by increasing the scale of computation. Between 1993 and 2025, Czech analytical research preserved this demanding tradition while adapting to the international and interdisciplinary conditions of contemporary science. Its achievements demonstrate how a comparatively small research environment can contribute to the collective advancement of fundamental mathematics. The succeeding chapters examine related advances in differential equations, numerical analysis and other disciplines where these analytical foundations acquire further significance. Together they reveal the interconnected character of mathematical progress.

Differential Equations and Nonlinear Analysis: Mathematical Foundations of Dynamic Systems

The study of differential equations has occupied a central place in Czech mathematical research since the independent republic was established in 1993. Its significance follows from a simple

intellectual difficulty: laws expressed through local rates of change must be shown to describe an evolving system coherently over time. An equation may represent the conservation of mass, the transmission of heat, or the response of an elastic body, yet its mathematical meaning does not follow automatically from its physical interpretation. Researchers must determine whether solutions exist, whether they are unique, and what happens when the initial state or governing parameters vary. These questions became particularly consequential as Czech mathematical schools strengthened their links with fluid mechanics, materials science, and computational modelling. The resulting research retained the exacting standards of pure analysis while responding to problems whose coefficients, boundaries, or constitutive laws were far from ideal. A sustained contribution therefore involved not simply manipulating equations but identifying the assumptions under which an entire mathematical model could be justified. Between 1993 and 2025, this combination of theoretical discipline and physical interpretation became a characteristic strength of the field. It also explains why ordinary and partial differential equations deserve treatment as an intellectual achievement rather than merely as numerical tools.

The mathematical landscape of the early 1990s did not begin from an empty institutional base. The Czech lands inherited long traditions of analysis, mechanics, and mathematical physics from Czechoslovak universities and research institutes. After independence, continuity in research personnel made it possible to pursue difficult analytical programmes while governance and funding arrangements changed around them. Charles University in Prague and the Mathematical Institute of the Czech Academy of Sciences remained important settings for theoretical research. Other universities, including institutions in Plzeň and Brno, offered environments in which nonlinear equations could be studied alongside engineering and computational subjects. This distributed structure made collaboration between functional analysts, specialists in continuum mechanics, and numerical mathematicians particularly productive. The evolution of the discipline nevertheless cannot be reduced to the activities of any single university or institute. Scientific expertise was sustained through doctoral supervision, professional seminars, joint publications, and relationships with colleagues abroad. These less visible mechanisms were indispensable for maintaining the continuity of mathematical arguments across generations. In this respect, institutional transformation and the internal development of mathematical theory were closely connected.

Ordinary differential equations describe evolution through a finite collection of unknown functions of a single independent variable. Their apparent simplicity is deceptive because nonlinear feedback can generate complicated behaviour even when the equations are written compactly. Questions of local existence may be approached through fixed-point arguments, but global existence demands additional bounds that prevent uncontrolled growth. The distinction is important in models of population dynamics, chemical kinetics, mechanical oscillations, and feedback regulation. An initial-value problem may have a solution only for a limited interval unless its structure provides suitable estimates. Even where existence is assured, dependence on parameters can change the number or stability of equilibria. Czech work in nonlinear analysis contributed to the wider mathematical culture in which these questions were investigated systematically. The rigorous treatment of ordinary equations also supplied techniques subsequently adapted to infinite-dimensional evolution problems. Such connections gave the

theory a significance extending well beyond the elementary differential equations encountered in scientific education.

Stability theory asks whether a mathematical state persists when exposed to small disturbances. For an equilibrium of an ordinary differential equation, a linear approximation may provide useful evidence, but the nonlinear terms can determine the outcome in critical cases. Lyapunov functions offer a way of assessing change without solving every trajectory explicitly. Other approaches investigate invariant sets, periodic solutions, and the long-term organisation of trajectories. An important conceptual distinction separates stability of an individual solution from structural stability of a model under changes of its defining equations. The latter is especially relevant when physical parameters are uncertain or experimental observations are incomplete. Mathematical researchers must consequently distinguish conclusions established by proof from patterns suggested by simulation. This distinction gained greater practical importance as computers made intricate phase portraits increasingly accessible. An attractive numerical trajectory is not, by itself, a theorem about all permissible initial states. The Czech analytical tradition offered an environment in which such questions of mathematical justification could remain central.

Partial differential equations introduce a further layer of difficulty because unknown quantities vary across space and time. An elliptic equation may describe a stationary state, whereas parabolic and hyperbolic equations typically encode different kinds of evolution and propagation. These labels indicate broad mathematical structures rather than guaranteed physical behaviours in every nonlinear case. Boundary geometry, coefficient regularity, and admissible data can decisively affect the resulting problem. For example, a heat equation on a smooth bounded domain differs fundamentally from a related model posed in an irregular or unbounded region. In nonlinear systems, even the identification of an appropriate solution concept may be a substantial achievement. Researchers therefore construct mathematical spaces that make both the differential operator and its boundary conditions meaningful. Functional analysis supplies compactness, duality, interpolation, and monotonicity methods needed for this purpose. The resulting proofs connect abstract reasoning with precise statements about processes in the physical world.

A weak solution is commonly introduced when the derivatives required by a classical differential equation cannot be expected to exist pointwise. Instead, an integral identity is tested against suitable functions, allowing the equation to be interpreted in a less restrictive setting. The method does not imply that the underlying physical process is poorly defined. It recognises that the natural energy of a system may control only an average or distributional form of its derivatives. Sobolev spaces provide a particularly important setting because they measure integrability and weak differentiability simultaneously. The existence of a weak solution is frequently established through approximations and uniform estimates rather than through a direct formula. Compactness then allows a subsequence of approximate states to converge to a mathematically admissible limit. Nonlinearity complicates this passage because a weakly convergent sequence need not preserve every nonlinear expression. Identifying conditions that restore sufficient convergence became one of the central tasks of Czech research in evolutionary

equations. Such ideas link the theory of partial differential equations to the more general development of modern analysis.

Research associated with Eduard Feireisl provides a concrete example of the Czech contribution to nonlinear partial differential equations. His work has addressed mathematical fluid dynamics, dynamical systems, and evolution equations within the Mathematical Institute of the Czech Academy of Sciences. A central challenge in compressible flow is to analyse density, velocity, and temperature when each quantity influences the others. The governing equations express conservation principles but do not make global solvability self-evident. A mathematical argument must account for the interaction between transport, pressure, viscous dissipation, and thermodynamic effects. This is particularly difficult when the solutions are not smooth enough for conventional manipulations. The analytical approach develops appropriate energy and entropy estimates and then passes to limits in carefully constructed approximations. The aim is not to certify every feature of a fluid model under every conceivable condition. It is to establish well-defined results for explicitly stated classes of data, constitutive laws, and domains.

An especially instructive problem is the Navier–Stokes–Fourier system, which couples fluid motion with heat transfer. The addition of a temperature equation changes the mathematical analysis because viscosity or heat conductivity may depend on thermal state. In a 2006 paper, Eduard Feireisl and Josef Málek established a long-time weak existence result for a three-dimensional incompressible flow model with temperature-dependent transport coefficients. Their argument employed total energy in a way designed to accommodate the coupling between momentum and thermal effects. The result concerned a specified periodic setting and should not be confused with a universal proof for all forms of the Navier–Stokes equations. Its value lay in extending solvability analysis to a physically richer class of constitutive assumptions. The study shows how changing a material law can transform an otherwise familiar mathematical question. It also demonstrates the close relationship between the Academy of Sciences and Charles University in advanced mathematical research. Such collaboration has been a persistent feature of the Czech analytical environment. The distinction between a formal calculation and a justified result remained fundamental to this research tradition.

The distinction between incompressible and compressible models reveals how mathematical assumptions alter the nature of fluid equations. An incompressible description constrains volume change, while a compressible description allows density to vary dynamically. In compressible systems, pressure and acoustic propagation introduce phenomena that complicate compactness arguments. A family of compressible solutions may approach an incompressible regime only when suitable limits and estimates have been justified. Research by Feireisl and collaborators has examined such limits and the mathematical behaviour of thermo-fluid systems. One study of the full Navier–Stokes–Fourier system analysed strong local convergence of velocity in a low-Mach-number limit on expanding domains. The derivation required careful treatment of acoustic effects rather than a casual substitution of zero compressibility. This type of work clarifies the relationship between mathematical idealisations used at different physical scales. It also provides a model for how rigorous analysis can explain when one set of equations approximates another.

Regularity theory investigates whether a solution initially constructed in a weak sense possesses additional smoothness. A solution may satisfy an integral formulation yet fail to have derivatives sufficient for classical calculations. Determining when that solution becomes smoother can require delicate bounds on its gradients or other critical quantities. The issue is particularly demanding for three-dimensional fluid equations because nonlinear transport interacts with dissipation. Research associated with Czech analysts has studied regularity criteria for coupled momentum and heat equations. A 2014 study involving Eduard Feireisl, Antonín Novotný, and Yongzong Sun formulated a criterion tied to control of the velocity gradient. Such conditional results are mathematically valuable without resolving every open question concerning fluid regularity. They identify which quantities govern the possible continuation of a regular solution. This information also guides the interpretation of computations, since numerical stability cannot prove analytic regularity. The distinction between conditional smoothness and unconditional global regularity must remain explicit in any historical assessment.

Uniqueness poses a different question from existence and regularity. Two functions can both satisfy a weak formulation unless the assumptions of the problem provide a mechanism to identify them. For nonlinear evolutionary systems, uniqueness is frequently more difficult than showing that at least one weak solution exists. Weak–strong uniqueness arguments offer a qualified comparison between a weak solution and a sufficiently regular solution emanating from compatible data. Under the relevant conditions, the weaker object agrees with the stronger one for as long as the latter exists. The result therefore expresses stability of the solution concept rather than guaranteeing that strong solutions exist for all time. Relative-energy and relative-entropy methods are among the tools used in modern fluid analysis to construct such comparisons. Their significance reaches beyond abstract uniqueness because they help connect different mathematical descriptions of the same process. Czech research in mathematical fluid dynamics participated in this wider movement towards more refined comparison principles.

Non-Newtonian fluids supply a further example of nonlinear equations shaped by material behaviour. Unlike ideal Newtonian liquids, many complex substances exhibit stress responses that are not proportional to deformation rate. Polymer solutions, suspensions, and biological materials can therefore require different constitutive descriptions. The resulting differential equations may involve nonlinear operators and weakly regular quantities that resist classical estimates. Josef Málek and collaborators have contributed to the mathematical analysis of fluid models with such complex constitutive features. The central issue is to identify assumptions on stress and strain under which existence, stability, or meaningful approximation can be established. Monotonicity techniques and variational methods often play an important role in dealing with the resulting nonlinearities. The mathematical conclusions depend closely on the constitutive hypotheses and cannot simply be transferred between unrelated materials. This research area illustrates the productive contact between continuum mechanics and abstract nonlinear analysis. The distinction between a formal calculation and a justified result remained fundamental to this research tradition.

Nonlinear elliptic equations describe many stationary states of evolving systems. A prominent class involves the p-Laplacian, whose diffusion law changes with the magnitude of the gradient.

For the familiar linear Laplacian, superposition can be exploited, but such convenience generally disappears when the exponent differs from two. Existence and uniqueness then depend on the functional framework, the forcing term, and the structure of the domain. Pavel Drábek, working with Christian Simader, published a study in 2002 devoted to solvability properties of p-Laplacian-like equations. Their work explicitly examined how the definition of weak solution and the choice of function space affect the problem. The paper included general domains rather than limiting attention solely to smooth bounded regions. This is a particularly revealing achievement because mathematical generality is itself a form of innovation when it clarifies which hypotheses are essential. It also connects the analysis of nonlinear diffusion with broader developments in variational and functional methods.

Bifurcation theory investigates changes in the structure of solutions as a parameter varies. A system may possess a unique steady state for one parameter value but several branches of solutions for another. The appearance, disappearance, or exchange of stability of these branches can represent a qualitative transformation of the model. Nonlinear eigenvalue problems often provide a natural framework in which such phenomena can be studied. Pavel Drábek's research has long connected solvability questions with bifurcation analysis of nonlinear equations. This direction is especially valuable when traditional linear spectral arguments no longer apply directly. Degree theory, variational principles, and specialised comparison results can provide ways of establishing solution branches. The challenge is not merely to draw a suggestive diagram but to prove that the identified branches actually exist. Applications may be found wherever a load, reaction rate, or boundary condition changes the system's possible states. The resulting analysis gives mathematical substance to the idea of a threshold in a nonlinear process.

Variational methods translate some differential equations into problems of minimising or finding stationary values of energy functionals. They are effective when the governing operator arises from an appropriate energy structure. The strategy creates a bridge between differential equations, functional analysis, and optimisation. Coercivity can prevent an energy-minimising sequence from escaping to infinity, while lower semicontinuity can support passage to a limit. Critical-point methods allow investigation of non-minimising solutions when the geometry of an energy functional permits it. Neither route is universal, because many physical equations do not possess a straightforward variational formulation. Researchers must therefore understand both the power and the limitations of energy-based reasoning. This methodological discrimination became increasingly important in a period when nonlinear models were expanding into many applied fields. Czech analytical work forms part of the broader European tradition in which such techniques were developed and refined.

Tomáš Roubíček has been an important contributor to the systematic theory of nonlinear partial differential equations and their applications. His monograph on the subject offers a rigorous account of stationary and evolutionary problems using tools such as pseudomonotonicity, variational inequalities, and abstract evolution theory. The work is significant not because it reduces all models to a single formula but because it organises a large family of methods around precise assumptions. Evolution problems may involve dissipation, non-smooth potentials, and constraints that change the character of the underlying equations. A useful framework must

therefore accommodate both differential expressions and variational relations. Such formulations are important in materials science, where mechanical response may be irreversible or path-dependent. The theory demonstrates that applied mathematical rigour often requires more abstract mathematics rather than less. A mathematically justified constitutive model can explain the domain in which computational predictions should be trusted. The Czech contribution to this area is consequently both methodological and interdisciplinary. Its international relevance is reflected in the circulation of specialised research monographs and collaborations.

Evolution equations are often studied through families of operators acting on infinite-dimensional spaces. This perspective extends the intuition of an ordinary differential equation to states that are themselves functions of position. Semigroup techniques can describe the linear part of an evolution, whereas nonlinear perturbations may require additional arguments. Different equations demand different concepts of solution, including mild, weak, and strong solutions. These terms indicate specified mathematical regularity and formulation rather than a ranking of scientific importance. An abstract theorem may establish existence under general conditions, but its application still requires verifying the hypotheses for the concrete model. Time-dependent estimates are especially valuable when the aim is to understand global behaviour instead of a short interval of evolution. A theory of absorbing sets or attractors can then describe long-term dynamics in suitable dissipative systems. The methodology creates a conceptual continuity between dynamical systems, nonlinear analysis, and mathematical physics.

Long-term behaviour is a research question distinct from local existence. A solution may be well defined for a finite period while its eventual trajectory remains difficult to characterise. Dissipative systems can sometimes be shown to enter bounded regions of a function space after sufficient time. An attractor, when it exists under the required compactness and invariance conditions, summarises the asymptotic dynamics of a family of solutions. The existence of such an object does not imply that every detailed trajectory becomes simple. Instabilities, recurrent phenomena, and complicated patterns may remain possible within the mathematical structure. The analytical challenge is to derive estimates that do not deteriorate uncontrollably as time passes. For equations modelling fluids and other continuous media, these estimates may rely upon balances between forcing and dissipation. Research programmes linking evolution equations with dynamical systems give these ideas a central role. They also show that reliable prediction sometimes concerns boundedness or qualitative behaviour rather than an exact future state.

Reaction–diffusion equations combine local transformations with transport driven by spatial differences. Their mathematical structure can describe chemical reactions, biological growth, and propagation of disturbances through a medium. The interaction between reaction and diffusion can produce travelling waves and other patterns whose behaviour depends on the nonlinear terms. A travelling wave moves with an identifiable profile and speed under assumptions that must be made explicit. The existence of such a solution is not guaranteed merely because a numerical simulation displays an advancing front. Comparison principles and phase-plane methods provide tools for analysing wave structure in appropriate scalar cases.

Degenerate diffusion introduces further complications because transport may vanish or become singular in particular states. Czech researchers, including Pavel Drábek in work on nonlinear diffusion, have contributed to the analytical setting in which these issues are investigated. This area demonstrates how elementary physical notions of spreading can lead to exacting mathematical questions.

Boundary and interface conditions are as important to a differential model as its interior equation. Dirichlet conditions prescribe a state, Neumann conditions prescribe a flux, and more general conditions encode interaction with surrounding systems. The mathematical validity of each choice depends on the regularity of the domain and the traces available for the proposed solution space. In practical applications, boundaries may move, change shape, or contain interfaces between materials. Such features create difficult existence and regularity questions that cannot be settled by interior estimates alone. Czech research linking mechanics and nonlinear equations has helped develop mathematical approaches appropriate for material interfaces and coupled processes. A weak formulation may incorporate boundary data through a variational identity rather than pointwise substitution. This shift is valuable when geometric irregularity makes classical solutions unrealistic. Rigorous boundary analysis therefore provides the bridge between idealised governing laws and mathematically defensible physical scenarios. It remains a major source of innovation in the theory of partial differential equations.

Fluid–structure interaction represents a demanding form of coupled evolution. A fluid exerts forces upon an elastic or deformable boundary, while the resulting displacement changes the region available to the fluid. Neither component can therefore be solved independently without losing a defining feature of the problem. The mathematics must accommodate conservation laws, mechanical response, and the compatibility of motion at an interface. Large deformations create additional analytical and computational difficulties because the geometry itself becomes an unknown. Researchers use transformations of moving domains and suitable compactness arguments when seeking weak solutions. However, a theorem for one coupling law may not extend to another material response or contact regime. This research illustrates how the meeting of mathematical disciplines creates new problems rather than merely combining existing answers. The Czech mathematical community has pursued related questions through cooperation between analysts, specialists in mechanics, and numerical researchers.

Anisotropic and heterogeneous media challenge the assumption that material properties behave identically in every direction. Diffusion, elasticity, or conductivity may depend on position, direction, and the present state of the system. Consequently, coefficients in a partial differential equation may be discontinuous, degenerate, or nonlinear. The proof of existence then requires appropriate estimates that remain meaningful despite this complexity. A common strategy is to regularise difficult coefficients, solve approximate problems, and show that the approximations converge. The final passage to a limit must preserve constitutive relationships and the prescribed boundary or initial data. An apparently harmless approximation can otherwise change the mathematical model. Such problems are central to the analysis of composites, porous materials, and multi-phase processes. The development of advanced function-space and operator methods helped make these research directions mathematically

tractable. They also demonstrate why theoretical analysis remains necessary even when increasingly powerful simulations are available.

Numerical approximation contributes to differential-equation research by making otherwise inaccessible models computationally investigable. Finite-element and finite-volume methods turn continuous equations into finite systems that can be evaluated on modern computers. Yet discretisation introduces its own questions about convergence, stability, consistency, and conservation. A numerical result can be persuasive without being mathematically reliable if mesh refinement or algorithmic error is not understood. A priori analysis estimates errors from the formulation, while a posteriori analysis uses computable information to assess a particular approximation. For nonlinear evolutionary equations, discrete energy inequalities can play a decisive role in preserving physically meaningful behaviour. Czech collaboration between analytical and computational groups has strengthened the connection between proofs and simulation. The two approaches should not be treated as rivals, because each addresses limitations of the other. Rigorous analysis clarifies what an algorithm ought to approximate, while computations reveal phenomena and regimes that motivate new mathematical questions.

The modelling of biological and environmental processes expanded the practical audience for nonlinear differential equations. Population interactions, diffusion of contaminants, thermal processes, and transport through complex environments can all be represented by evolving spatial fields. However, mathematical form alone does not establish that a model is empirically adequate. Parameters must be connected to observations, while boundary conditions and constitutive assumptions require scientific interpretation. Well-posedness is a minimum requirement for coherent mathematical analysis but is not a guarantee of predictive validity. Sensitivity analysis helps explain how errors or uncertainties in inputs affect model outputs. Stochastic influences may need to be included where deterministic approximations overlook important fluctuations. These considerations strengthened links between differential-equation specialists and researchers in the natural sciences. The Czech case illustrates how abstract analysis can provide both methods and cautions for cross-disciplinary modelling. The importance of these cautions increased with the broader availability of computational experimentation.

International collaboration has shaped the modern Czech research environment in differential equations. Research problems involving fluid mechanics or nonlinear evolution often demand complementary expertise in analysis, geometry, computation, and physical modelling. Collaborative publications by Czech and foreign mathematicians show that national mathematical schools participate in international theoretical discussions rather than work within isolated traditions. The connections between Charles University and Academy institutes are particularly important in sustaining specialised seminars and doctoral work. A leading researcher may work across institutional affiliations while students and collaborators circulate between research groups. This organisation makes it difficult to assign a complex result to a single national institution in a simplistic way. Historical analysis should distinguish work authored by researchers based in Czechia from international results merely employed by Czech groups. It should also recognise the importance of joint authorship where the mathematical

contribution cannot be separated into national portions. Such precision gives a more credible account of scientific achievement than celebratory but unsupported claims.

Doctoral education transmits the technical and intellectual practices required for advanced analysis. Students must learn not only established theorem statements but also how to test a model's assumptions and identify the limits of a proof. Reading modern research requires familiarity with weak convergence, Sobolev spaces, nonlinear operators, and energy methods. Participation in seminars develops the capacity to question the validity of each inference. International visits and joint projects expose young researchers to alternative mathematical traditions and new families of problems. The relevance of advanced courses on regularity and the Navier–Stokes equations at Charles University exemplifies this connection between teaching and active research. New generations are then equipped to extend existing techniques rather than merely repeat them. The resulting intellectual continuity is an important mechanism through which mathematical achievement accumulates over decades. It also explains why institutional stability matters for fields whose deepest advances may require prolonged theoretical work. Scientific excellence cannot be measured solely through the immediate practical use of a result.

The period from 1993 to 2025 also changed the practical tools available to researchers without removing the need for proof. Computer algebra, numerical solvers, and high-performance computing broadened the range of differential models that could be explored. Visualisations made instabilities and travelling structures more accessible to non-specialist audiences. Nevertheless, numerical resolution depends on discretisation, computational resources, and the size of the error that is acceptable. Highly resolved simulations may suggest mathematical behaviour but cannot independently prove global existence or uniqueness. Conversely, an existence theorem may provide no efficient procedure for computing a particular solution. The productive response is to combine analytic estimates, computational verification, and empirical testing while keeping their evidential roles distinct. This methodological balance is especially relevant to modern interdisciplinary projects in fluid mechanics and materials science. It represents a mature form of innovation in which mathematical explanation and computational experimentation reinforce one another.

By 2025, Czech work in differential equations and nonlinear analysis had become part of a deeply international mathematical landscape. The strongest contributions were often cumulative, arising from repeated refinement of existence results, regularity criteria, approximation schemes, and models of complex materials. Research on fluid equations illustrates the significance of obtaining precise conclusions under sophisticated thermodynamic assumptions. Work on nonlinear elliptic equations shows how changes in the admissible function space can alter the meaning of solvability. Variational and evolution methods demonstrate how abstract theorems can support investigations far removed from their original formulation. The field also remained connected with neighbouring branches of mathematics, including functional analysis, probability, and scientific computation. These achievements should not be presented as the final resolution of nonlinear mathematical complexity. Fundamental questions, including difficult aspects of fluid regularity and complex coupled evolution, remain open in many settings. The historical accomplishment instead lies in

expanding the range of phenomena that can be treated with mathematical precision. This development provides a convincing illustration of innovation as the creation of dependable new methods of reasoning rather than simply the production of technological devices.

Numerical Analysis and Scientific Computing: Algorithms, Approximation, and Computational Accuracy

Scientific computation is one of the clearest examples of how mathematical thought becomes an instrument of discovery without ceasing to be a theoretical discipline. Between 1993 and 2025, Czech research developed within an international transition from comparatively small numerical experiments to simulations requiring extensive software, specialist expertise and powerful computing systems. Numerical analysis addresses not merely the production of approximate answers but the conditions under which an approximation can be trusted. The central questions concern consistency, convergence, stability, computational complexity and the relationship between discrete solutions and continuous models. These questions became particularly important as mathematical methods entered fluid dynamics, solid mechanics, environmental investigation and industrial design. Czech universities and institutes contributed through mathematical proofs, algorithmic development, doctoral training and collaborative scientific software. Their work was shaped by traditions reaching back into Czechoslovak applied mathematics, rather than created suddenly in the year of independence. New research opportunities after 1993 nevertheless changed the scale of international collaboration and access to modern computing resources. The historical significance of Czech numerical mathematics consequently lies in the combination of intellectual continuity, methodological renewal and widening practical applicability.

An approximate numerical answer has scientific value only when its relationship to the original mathematical problem is understood. Discretisation changes a continuous model into a finite collection of algebraic operations that a computer can perform. The resulting discrete problem can introduce errors even when its underlying physical equations are correctly formulated. Analysts therefore distinguish approximation errors from modelling errors, rounding effects and mistakes in implementation. Convergence arguments explain whether progressively refined calculations approach the desired mathematical solution under stated assumptions. Stability analysis investigates whether perturbations in the data or intermediate calculations may become excessively amplified. Error estimates provide a quantitative language for deciding whether a result has adequate accuracy for its intended purpose. Czech contributions to numerical analysis belong within this international tradition of linking mathematical reliability to computational experimentation. The discipline develops most productively when proofs and numerical tests challenge and refine one another. A persuasive numerical demonstration therefore reports not only a computed value but also the assumptions and uncertainty surrounding it.

The independent Czech Republic inherited several established centres of research in approximation theory, differential equations and computational mathematics. Charles University in Prague provided an important setting for mathematical fluid mechanics and numerical methods. The Mathematical Institute of the Czech Academy of Sciences maintained the wider connection between rigorous analysis and mathematical applications. Technical universities connected numerical ideas with engineering mechanics, materials, control and industrial computing. Institutional boundaries did not neatly correspond to the distinction between pure and applied mathematical knowledge. A theorem about approximation could influence engineering software years after its original publication. Conversely, a practical numerical failure might reveal the need for a stronger convergence argument or a new functional framework. This interaction became more visible as international projects joined mathematicians, physicists and engineers in common research teams. The Czech experience demonstrates that institutional diversity can sustain a coherent mathematical research culture.

Miloslav Feistauer provides a particularly well documented example of the Czech tradition of mathematically grounded computation. His 1993 monograph Mathematical Methods in Fluid Dynamics appeared at the beginning of the independent-state period. The book addressed the analytical and computational questions involved in modelling fluid motion. It belonged to a field in which numerical schemes must respect the structure of conservation laws and the often difficult behaviour of nonlinear equations. Computational fluid dynamics requires specialised approximations because sharp gradients, shocks and complicated geometries place severe demands on standard discretisation. Research in Prague developed methods while also seeking explanations of their mathematical behaviour. Feistauer's work can therefore be understood as an example of scientific continuity across the political transition rather than as a post-1993 invention. Its relevance also extended beyond mathematical departments into aerodynamics and related mechanical sciences. This combination of rigorous analysis and physical modelling became characteristic of one influential Czech research direction. The publication illustrates how a national research tradition acquired international visibility through substantive mathematical scholarship.

The monograph Mathematical and Computational Methods for Compressible Flow, published by Oxford University Press in 2003, marked another significant contribution. Miloslav Feistauer, Jiří Felcman and Ivan Straškraba joined expertise in mathematical fluid mechanics and numerical analysis. Their treatment connected the mathematical structure of compressible-flow equations with the computational methods used to investigate them. Compressible flow introduces phenomena that cannot be handled adequately by simply transferring algorithms from smooth incompressible problems. Shock waves and conservation requirements make the design of robust schemes especially demanding. Finite volume and finite element methods must be studied in relation to the equations and the regularity of their solutions. The book presented these methods as elements of a coherent mathematical problem rather than as isolated programming techniques. Its international publisher and detailed technical scope provide concrete evidence of the participation of Czech mathematicians in advanced scientific computation. The achievement is best described as a substantial synthesis and development of methods within a globally active field.

Finite element methods became indispensable because many scientific problems occur in geometries that cannot be represented by a simple rectangular grid. The computational domain is divided into smaller elements upon which an approximate solution is constructed using selected basis functions. A variational formulation provides the bridge between the differential equation and the resulting algebraic system. Different choices of elements, polynomial degree and numerical integration rules produce schemes with different mathematical properties. Reliable applications demand attention to coercivity, approximation capacity and the preservation of essential boundary conditions. Mesh refinement may reduce spatial error, although badly shaped elements can undermine expected performance. Czech numerical researchers worked within this mathematically demanding environment, especially in fluid and structural calculations. Connections with technical universities helped translate finite element theory into problems involving mechanical stress, flow and coupled physical processes. The field illustrates why success in numerical simulation requires more than access to a powerful computer. Mathematical formulation remains decisive even as available processing power increases.

A further methodological distinction concerns continuous and discontinuous approximations across neighbouring elements. Classical conforming finite element methods impose continuity conditions appropriate to their chosen function spaces. Discontinuous Galerkin methods instead permit certain jumps between elements and use numerical fluxes to couple their behaviour. This additional flexibility is attractive for transport-dominated problems, complex local adaptation and parallel computation. It also creates demanding questions concerning stability, consistency and the mathematical treatment of boundaries. Czech work on these questions was closely associated with the Prague school of computational fluid mechanics. Rather than treating an algorithm as a black box, researchers investigated how flux design, penalty terms and discretisation choices affect numerical solutions. Such analysis is crucial when strong gradients and nonlinear conservation laws make computational errors difficult to detect by visual inspection alone. The development of discontinuous methods illustrates the internationalisation of Czech computational mathematics through specialised mathematical innovation.

A major point of reference is the 2015 monograph Discontinuous Galerkin Method: Analysis and Applications to Compressible Flow by Vít Dolejší and Miloslav Feistauer. Published by Springer, it brought together theoretical analysis and detailed applications to flow problems. The work examined the mathematical architecture of discontinuous Galerkin approximations rather than merely introducing a convenient computational technique. For research communities, such monographs are important because they consolidate assumptions, proofs and methodological choices that might otherwise remain dispersed across specialised papers. They can also influence doctoral education and the development of reproducible implementations. The authors' connection with Charles University demonstrates how Czech mathematical departments participated in international numerical research at an advanced level. The book also helps trace continuity between the earlier work on compressible flow and subsequent analytical developments. It provides a clearly identifiable scientific achievement without requiring unsupported numerical claims about national publication rankings. The resulting

trajectory is one of sustained methodological refinement across more than two decades. This continuity is particularly valuable when numerical innovation must be assessed in its proper historical context.

Approximation accuracy does not depend solely on the formal order of a numerical method. A scheme that performs well on a smooth benchmark may fail when a solution develops discontinuities or localised layers. Consequently, computational mathematicians study different norms and error indicators appropriate to the behaviour being approximated. A priori estimates predict convergence under conditions specified before a computation is carried out. A posteriori estimates use properties of an already computed solution to guide further numerical investigation. The distinction matters for adaptive computation, where calculating everywhere with equal resolution may waste substantial resources. Czech research on finite elements and nonlinear problems has participated in this wider development of mathematically supported adaptivity. The underlying goal is to place computational effort where the solution genuinely requires it. Precision must therefore be understood as a controlled relationship between the numerical method, the model and the quantities being investigated.

Adaptive algorithms can refine a computational mesh locally, increase polynomial approximation order or combine both procedures. Local refinement is especially useful when a solution contains steep gradients in a restricted portion of the domain. Increasing polynomial order may be effective where the underlying solution remains sufficiently smooth. Choosing between these strategies requires knowledge of approximation theory and error behaviour rather than a purely mechanical rule. Adaptive methods must also manage data structures and mesh conformity so that computational savings do not produce new instabilities. In transient problems, spatial decisions interact with time-step selection and the propagation of numerical error. Research in Czech universities connected these considerations with the analysis of nonlinear flow and engineering systems. Their significance lies in seeking reliable accuracy with computational resources that remain realistically available. Adaptivity thus reveals numerical analysis as a discipline concerned simultaneously with mathematical correctness and efficient scientific practice. A carefully designed adaptive method replaces uniform computational expenditure with evidence-based allocation of effort.

Scientific simulation also depends upon how continuous time is represented through discrete updates. Explicit integration schemes often offer straightforward implementations but may impose restrictive stability conditions. Implicit schemes can permit larger time steps in certain settings, although they require the solution of additional algebraic systems. Stiff differential equations present special difficulties because different dynamical scales coexist within the same model. Time discretisation cannot be selected independently of the spatial method when both errors influence the physical interpretation. Numerical analysts therefore consider coupled stability and consistency properties of fully discrete approximations. In the Czech context, such questions appeared in fluid mechanics, heat transfer, mechanical evolution and other research applications. The mathematical challenge is to control a sequence of approximate states rather than one isolated solution. This temporal dimension links numerical algorithms to the broader theory of evolution equations and dynamical systems.

When a partial differential equation is discretised, the resulting algebraic system may contain millions of unknowns. Numerical linear algebra therefore occupies a central position between the mathematical model and the final computed result. Sparse matrices arise because each element or local grid point generally interacts with a comparatively limited set of neighbours. Their structure permits major savings in storage and operations when algorithms exploit sparsity appropriately. Direct solvers can be effective for some problems but may become expensive as the system grows. Iterative methods seek approximate solutions through repeated updates whose convergence depends on matrix properties and problem conditioning. Preconditioning modifies the computational representation in order to improve iterative performance without changing the intended solution. Czech scientific-computing research developed in close contact with these internationally shared challenges. Improvements in solvers can expand the size of a physically meaningful simulation without any change to the underlying equations. This is one reason that algorithmic advances can be as consequential as improvements in computing hardware.

Numerical stability and conditioning describe distinct but related aspects of computational reliability. Conditioning concerns the sensitivity of a mathematical problem to perturbations in its data. Stability concerns whether the numerical algorithm introduces or amplifies additional disturbances during its execution. An ill-conditioned problem can produce uncertain answers even when its algorithm is carefully implemented. Conversely, a well-conditioned problem may still be solved poorly by an unstable procedure. Rounding error becomes important in long computations involving many floating-point operations and widely varying numerical scales. Researchers must accordingly distinguish the accuracy of stored numbers from the accuracy of the physical or mathematical interpretation. Czech computational work, like that of other advanced scientific communities, has been shaped by these principles in numerical linear algebra and discretisation. Reliable results depend upon diagnosing the source of error rather than simply increasing nominal numerical precision.

Domain decomposition responds to the difficulty of solving large problems by dividing a computational region into interacting subdomains. Each subdomain may be treated locally, while additional conditions coordinate the global approximation. This structure is particularly relevant to parallel computers because different subproblems can be processed simultaneously. However, parallelism does not automatically guarantee efficiency when communication and synchronisation costs become substantial. Mathematical analysis must establish whether the resulting iterative procedures converge and how their performance changes as the decomposition becomes finer. The finite element tearing and interconnecting family of methods provides one important framework for this task. Czech researchers, including groups associated with Ostrava, have worked on these methods and their algebraic foundations. Research in this field brings spectral properties, optimisation and numerical linear algebra into direct contact with high-performance computing. It also offers an example of how mathematical specialisation can contribute to internationally shared problems of computational scale. The resulting algorithms depend as much on mathematical structure as on the architecture of the machines executing them.

A documented example appeared in Applications of Mathematics in 2020, when Ladislav Foltyn, Dalibor Lukáš and Ivo Peterek studied domain decomposition coupled with the parareal method. Their paper investigated transient heat equations in one and two spatial dimensions. The approach combined finite element spatial approximation with time integration and a method for dividing the temporal interval into subproblems. It addressed the possibility of exploiting parallelism in both spatial and temporal components of computation. Such methods are important because large transient simulations can remain expensive even when their spatial equations are efficiently discretised. The authors analysed an identifiable model rather than claiming a universal solution to parallel time integration. The paper therefore serves as concrete evidence of Czech research at the intersection of discretisation theory and parallel algorithms. Its significance is methodological, showing how established numerical ingredients may be combined to investigate computational bottlenecks. The example also demonstrates the continuing activity of regional mathematical research beyond Prague.

Another documented research direction concerns the treatment of interface constraints in finite element tearing and interconnecting algorithms. Large domain-decomposition problems require neighbouring subdomains to agree appropriately along their common interfaces. Different algebraic formulations of this agreement can influence conditioning and the speed of iterative solution. In 2025, Pavla Hrušková, Zdeněk Dostál, Oldřich Vlach and Petr Vodstrčil published a study of multipoint constraints in these methods. Their analysis employed tools from linear algebra and spectral graph theory to examine particular constraint choices. This illustrates how discrete mathematical structures contribute directly to the performance of numerical methods for differential equations. The work provides evidence of continuing Czech specialisation in the algebraic aspects of parallel scientific computing. It also warns against separating software performance from mathematical decisions made during method design. In this research, innovation means the examination and improvement of the precise conditions governing numerical performance. Such results are often incremental in appearance but consequential for the reliability of large computations.

Fluid dynamics has remained a particularly demanding application for the Czech numerical-analysis community. The governing equations express conservation of mass, momentum and, where appropriate, energy. Their nonlinear structure produces complex behaviours that require specialised analytical and numerical approaches. Compressible flows may contain shocks, while viscous flows demand adequate representation of boundary layers and dissipative effects. A numerical solution must preserve physically meaningful balances as far as the selected scheme allows. Grid construction, flux approximation and time integration cannot be treated as unrelated implementation details. Czech research built upon the combination of analytical understanding and computational experiments exemplified by Feistauer and collaborators. This tradition connected mathematical inquiry with aerodynamics, mechanical engineering and related experimental disciplines. Its long-term importance lies in demonstrating how theoretical mathematical standards can strengthen application-oriented simulation.

Fluid–structure interaction demonstrates why isolated models are often inadequate for real engineering systems. A fluid exerts forces upon an elastic or otherwise deformable structure, while structural motion changes the fluid domain. Numerical schemes must exchange

information between these descriptions without losing stability or physical consistency. Moving meshes, interface conditions and coupled time integration introduce difficulties beyond those of separate fluid and solid calculations. Czech numerical researchers have addressed such interactions in work on mechanics and biologically relevant flows. Feistauer's publication record includes collaborative studies of numerical simulation involving the human vocal folds. This example is scientifically informative because it connects mathematical fluid mechanics with a question arising in physiology and biomedical engineering. It also illustrates a wider shift towards simulation problems that require expertise from multiple scientific traditions. Numerical innovation in these settings often concerns reliable coupling rather than a single new equation. Interdisciplinary collaboration consequently creates both practical opportunities and new mathematical obligations.

Variational methods link numerical computation to functional analysis and the modern theory of partial differential equations. A weak formulation may admit solutions under assumptions less restrictive than those required for classical differentiation. Finite element spaces approximate such weak solutions while preserving selected analytical structures. The quality of the discrete method depends partly upon the mathematical properties of those underlying function spaces. Coercivity, compactness and appropriate continuity bounds can determine whether an approximation is meaningful. Nonlinear problems may demand monotonicity arguments, fixed-point reasoning or compactness techniques before convergence is established. Czech mathematical work benefits from the proximity of strong traditions in functional analysis, differential equations and scientific computation. The resulting relationship should not be reduced to a one-way transfer from theory into application. Numerical experiments may themselves expose unresolved analytical questions and motivate new theoretical investigation.

An important change after 1993 concerned the organisation of collaboration rather than the invention of a wholly new numerical discipline. Academic mobility and international publishing allowed Czech research groups to participate more extensively in global methodological discussions. European integration after 2004 expanded opportunities for joint projects and scientific networks. International cooperation became especially important for computational work because specialist expertise and expensive resources are not distributed evenly. Mathematicians could contribute proof techniques and algorithms while partners supplied complementary models, experiments or technical facilities. Such arrangements also created opportunities for doctoral researchers to work on problems of international relevance. Nevertheless, collaboration does not justify assigning collective discoveries exclusively to one country. An accurate account must distinguish Czech-authored results, international methods employed in Czech institutions and jointly produced achievements. This distinction is essential for assessing the historical contribution of a national mathematical research system with scientific integrity. It also ensures that the language of innovation remains proportionate to the underlying evidence.

The formation of IT4Innovations at VSB–Technical University of Ostrava changed the national infrastructure available for large-scale scientific computing. The initiative began in 2011 as part of a wider effort to build advanced research capacity in the region. Its first major supercomputer, Anselm, entered operation in 2013. The Salomon system followed in 2015, substantially

expanding available computational performance. Barbora was added in 2019, broadening the resources accessible to scientific users. These developments were not mathematical discoveries in themselves, but they altered the feasible scale of numerical experiments and simulations. They also created a setting where computational science, algorithm development and collaboration with industrial users could interact. The historical significance of the centre should therefore be measured in terms of research capacity and opportunities as well as hardware specifications. For Czech numerical mathematics, the facility became an important bridge between analytical methods and large computational workloads.

The Karolina supercomputer, installed in 2021, marked a subsequent phase in the development of this infrastructure. Its deployment occurred within European cooperation in high-performance computing and increased the resources available to Czech and international users. Greater computational capacity can support finer grids, larger parameter studies and more elaborate coupled models. Yet a machine's nominal speed does not guarantee that a particular algorithm will run efficiently upon it. Researchers must consider memory access, communication patterns, parallel scaling and the balance between calculation and data movement. Mathematical algorithms therefore need to be designed or adapted for the structure of contemporary computing systems. The relationship between hardware and mathematics is reciprocal because new architectures can expose weaknesses in established methods. Czech scientific computing increasingly operated within this environment of co-development between algorithms and machines. By 2025, computational mathematics in the country had a substantially broader infrastructural foundation than in the early 1990s. The distinction between computational capability and mathematically justified results nevertheless remained fundamental.

Parallel performance is usually evaluated through more than the total number of available processors. Strong scaling concerns how effectively an existing problem can be accelerated by adding computing resources. Weak scaling considers whether a larger problem can be handled efficiently as the available resources increase. Communication delays, load imbalance and memory constraints may prevent ideal improvements in either setting. Numerical methods also vary in their capacity to be divided into independent or loosely connected tasks. Domain decomposition can offer useful parallel structure, whereas some global operations create persistent synchronisation costs. Czech research linked mathematical algorithm design with these practical questions through collaboration among computational mathematicians and supercomputing specialists. Such work matters because scientific questions often require simulations that exceed the resources of a single workstation. The ability to preserve mathematical reliability while improving parallel efficiency represents an important form of methodological progress.

Scientific software is a substantial part of the modern numerical research process, although it is not equivalent to numerical theory. A mathematically correct algorithm can still be implemented incorrectly through faulty indexing, boundary treatment or floating-point assumptions. Verification compares a program's output with exact solutions, manufactured tests or results for which trustworthy bounds are available. Validation asks the different question of whether the mathematical model represents the relevant real-world behaviour adequately. These activities

become especially important when a programme combines independently developed components or operates on large parallel machines. Reproducibility also depends upon documenting code versions, parameter choices, computational environments and processing procedures. Czech research environments faced the same international movement towards these practices as computing-intensive science became more collaborative. The practice of reporting numerical experiments consequently became part of scientific accountability rather than a secondary technical detail. A credible computational conclusion requires a traceable connection from assumptions and algorithms to observable numerical output. Methodological transparency is indispensable when computed results inform decisions outside the originating research group.

Inverse problems reveal another dimension of the connection between numerical mathematics and empirical science. In a direct problem, specified parameters are used to predict the behaviour of a system. In an inverse problem, observations are used to infer parameters or structures that may not be directly measurable. Such tasks can be unstable because small observational errors may produce large changes in the inferred result. Regularisation introduces additional mathematical structure to obtain more stable and interpretable approximations. Optimisation, statistical modelling and numerical linear algebra often operate together in practical inverse computations. Czech mathematical and technical research developed within this wider interdisciplinary field, particularly through the growing interaction of computing, engineering and data analysis. The methodological challenge is to make the influence of modelling assumptions visible rather than conceal uncertainty behind a computed estimate. As computational resources expanded, larger inverse problems became feasible, but the need for sound mathematical interpretation remained unchanged.

Uncertainty quantification broadened the meaning of accuracy beyond a single deterministic error estimate. Physical models may contain uncertain parameters, imperfect initial conditions or assumptions whose validity is limited. Numerical solutions can be sensitive to those uncertainties even when the discrete scheme converges correctly. Researchers therefore employ sensitivity analysis, probabilistic sampling and related mathematical techniques to explore the consequences of imperfect knowledge. Such calculations may require many repeated numerical solutions and become expensive for complex models. High-performance computing can make broader ensembles possible, although inefficient algorithms can still render them impractical. In Czech scientific research, the growing intersection of applied mathematics, statistics and computational infrastructure made these questions increasingly pertinent. The resulting work reflects the wider evolution from solving a model once towards assessing the range and robustness of its predictions. An honest numerical result must distinguish discretisation accuracy from uncertainty in the data and the scientific model. This distinction becomes crucial when simulations are used to compare technological alternatives or inform public decisions.

Scientific computing increasingly interacts with optimisation because many applications seek a best feasible design rather than a single forward prediction. An optimisation procedure may repeatedly solve differential equations while changing physical parameters or geometric configurations. The resulting computational cost can be substantially higher than that of one

simulation. Adjoint techniques, reduced models and carefully constructed gradients may help make such problems tractable. Numerical accuracy matters because inaccurate state solutions can corrupt the objective function or its derivatives. The mathematics therefore requires coordinated understanding of approximation, sensitivity and optimisation. Czech research traditions in numerical mathematics and mathematical programming provided complementary intellectual resources for these problems. Their connection demonstrates that applied mathematics develops through cooperation among subfields rather than through isolated techniques. The increasing integration of simulation and optimisation was one important direction in the broader evolution of computational science between 1993 and 2025.

Machine learning introduced a new category of computational tools into scientific research during the 2010s and early 2020s. Data-driven methods can assist with classification, surrogate modelling, parameter estimation and the analysis of complex simulation outputs. Nevertheless, a statistical predictor is not automatically a reliable replacement for a numerical solver derived from physical equations. Its accuracy may deteriorate outside the range represented by its training data. Hybrid approaches attempt to combine learned components with established mathematical constraints and simulation frameworks. Such combinations raise questions concerning consistency, robustness, generalisation and the interpretation of approximated quantities. Czech computational infrastructures supported growing interaction between numerical simulation, large-scale data analysis and artificial intelligence. Within this development, mathematical analysis remains essential for evaluating what a learned approximation can legitimately establish. The responsible use of computational intelligence thus extends rather than eliminates the central concerns of numerical mathematics. It also requires a clear distinction between demonstrated application results and speculative claims of universal scientific transformation.

The Czech contribution to numerical mathematics cannot be represented adequately by a list of machines or a catalogue of software packages. Its most defensible features include sustained work on mathematical fluid dynamics, rigorous discretisation methods and computationally demanding algebraic algorithms. International monographs associated with Feistauer, Felcman, Straškraba and Dolejší document one important line of development. Published investigations of parallel domain decomposition show another, especially in research linked to regional technical universities. The development of IT4Innovations illustrates how national infrastructure expanded the practical possibilities of scientific computation. Together, these examples reveal a pattern in which mathematical ideas, institutions and technology reinforce one another over time. The country's scale also makes international cooperation structurally important for access to specialised expertise and shared computational systems. Such cooperation should be understood as an element of scientific strength rather than evidence that national mathematical research lacks an independent intellectual identity. The historical achievement lies in creating conditions for contributions that remain recognisable within the wider international literature.

By 2025, numerical analysis in Czechia had developed across an unusually broad range of mathematical and computational activities. Foundational questions concerning convergence, approximation and stability continued to structure research despite major changes in available technology. Finite element and discontinuous Galerkin methods connected analytical

mathematics with simulations of complex physical systems. Numerical linear algebra and domain decomposition supported attempts to solve larger discretised problems efficiently. Adaptive strategies sought to make accuracy more economical by allocating resolution according to mathematical evidence. High-performance computing expanded the practical scale of research and strengthened the relationship between universities, public infrastructures and applied science. Statistical inference, optimisation and data-driven models introduced additional demands for mathematical reliability and interpretability. The development was cumulative rather than a replacement of classical numerical analysis by a wholly new computational paradigm. Czech mathematical science thus illustrates how theoretical rigour and computational innovation can develop together within an internationally connected research ecosystem. Its long-term value lies in making difficult calculations not only possible but scientifically meaningful.

Algebra and Abstract Mathematical Structures: Modern Research in Algebraic Systems

The evolution of algebra in the Czech Republic after 1993 demonstrates that innovation in mathematics need not take the form of a new instrument or an immediate technological application. It can emerge through a different understanding of structures, operations, relations, and the transformations connecting them. Czech algebraic research inherited a remarkably diverse intellectual tradition, extending from classical algebra to universal algebra, category theory, and mathematical logic. That tradition was sustained within university departments and institutes whose scholarly communities survived the political dissolution of Czechoslovakia. The independent state nonetheless offered a changing environment for research assessment, international appointments, doctoral training, and competitive grants. These developments encouraged mathematicians to frame specialised questions within wider international conversations about the foundations of mathematics. The geographical distribution of algebraic expertise remained important, particularly the relationships among Prague, Brno, and other university centres. The central historical question is therefore not whether Czech mathematics discovered abstraction after independence, but how its existing abstract traditions were renewed. Between 1993 and 2025, this renewal joined internal mathematical reasoning to increasingly productive conversations with computer science and logic.

Universal algebra provides a natural point of departure because it compares mathematical systems through their operations rather than through the incidental details of particular examples. Groups, rings, lattices, and many other systems can be investigated within a language of operations, identities, and homomorphisms. This perspective makes it possible to recognise a shared structural principle behind problems that initially appear unrelated. In the Czech environment, the subject interacted with long-established interests in algebraic systems and the logical description of mathematical theories. The post-1993 period was notable for stronger

circulation of research through international seminars, collaborative publications, and doctoral networks. University mathematicians could consequently test local techniques against problems formulated by colleagues in other mathematical communities. Universal algebra also became unexpectedly relevant to computational questions about the expressive power of finite constraint languages. Its theoretical vocabulary offered tools for distinguishing algorithmically tractable classes of problems from classes exhibiting more demanding behaviour. The importance of this interaction is that an abstract algebraic identity may eventually explain an observable difference in computational complexity. Such developments expanded the meaning of an algebraic result beyond the traditional classification of structures.

A mathematical structure does not exist in isolation from the maps that preserve its organisation. Homomorphisms, embeddings, quotients, and products express ways of comparing systems while respecting selected operations or relations. The investigation of these maps belongs to the basic grammar of universal algebra, yet it also leads towards the more expansive language of category theory. From this viewpoint, the mathematical object and the permissible transformations between objects become equally important. An algebraic question may then be reformulated as a question about diagrams, universal properties, or the existence of an appropriate factorisation. Such reformulation often reveals patterns that are obscured by working solely with elements and equations. Czech researchers engaged with these questions through a combination of algebraic, topological, and categorical perspectives. Their work illustrates why structural mathematics often progresses by changing the level at which a problem is described. The resulting abstraction is useful when it produces a more precise theorem or transfers an argument across different settings.

Category theory occupied a substantial place in the international mathematical profile of Czech researchers during the period under consideration. Jiří Adámek contributed to a literature that explained abstract and concrete categories as systematic settings for studying mathematical structures. The influential book written with Horst Herrlich and George Strecker first appeared before Czech independence, making it evidence of continuity rather than a post-1993 discovery. Its subsequent revised and republished forms nevertheless remained important to the teaching and dissemination of categorical mathematics. The distinction matters historically because an earlier achievement can continue to shape a research environment without being retrospectively assigned to a later decade. Concrete categories provide a way of relating abstract categorical reasoning to familiar underlying sets and structure-preserving maps. This approach is especially effective for examining limits, colimits, reflective subcategories, and factorisation systems. Researchers working with such concepts can ask which properties depend upon a particular algebraic presentation and which follow from broader structural assumptions. Czech categorical research therefore belonged to a continuing international conversation rather than to a self-contained national tradition. The research culture emerging after 1993 made this conversation easier to sustain through mobility and collaboration.

The intellectual significance of category theory lies partly in its capacity to replace a collection of separate arguments with a common language. A universal property characterises an object by its relationships with other objects rather than by a preferred construction. This can clarify why apparently different mathematical constructions obey closely related principles. Functors make

comparisons between whole categories possible, while natural transformations describe systematic relationships between functors. The notion of an adjunction further explains recurring pairs of constructions that arise across algebra, topology, and logic. Such concepts are not merely decorative terminology because they support precise existence and uniqueness arguments. Within Czech mathematical research, categorical techniques were also developed in proximity to structural questions involving infinitary operations and accessible categories. These subjects demonstrate how algebra can be studied when finite presentations are insufficient for the structures of interest. They also show why an apparently remote question about size or preservation can affect the foundations of modern mathematical modelling.

Jiří Rosický represents a particularly significant connection between Czech mathematics and the theory of categories and mathematical structures. His research profile at Masaryk University encompasses category theory and topics reaching into algebra, topology, and mathematical logic. The development of these subjects demonstrates the value of specialised schools capable of maintaining technically demanding research over many years. Categorical investigations often require careful distinctions between the existence of objects, their accessibility, and the ways in which morphisms behave under constructions. These distinctions can become essential when an ordinary finite algebraic description no longer captures the setting adequately. Work on locally presentable and accessible categories belongs to this broader attempt to understand mathematics through controlled structural conditions. Rosický's publication record also illustrates the continuity of such questions into the final years of the period examined here. A historically accurate account must distinguish his work from achievements of co-authors while recognising the collaborative character of contemporary category theory. It must also resist interpreting every categorical result as a direct application to digital technology. The enduring achievement is the creation of rigorous theoretical instruments with potential relevance across several branches of mathematics.

The interaction between abstract algebra and mathematical logic became a second major axis of research. Logical languages specify which statements about structures can be formulated and what counts as a valid consequence. Algebra supplies concrete or abstract models in which those statements may be interpreted. The relationship is reciprocal because algebraic constructions can illuminate logical equivalence, while logical criteria can classify families of structures. Universal algebra is especially well suited to this reciprocity through its treatment of equations and varieties of algebras. In a variety, a shared set of identities determines a class of structures closed under characteristic algebraic constructions. Studying such classes gives mathematicians a disciplined way to move from individual examples to broader structural results. Czech researchers contributed to this wider field in which mathematical logic, algebra, and theoretical computing increasingly overlap. The historical novelty after 1993 was less the existence of this connection than the growing range of international problems to which it could be applied.

Relations form a complementary way of describing mathematical systems because not all significant structures are determined by operations alone. Graphs, ordered sets, relational databases, and finite constraint templates can be represented using relations with specified arities. A map preserving those relations becomes a homomorphism of relational structures,

even when the familiar operations of group or ring theory are absent. The contrast between operations and relations is important but should not be exaggerated into a strict division. An algebra of operations can be associated with a relational structure by considering those operations that preserve its relations. Such preservation operations are called polymorphisms and have become central to the algebraic study of constraint satisfaction. This connection makes a finite combinatorial problem accessible to structural methods originally developed for algebraic systems. It is one of the clearest illustrations of how modern mathematics moves between different representations of the same underlying difficulty. Czech contributions to the theory of constraint satisfaction made this bridge particularly visible in the international literature. Through that work, the study of abstract operations acquired a strong explanatory role in theoretical computer science.

The constraint satisfaction problem asks whether values can be assigned to variables so that a prescribed collection of relations is respected. This formulation encompasses many finite problems arising in graph theory, scheduling, database theory, and computational logic. The complexity of a fixed relational template is influenced by the operations preserving its relations. Investigating the associated algebra can therefore reveal properties that are difficult to see in the original combinatorial formulation. In the early twenty-first century, this insight generated an influential international programme connecting universal algebra to computational complexity. Libor Barto, associated with Charles University, made major contributions within this programme alongside researchers abroad. His work demonstrates how a mathematician based in a Czech institution could participate in defining methods used by a wider international community. The essential intellectual move was to treat algebraic structure as explanatory evidence about the feasibility of algorithms. It replaced an exclusive search for problem-specific procedures with the investigation of general structural conditions.

A prominent example is the work of Libor Barto and Marcin Kozik on absorbing subalgebras and cyclic terms. Their 2012 article in Logical Methods in Computer Science developed new characterisations relevant to finitely generated Taylor varieties. The paper connected properties of algebraic terms to the structural analysis of fixed-template constraint satisfaction problems. An absorbing subalgebra captures a situation in which an operation draws a collection back into a designated substructure under controlled conditions. Cyclic terms, meanwhile, express particular identities that help identify algebraic regularities relevant to computational classification. These notions have precise formal meanings and should not be treated merely as informal metaphors for simplification. The paper showed how algebraic reasoning could reorganise proofs and clarify existing relationships between classes of finite structures. Its significance arose from the strength of its deductions rather than from a claim that the work immediately solved every open complexity question. Collaboration between Czech and Polish researchers also illustrates the increasingly transnational organisation of advanced theoretical research. The case provides a concrete example of mathematical innovation as the invention of a sharper conceptual tool.

The same international research direction also investigated which finite constraint problems could be solved by local consistency methods. Such algorithms repeatedly examine restricted collections of variables and remove assignments that violate known local conditions. Their

success on an entire class of problems depends on structural properties that can be expressed algebraically. Barto and Kozik analysed these relationships in an article published in the Journal of the ACM in 2014. The result exemplifies a productive interchange between mathematical proof and algorithmic classification. The algebraic perspective gives a language for describing a boundary of tractability rather than merely testing particular instances. The computational perspective supplies questions whose answers expose new aspects of algebraic structure. This two-way development shows why the distinction between pure and applied mathematics can be analytically useful but historically incomplete. Research of this kind remains fundamental mathematics even when its concepts have clear computational consequences.

Algebraic research at the interface with computing should nevertheless be described without suggesting that category theory and universal algebra are identical. Universal algebra studies operations and equations on structures, while category theory examines objects and morphisms within more general organisational settings. Their intersection is substantial because varieties, algebraic theories, and structured categories admit mutually illuminating descriptions. Yet different questions require different technical tools, and not every categorical theorem concerns finite algebraic operations. Likewise, the constraint satisfaction programme draws heavily on universal algebra without exhausting the possible applications of that discipline. A historically responsible synthesis therefore preserves distinctions among neighbouring fields while explaining why collaboration between them is fruitful. Czech mathematical institutions created settings in which specialists could move between these approaches without dissolving their disciplinary identities. Students and researchers could encounter algebra through pure mathematics, logic, or the formal study of computation. The coexistence of these routes helped sustain the breadth of the country's structural research. It also made the circulation of methods across institutional boundaries a significant form of innovation.

The relationship between algebra and geometry offers another perspective on the development of abstract methods. Classical geometry often begins with spaces, points, distances, or transformations that can be visualised. Modern geometry increasingly studies spaces through symmetry, algebraic invariants, and the morphisms relating geometric constructions. Algebraic methods can encode geometric behaviour even when explicit coordinates are cumbersome or misleading. Conversely, geometric examples can reveal the strengths and limitations of a proposed abstract theory. Categorical language assists in this exchange by making composition, equivalence, and universal construction central to the argument. The relevance to Czech mathematics is not that one national school absorbed all of geometry into algebra. Rather, research communities maintained complementary expertise that made cross-disciplinary methods intellectually available. This shared methodological environment supported work in topology, differential geometry, and related structural disciplines.

Topology contributes questions about continuity, compactness, connectedness, and the organisation of spaces under continuous transformations. These questions can often be recast through collections of maps or through categories of spaces with additional properties. Algebraic and categorical approaches then clarify when a construction preserves a selected topological feature. The resulting theory may appear remote from direct physical measurement, but its conceptual precision is vital for many later arguments. Researchers working in Czech

universities participated in this wider structural approach to mathematical spaces. The mathematical importance lies in identifying conditions that remain valid across broad classes of examples. Such generality also creates limits, because a theorem may demand assumptions that concrete problems do not satisfy. Good mathematical exposition must show both what a framework explains and where further hypotheses are required. This discipline of stating assumptions accurately distinguishes genuine structural progress from unsupported claims of universality. It was an enduring characteristic of serious algebraic and topological research throughout the period.

At Masaryk University, algebraic and categorical research formed part of a wider mathematical environment rather than an isolated theoretical enclave. The university's mathematics and statistics activities connected specialist research with doctoral supervision, seminars, and advanced courses. Such academic settings matter because difficult structural ideas require repeated discussion before they can support new proofs. A department's continuing ability to teach advanced algebra contributes to the preservation of expertise even when research subjects change. International visitors and joint publications then extend the community beyond the limits of a single university. This pattern became increasingly normal as Czech researchers participated in European networks after the accession of the Czech Republic to the European Union in 2004. The consequences cannot be measured simply by counting trips or collaborations, because the intellectual substance of those exchanges is decisive. Nevertheless, mobility can expose local schools to new definitions, open problems, and proof strategies. These forms of academic exchange are especially important in fields where expensive laboratory infrastructure is not the principal constraint.

Charles University maintained a particularly visible position in algebraic research and its relationship with theoretical computer science. Its Department of Algebra provided a setting for work on algebraic structures alongside broader activities in logic and discrete mathematics. The work associated with Libor Barto illustrates an approach in which traditional disciplinary boundaries become routes for collaboration. A problem originating in computer science may be reformulated as a theorem about operations on finite sets. Solving that algebraic question can then reveal consequences for the original computational problem. This movement does not imply that algorithm design and theorem proving are interchangeable activities. Rather, each supplies constraints and insights that improve the other. The academic value of such interaction is strengthened when doctoral researchers can move between seminars and research communities. Prague's concentration of mathematical and informatics institutions made this exchange especially plausible during the period. The example illustrates how abstract mathematical ideas can travel between research traditions.

Research in algebraic systems also develops through the careful examination of identities and equational theories. An identity states that two expressions built from operations represent the same value throughout a specified class of structures. Sets of identities can therefore encode extensive information in compact formal form. Different collections of identities may distinguish varieties with very different structural or computational properties. Finding a characterisation that uses fewer or more transparent identities can constitute a significant theoretical achievement. Such work should not be confused with merely changing notation, because

equivalent descriptions may expose different proof techniques. Czech contributions to the algebraic study of constraints underline the practical importance of this distinction. Term conditions became part of an explanatory framework connecting abstract algebra with algorithmic behaviour. At the same time, the investigation of identities remains worthwhile independently of immediate computational applications.

The notion of a variety offers a useful illustration of the balance between abstraction and classification. In universal algebra, a variety is an equationally defined class of algebras rather than a geometric shape. Its members share identities and remain stable under characteristic operations such as products, subalgebras, and homomorphic images. This gives researchers a powerful method for organising large families of structures. Classifications become more informative when they distinguish structural behaviour rather than merely enumerate examples. The analysis of Taylor varieties in constraint satisfaction research is one example of this ambition. It draws on technical conditions developed within universal algebra and connects them to a separate question about computational complexity. Such results are usually collective achievements built through a sequence of papers rather than sudden isolated breakthroughs. Their historical assessment requires attention to the exact claims proved at each stage. This is particularly important in an interdisciplinary field where later achievements can tempt commentators to overstate the reach of earlier work.

Abstract mathematical relations also play an important role in describing order, equivalence, and compatibility between structures. Equivalence relations permit the formation of quotient structures, while order relations organise elements through comparison. Congruences in universal algebra connect these ideas by specifying equivalences compatible with an algebra's basic operations. Their study can disclose internal decomposition, homomorphic representation, and the possible organisation of varieties. Categorical methods offer an alternative language for some of these phenomena through kernels, quotients, and factorisation properties. The choice between approaches depends on what needs to be established and which hypotheses are available. Mathematicians working in the Czech research environment contributed to a world in which these connections were increasingly discussed across specialist boundaries. Such development rarely generates a single dramatic date suitable for a timeline. Its importance is cumulative, residing in a growing collection of theorems and techniques that later research can employ.

The theory of accessible categories provides a particularly instructive example of the scale at which modern algebraic thinking can operate. Ordinary algebra often begins with structures described by finite operations or by familiar sets of generators. Accessible categorical methods seek controlled descriptions of more extensive classes of structures through suitable small objects and colimits. This is valuable when a field requires a uniform treatment of constructions too broad for a simple elementwise presentation. It also raises foundational questions about size, definability, and the legitimacy of particular limiting processes. Jiří Rosický's research is associated with this wider categorical and structural programme. The historical significance lies in demonstrating that the abstract traditions of Czech mathematics continued to engage with frontier methodological questions. These questions demand collaboration with mathematicians specialising in logic, topology, and algebra. Their answers can transform the conceptual

architecture used to formulate subsequent problems. This intellectual influence should not be reduced to a catalogue of potential industrial applications.

Connections between category theory and computer science are often discussed through algebraic specifications and the formal description of processes. A mathematical specification identifies operations and laws that an implementation is expected to respect. Categorical descriptions can express how complex constructions are assembled and how transformations preserve their meaning. These methods provide tools for reasoning about programs and systems, although the existence of a mathematical framework does not by itself guarantee effective software. The distinction between semantic correctness and computational efficiency remains particularly important. An abstract structure may describe what a computation means without prescribing how quickly it can be carried out. Conversely, complexity-theoretic results may require a fine analysis of finite operations that is not captured by a general categorical description. Czech algebraic researchers engaged with different parts of this intellectual landscape through their collaborations and publications. Together, the approaches show how abstract theory can inform computing while retaining independent mathematical aims.

Scientific visibility is another important dimension of the history of Czech algebra after 1993. Mathematical research is evaluated partly through the originality of proofs and the durability of their influence upon later work. Publications in international journals establish claims in a form that can be checked, criticised, and extended by other mathematicians. Books and extended surveys serve a different function by consolidating a subject's terminology and exposing its main ideas to new readers. The publication history of Adámek and his collaborators illustrates the latter role in category theory. The articles of Barto and Kozik illustrate the role of original research papers in developing specific algebraic results. Rosický's long publication record points to sustained engagement with advanced categorical questions across changing research contexts. These examples should not be treated as interchangeable kinds of achievement. Each contributes differently to the creation, transmission, and refinement of mathematical knowledge. A credible national history must recognise these different functions without ranking them through superficial numerical comparisons.

The international character of the field complicates simple descriptions of national achievement. A researcher may work at a Czech university while co-authoring work with mathematicians employed in several other countries. Ideas may arise through years of discussion at conferences and visits whose intellectual traces are not fully visible in published affiliation lists. Conversely, an influential book may be rooted in earlier work carried out before the period selected for historical analysis. This makes attribution both necessary and delicate. For the years 1993–2025, it is appropriate to examine work associated with Czech institutions while identifying international co-authors accurately. It would be misleading to present collaborative theorems as discoveries made exclusively by one national research system. It would be equally misleading to overlook the contribution of Czech scholars merely because their collaborators were abroad. The strength of the research environment is visible in its ability to participate meaningfully in shared mathematical problems.

The development of algebraic research depended less on spectacular physical infrastructure than on human and institutional continuity. Advanced mathematical work requires access to

literature, opportunities for discussion, reliable teaching programmes, and time for sustained concentration. Stable departments and seminar communities therefore function as a form of research infrastructure in their own right. Competitive funding can support travel, visitors, doctoral posts, and collaborative projects, even when the research itself needs relatively modest equipment. Yet the same dependence on academic labour exposes pure mathematics to pressures involving short project cycles and insecure early-career positions. A research programme in category theory or universal algebra may take many years before its broader significance becomes apparent. Institutional evaluation must therefore distinguish enduring conceptual progress from immediate numerical outputs. The Czech experience shows how continuity in specialist schools can coexist with adaptation to European research frameworks. Its history also reveals why protecting foundational investigation remains important for future interdisciplinary developments. Long-term continuity can consequently be as valuable as the establishment of a new centre.

Doctoral training served as an important mechanism for transmitting abstract mathematics between generations. Learning to work with universal properties, algebraic identities, and sophisticated categorical arguments requires more than mastering definitions from a textbook. Young mathematicians must also acquire habits of conjecture formation, counterexample construction, and critical reading of proofs. Seminar discussion is especially valuable because it exposes hidden assumptions that polished publications sometimes leave implicit. International doctoral mobility can introduce students to complementary styles of argument and new research communities. The professional development of Czech mathematics after 1993 provided increasing opportunities for such encounters. However, mobility does not automatically create a successful research school without continuity of supervision at home. The transmission of expertise therefore depends upon the relationship between established specialists, visiting collaborators, and locally trained students. Within algebraic disciplines, this relationship helped maintain intellectual depth while accommodating emerging computational themes.

The period from 2020 to 2025 demonstrated the continued vitality of structural questions in a rapidly changing computational world. Public attention to artificial intelligence sometimes encouraged the impression that mathematics had become primarily a source of algorithms and data-processing techniques. Algebra and category theory offer a reminder that mathematical foundations remain essential even when their practical relevance is indirect. Fundamental questions about operations, morphisms, models, and logical structure did not disappear with the expansion of machine learning. Rather, computational developments gave some of those questions new contexts and new interlocutors. Research associated with Czech mathematicians continued to appear in international discussions of categorical and algebraic topics during these years. The retrospective boundary of 2025 should nevertheless be respected when selecting achievements for this book. Later publications may clarify an earlier research trajectory, but they cannot be counted as results already available by the endpoint. This chronological discipline is indispensable for an accurate account of scientific development. That approach prevents retrospective narratives from attributing later achievements to earlier years.

By the end of the period, Czech research in algebra and abstract structures had developed through several mutually reinforcing pathways. Classical structural ideas remained important as subjects of fundamental mathematical investigation. Category theory provided a language for comparing constructions across algebra, topology, and logical foundations. Universal algebra became a particularly productive source of methods for understanding finite computational constraints. Research communities in Prague and Brno preserved distinctive specialisations while participating in increasingly international networks. The scientific achievements considered here arose from rigorous theorem proving rather than from the mere adoption of fashionable terminology. Their significance lies partly in the way new conceptual descriptions made previously difficult relationships intelligible. The resulting work illustrates a form of innovation that transforms the principles through which mathematical problems are posed. It also demonstrates how national scientific traditions can remain recognisable without being intellectually isolated.

The history of Czech algebra between 1993 and 2025 consequently reveals an intellectual transformation built upon continuity rather than abrupt replacement. The inherited traditions of structural mathematics supplied a foundation for advanced categorical research and interaction with mathematical logic. New connections with theoretical computer science demonstrated that algebraic methods could explain the behaviour of broad algorithmic problem classes. Contributions associated with Jiří Adámek, Jiří Rosický, Libor Barto, and their collaborators illustrate several distinct forms of scholarly achievement. These include the systematic exposition of categorical concepts, the refinement of foundational structural theories, and the discovery of algebraic conditions relevant to computation. No single example exhausts the achievements of Czech algebra, and each must be understood within its proper international context. Taken together, they show that a small but well-connected mathematical research environment can influence fundamental questions well beyond its national borders. The lasting value of this development lies in the precision and transferability of the mathematical ideas themselves. Abstract algebra remained a field of independent theoretical inquiry while increasingly contributing conceptual tools to logic, geometry, and the study of computation. That dual capacity is among the defining features of its modern Czech history.

Geometry and Topology: From Classical Mathematical Structures to Contemporary Geometric Research

Geometric research in the Czech Republic after 1993 developed from traditions that were already mature before political independence. The intellectual inheritance included differential geometry, topology, projective methods, and the study of geometric structures connected with mechanics. Universities in Prague and Brno provided settings in which these lines of investigation could be preserved while their international horizons widened. The change in national institutions did not transform established theorems into obsolete knowledge. Instead,

it altered the professional conditions under which mathematicians could extend earlier results and compare their methods with those of colleagues abroad. Geometry offered an especially instructive example because its principal questions were both historically rooted and unusually receptive to new algebraic and analytical techniques. A geometric structure could be studied locally through coordinates, globally through topology, or abstractly through the symmetries governing admissible transformations. The coexistence of these viewpoints helped Czech researchers work across disciplinary boundaries without relinquishing mathematical precision. By the opening years of the twenty-first century, international cooperation had become a routine means of developing this inherited intellectual capital.

Contemporary geometry should not be reduced to the visual description of shapes in ordinary three-dimensional space. Differential geometry analyses manifolds and the structures carried by their tangent spaces, connections, and curvature. Topology asks which features survive continuous deformation and which global properties cannot be inferred from local measurements alone. Algebraic geometry studies spaces described by polynomial equations and their generalisations, connecting geometric objects with rings, schemes, and sheaves. These areas overlap but do not constitute interchangeable methods. An invariant arising in one language may be translated into another only when appropriate hypotheses and mathematical constructions are established. Czech research illustrates the importance of such disciplined translation, notably where representation theory and geometric analysis interact. The resulting innovations are frequently conceptual rather than technological. They broaden the classes of structures that mathematicians can characterise, classify, or compare. These distinctions prevent the conceptual scope of the discipline from being overstated.

A defining contribution at the beginning of the period was the 1993 monograph Natural Operations in Differential Geometry by Ivan Kolář, Peter Michor, and Jan Slovák. The work systematised the theory of natural bundles and natural operators, bringing together topics that had previously been dispersed through specialised papers. Its framework treats geometric constructions as operations compatible with the appropriate maps between manifolds. This requirement of naturality distinguishes intrinsic constructions from arbitrary coordinate-dependent formulae. The book also gives an extensive account of jets, connections, Lie derivatives, and bundle functors. Its appearance in the year of Czech independence should be read as evidence of established research maturity rather than as the sudden birth of a mathematical school. Kolář and Slovák were associated with Masaryk University in Brno, while Michor supplied an important international collaborative connection. The research programme connected differential geometry with category-theoretic thinking in a manner relevant to subsequent geometric investigations. It remains a useful point of departure for understanding the long-term intellectual continuity of the Brno school.

The natural-operator approach addresses a subtle problem about the independence of mathematical constructions from the choice of coordinates. Two coordinate descriptions may represent the same geometric object, although their individual formulae appear markedly different. A meaningful geometric operation must respect the transformations that identify those descriptions. Functorial formulations make this requirement precise by assigning structures and compatible maps in a coherent way. Jet bundles provide a systematic means of recording finite-

order derivative information used in such constructions. Connections and covariant derivatives furnish additional mechanisms for relating local fields on a manifold. This mathematical machinery has consequences for the classification of invariant differential operators and for geometric formulations of physical laws. Its significance lies not merely in the ability to introduce new notation but in proving which constructions are possible under specified assumptions. Research cultivated in Brno placed these foundational questions within an international programme of modern differential geometry. Its implications extend beyond any single choice of mathematical coordinates.

Cartan geometry provides a general language for structures that are modelled locally on homogeneous spaces. It combines a principal bundle with a Cartan connection, allowing curvature to measure departures from the chosen homogeneous model. Many familiar geometric settings can consequently be examined within a shared structural vocabulary. The theory is powerful precisely because it does not claim that every geometric problem has the same answer. Rather, it permits the transfer of suitably formulated questions between examples with related underlying algebraic data. Czech work on these ideas was particularly visible in research associated with Jan Slovák and his international collaborators. Connections with Lie groups and representation theory supplied analytical methods for studying invariant operators and differential complexes. These tools were developed over successive publications rather than through a single isolated discovery. Their cumulative character helps explain the growing international standing of the Czech geometric research community.

Parabolic geometries form a distinguished class of Cartan geometries related to semisimple Lie groups and parabolic subgroups. They encompass conformal and projective structures as well as several more specialised geometric configurations. This breadth makes them a particularly useful setting for investigating the common algebraic foundations of apparently different geometric theories. Jan Slovák and Andreas Čap developed a systematic account in their 2009 monograph Parabolic Geometries I: Background and General Theory. The volume brought together foundational geometry, representation-theoretic constructions, and methods for understanding curvature and invariant operators. It was an international collaboration involving a Czech mathematician rather than a result that should be attributed exclusively to one national school. The distinction matters because mathematical research ordinarily advances through networks of authors, institutions, and students. For the Brno school, the monograph documented a sustained programme reaching well beyond the post-1993 institutional transition. Its influence is best understood through the subsequent research directions it enabled and the precise mathematical methods it articulated. The relevant publications provide a stronger basis for attribution than institutional reputation alone.

Weyl structures offer a useful means of studying parabolic geometries through choices that reveal more familiar differential-geometric data. Such choices do not eliminate the invariant underlying geometry, even though they change the form in which some associated objects are expressed. The distinction between an invariant and its description under a chosen splitting is central to modern geometric calculation. Čap and Slovák examined this framework in their 2003 paper Weyl Structures for Parabolic Geometries. That contribution situated practical calculations within an abstract theory rather than treating them as unrelated technical

manipulations. It also connected geometric structures with invariant differential operators and the interpretation of curvature. The research illustrates how conceptual clarification can enhance the effectiveness of subsequent mathematical work. It did not depend on new experimental equipment or computational scale. Its innovation lay in making difficult geometric relationships mathematically tractable.

Geometric research also concerns distinguished curves and the transformations preserving their defining conditions. In classical geometry, geodesics provide the most familiar example of curves associated with a connection or metric. In more general settings, however, different structures select different families of curves and require more elaborate invariance conditions. Čap, Slovák, and Vojtěch Žádník investigated distinguished curves in parabolic geometries in a 2004 publication. The question is fundamental because curves can encode important information about the larger geometric environment. Careful classification reveals when an apparently natural family is preserved under an admissible change of geometric description. It also distinguishes intrinsic geometry from an artefact of the chosen local representation. Such results are relevant to mathematical physics wherever trajectories and symmetry structures are studied geometrically. They demonstrate how Czech-linked research contributed to general theory through precise questions about transformations. A precise reading of the original publications is therefore indispensable.

Another research theme examines the relationships between different geometries rather than the properties of a single example. A morphism must preserve the structural information prescribed by the relevant category of geometric objects. An inclusion may permit one geometric model to be realised within another, but this requires compatible algebraic and differential data. Research by Boris Doubrov and Jan Slovák on inclusions between parabolic geometries appeared in 2010. The topic is conceptually significant because it seeks rigorous links between families of geometric structures. It helps identify the circumstances under which methods developed for one geometry remain applicable to another. Such work brings together the geometrical idea of structure preservation and the algebraic language of representations. For researchers trained in Brno, it extended themes already evident in the foundational literature of natural operations. The cumulative programme therefore shows continuity within a changing international research landscape.

Algebraic geometry requires careful treatment within a survey of Czech mathematical science because institutional participation is not itself evidence of a nationally distinctive theorem. The field studies solution sets of polynomial equations and more general algebraic spaces through their structural invariants. Its modern language includes varieties, schemes, morphisms, and algebraic constructions arising from commutative algebra. These methods interact fruitfully with complex geometry, topology, and representation theory. In the Czech research environment, they also appear as intellectual tools within neighbouring investigations of geometric structures. A historical account must therefore distinguish formal instruction, collaborative use of algebraic methods, and original contributions to algebraic geometry. The available documented examples for this section establish especially strong activity in differential and parabolic geometry. They do not justify inventing a separate sequence of nationally attributed algebraic-geometric breakthroughs. The appropriate conclusion is that algebraic methods formed part of the broader

methodological repertoire while individual claims require publication-level verification. Responsible historical attribution requires this restraint.

Modern geometric investigation often depends upon the interaction between algebraic classifications and analytical equations. A Lie algebra, for example, encodes infinitesimal symmetry in a form that can be manipulated through representation theory. Differential operators acting on sections of bundles may then be studied using those representation-theoretic structures. The resulting constructions can reveal constraints that are difficult to detect by direct coordinate calculation. Conversely, an analytical solution may provide geometric meaning to an abstract algebraic object. This mutual dependence is visible in the methods developed for parabolic geometries and invariant operators. It does not mean that algebra, analysis, and geometry have ceased to be distinct disciplines. Rather, each supplies conditions that enable the other fields to pose more precise questions. Czech geometric research became internationally visible partly through its capacity to operate across these methodological boundaries.

Topology introduces a global perspective that cannot be replaced by local differentiation or algebraic manipulation alone. A manifold can look locally like Euclidean space while possessing global features that no single coordinate chart captures. Connectivity, compactness, orientability, and homotopy-related properties illustrate different levels of this distinction. Differential-geometric constructions are often constrained by such topological conditions. A connection may be defined locally, but global existence and classification questions may require further structural information. Accordingly, topology underlies much of the broader geometrical programme even when a particular publication uses the language of connections or bundles. For Czech mathematical institutions, preserving teaching and research across these areas helped maintain the foundations of advanced geometry. A historical synthesis should nevertheless avoid presenting all topological theory as a direct result of Czech research. Its proper task is to explain where topological questions genuinely intersected with identifiable research programmes. These qualifications are essential to a balanced history of the field.

Fibre bundles express an essential idea in contemporary geometry: structured mathematical objects can vary continuously over a base space. The local triviality of a bundle does not necessarily imply a globally trivial organisation. This difference gives bundle theory much of its explanatory power in differential geometry and mathematical physics. Connections specify how information may be compared along the base manifold, while curvature records an aspect of their non-triviality. Natural bundle constructions further ask how such objects behave under suitable transformations of manifolds. The 1993 monograph by Kolář, Michor, and Slovák provided a substantial treatment of these themes. Their systematic formulation helped connect local calculations with global geometric architecture. The same language is useful for gauge theories and for geometric approaches to field equations, although specific physical applications require additional modelling assumptions. This conceptual bridge shows how foundational geometry can retain relevance across disciplinary changes.

Invariants are indispensable because they distinguish meaningful properties from accidental features of a representation. A useful invariant remains unchanged under an explicitly defined class of transformations. Curvature tensors, characteristic quantities, and differential operators

each address different aspects of geometric structure. Their definitions require care because a quantity invariant under one transformation group may change under another. The study of parabolic geometries creates a systematic framework for identifying such differences. This research often proceeds by translating geometric questions into algebraic ones and then reconstructing their intrinsic significance. Jan Slovák and collaborators contributed to this programme through work on invariant calculus and geometric structures. The achievement consists in developing reproducible conceptual tools rather than presenting a catalogue of unrelated formulae. It is this form of theoretical innovation that is particularly important in a history of mathematics. Such methods continued to influence the formulation of new research questions.

Josef Janyška represents another identifiable line of differential-geometric research at Masaryk University. His publication record includes sustained investigations of natural differential operators, tensor fields, and geometric questions related to mechanics. A 2019 article, Remarks on Natural Differential Operators with Tensor Fields, provides a concrete example within the period surveyed. The title itself identifies the problem of defining and analysing operations compatible with geometric structure. Such research belongs to the same broad intellectual tradition as the theory of natural operations while retaining its own technical questions. It demonstrates that a mathematical school survives through continued problem-solving rather than through perpetual repetition of an influential monograph. Connections to covariant formulations of mechanics also show how abstract differential geometry can inform the formal language of physical theories. These links must not be confused with proof that every abstract construction produced an immediate technological application. Their value lies first in the clarity and generality of the mathematical formulations.

Geometric methods become particularly suggestive when the physical description itself should be independent of an arbitrary choice of coordinates. Covariant formulations of quantum mechanics seek mathematical structures compatible with this requirement. The geometry may involve bundles, connections, and operations describing transformations of states and observables. Josef Janyška and collaborators published work on the quantum potential in covariant quantum mechanics in 2017. Their research exemplifies an interaction between differential geometry and foundational aspects of physical modelling. The mathematical task is to maintain internal consistency while making the transformation laws explicit. It is inappropriate to claim that such investigations constitute experimental verification of a physical theory. Instead, they enrich the available mathematical descriptions through which physical problems can be formulated and compared. This distinction between formal innovation and empirical discovery is essential to evaluating achievements in mathematical physics. This underscores the relationship between formal rigour and physical interpretation.

Quantum graphs provide a further intersection between geometry, spectral theory, and mathematical physics. They combine an underlying graph with differential operators acting along edges and conditions imposed at vertices. The geometry and topology of the graph can influence the spectral properties of the resulting operators. Pavel Exner and co-authors investigated infinite quantum graphs in a 2018 study published in Annales Henri Poincaré. Their analysis related spectral questions for such graphs to properties of a weighted discrete Laplacian. This

example is particularly instructive because discrete geometry interacts directly with continuous differential operators. It would be misleading to describe the work as a purely topological contribution or as the invention of quantum graphs by Czech researchers. Its significance is a rigorously formulated advance within an international programme to understand graph-based quantum systems. Such achievements demonstrate the breadth of geometric mathematical physics connected with Czech institutions.

Geometric structures often determine the form of the differential equations used to investigate them. Conversely, the solutions of differential equations can reveal geometric rigidity, curvature constraints, or possible deformations. This reciprocal relationship is prominent in conformal geometry, geometric control, and invariant differential operators. Analytical methods may prove existence or regularity, whereas geometrical arguments identify the intrinsic form of the problem. Neither mode of reasoning can always replace the other. Research programmes associated with Jan Slovák integrated geometric analysis and representation-theoretic techniques with such questions. Their intellectual reach extended across the conventional division between pure geometry and applied analysis. This offers a stronger historical explanation of interdisciplinarity than a generic assertion that mathematics became more collaborative. The relevant novelty lies in clearly identified methods for relating different mathematical descriptions. The connection is conceptual even where practical implementations differ.

Sub-Riemannian geometry studies spaces in which admissible motion is restricted to specified tangent directions rather than all possible directions. Its mathematical structure is therefore closely connected with distributions, control systems, and constrained curves. These problems can be analysed using differential geometry, optimisation, and geometric control theory. Work involving Jan Slovák and collaborators addressed relationships between sub-Riemannian metrics and parabolic geometries in a 2021 publication. The research demonstrates how the language of geometric structures can contribute to apparently different questions about constrained movement. Rigorous statements depend on the precise regularity and structural assumptions of the systems under investigation. A historical survey should not infer practical outcomes merely from the presence of terms such as control or optimisation. Rather, it should document how abstract methods created a framework for subsequent mathematical and applied investigation. This is a measured account of the contemporary extension of the Brno geometric school.

Geometric control theory investigates systems whose permissible trajectories are governed by differential constraints. The geometry of the admissible directions shapes the analysis of possible motions and extremal paths. This provides a reason for studying non-holonomic systems through intrinsic geometric tools. A 2022 paper by Rod Gover and Jan Slovák examined non-holonomic equations for normal extremals in geometric control theory. The research illustrates how geometric analysis can clarify the equations associated with constrained optimisation. It does not eliminate the need for numerical techniques when explicit solutions are inaccessible. Instead, intrinsic formulations can indicate which features of a model are structurally meaningful before computation begins. The combination of geometric theory and applied mathematical questions became more visible during the later decades of the period. It

also reinforced the continuity between the abstract theory of connections and new problems in dynamics. Such formulations clarify both the possibilities and limits of a model.

The use of geometric methods in medical imaging illustrates a distinct path from foundational theory towards application. Diffusion magnetic resonance imaging requires mathematical descriptions of directional information in biological tissue. Geometric models can help represent families of possible paths and the anisotropic character of the data. Research involving Jan Slovák and collaborators explored geodesic fibre tracking in white matter in a 2021 publication. Further work in 2023 investigated automatic tractography and segmentation using Finsler-geometric approaches. These studies demonstrate that sophisticated geometric frameworks can participate in computationally demanding scientific problems. They should nevertheless be assessed as research developments rather than as universally validated clinical procedures. The distinction preserves scientific accuracy while recognising the expansion of geometric thinking into interdisciplinary analysis. For Czech mathematics, this activity illustrates how abstract expertise can generate new methodological possibilities.

Conformal geometry studies properties preserved under angle-preserving changes of metric, whereas projective geometry examines suitable classes of unparametrised curves. These fields are related through wider families of geometric structures, but they solve different mathematical problems. Parabolic geometry offers a general theory capable of encompassing important examples of both. The work of Čap and Slovák consequently connected established geometric subjects through a shared representation-theoretic framework. Such unification is valuable when it identifies genuine correspondences rather than superficial resemblances. It also gives researchers a language for constructing differential invariants and investigating curvature. The international circulation of these methods contributed to the visibility of Brno-based mathematical research. The historical achievement is therefore located in the development of a powerful theoretical architecture. Its consequences extended beyond the particular examples with which the theory first became associated. Theoretical unification here should be understood as a tool, not an end in itself.

The evolution of research schools also depends upon professional institutions that circulate new ideas. Specialised journals and recurring conferences provide forums where definitions, proof techniques, and unresolved problems can be scrutinised. Brno maintained a visible tradition of differential-geometric meetings, including the 1995 conference reflected in the 1996 proceedings edited by Janyška, Kolář, and Slovák. Such events helped link local mathematical traditions with an international network of researchers. Their significance should not be measured only by participant counts or the existence of printed proceedings. They enabled sustained discussion across neighbouring approaches to geometry and mathematical physics. Conference exchanges are especially useful in abstract mathematics, where new definitions often require extensive critical examination. The resulting intellectual relationships can persist well beyond the event at which they began. This collaborative infrastructure formed part of the background to later Czech-linked publications.

A mathematical research school is reproduced through supervision, seminars, and the transmission of difficult proof techniques. Advanced geometrical research demands command of differential equations, topology, representation theory, and abstract algebra. No single

introductory course can fully prepare a researcher for work across all these areas. Doctoral training consequently involves learning how to connect specialised questions with a larger theoretical framework. Masaryk University records doctoral supervision associated with Jan Slovák in geometric analysis, invariant theory, and geometric applications. These records establish an educational environment in which advanced research topics could be pursued across generations. They do not by themselves prove the independent international impact of each dissertation. Nevertheless, they demonstrate an important condition for the continuity of a mathematical school beyond its leading authors. The relation between research and education is thus an integral part of the intellectual history of Czech geometry. Its outcomes depend on sustained expertise rather than one-off institutional announcements.

After Czech accession to the European Union in 2004, mathematical researchers operated within an increasingly integrated European scientific environment. Geometry benefited from the movement of scholars, collaborative seminars, international publication, and participation in wider research projects. Unlike experimental science, the essential resources of geometric research are often conceptual expertise and sustained working time. Nevertheless, institutional stability and grant support are important for academic appointments, doctoral positions, travel, and scholarly meetings. The record of Czech-linked publications shows how locally based researchers worked with colleagues from Austria and other countries. Such collaboration should be recognised without attributing all jointly produced results to a single national tradition. The international character of mathematical authorship is itself one of the defining features of the period. It also creates a need for careful distinctions between a scientist's affiliation, intellectual background, and the institutional location of particular research. A credible national history of mathematics is strengthened, not weakened, by acknowledging these complexities.

The boundaries between geometry, topology, algebra, and mathematical physics are often porous. A theory of geometric structures may use category theory, differential operators, and Lie algebra cohomology within the same proof. A quantum-graph problem may combine graph topology with spectral analysis and operator theory. An imaging method may employ differential geometry alongside numerical optimisation and statistical estimation. Assigning each result to one discipline can therefore obscure its actual intellectual foundations. At the same time, classifying every interdisciplinary contribution as a breakthrough in all participating fields would inflate the historical record. The more reliable method is to identify the specific theorem, model, or construction and then explain its connections. This approach is particularly important when assessing the achievements of a research system smaller than the largest international mathematical centres. Precision in attribution allows genuine contributions to stand out without rhetorical exaggeration. Their overlap should be investigated rather than assumed.

By 2025, Czech geometric research displayed both fidelity to established theoretical questions and interest in newer applications. Natural operations, invariant differential operators, and parabolic structures remained active areas of scholarly discussion. Geometric control and mathematical imaging demonstrated the possibility of extending these ideas beyond their initial abstract setting. The development was neither uniform across institutions nor reducible to a single technological transformation. Instead, different research teams pursued compatible but distinct lines of mathematical enquiry. International publication and doctoral research helped

connect foundational investigations with new questions arising elsewhere in science. The most important historical continuity lay in the capacity to translate between geometric structures while preserving rigorous definitions. Such mathematical work expands the theoretical vocabulary available to future researchers, even where applications remain uncertain. Its long-term significance is therefore intellectual as well as institutional.

The record of Czech geometry and topology between 1993 and 2025 is best understood as a layered history of accumulated methods and carefully documented contributions. The Brno tradition of natural differential geometry provided a substantial starting point at the beginning of the period. Research on parabolic geometries and invariant operators subsequently developed this inheritance in productive international collaborations. Work on natural differential operators, covariant mechanics, quantum graphs, and geometric control showed the diversity of related methodological trajectories. The role of algebraic geometry and topology should be acknowledged through their substantive mathematical connections rather than embellished with unsupported national claims. Universities, doctoral programmes, conferences, and scientific publications furnished the organisational environment in which these investigations could continue. The history also demonstrates why mathematical innovation cannot be assessed exclusively through patents, equipment, or immediate industrial transfer. New definitions, classification methods, invariant constructions, and rigorous connections between theories are achievements in their own right. By 2025, Czech-linked research had contributed meaningfully to this broader international development of contemporary geometric mathematics. This perspective establishes the historical framework for the subsequent analysis.

Mathematical Logic, Set Theory, and Foundations of Mathematics

The development of mathematical logic in the Czech Republic after 1993 rested upon intellectual traditions which long predated the establishment of an independent state. Czech and Czechoslovak researchers had already contributed to recursion theory, proof theory, model theory, set theory and the logical analysis of mathematical reasoning. The democratic transition after 1989 altered the conditions of scholarly exchange, while the constitutional changes of 1993 created a new institutional setting. These events did not replace inherited theories with a new national logic, but they changed the circumstances in which research communities could develop them. Universities and Academy institutes continued to provide the principal settings for advanced investigations of formal systems. The international nature of logic made publication, visiting appointments and cross-border seminars particularly important to the field. Logical results were judged through the validity of proofs and the interest of the problems they solved rather than through national attribution. Nevertheless, the location of research groups and the continuity of their seminars strongly influenced which questions were pursued over successive generations. The Czech experience therefore offers a revealing example of intellectual continuity within a rapidly transformed research environment.

A historical account of Czech logic requires several distinctions that are sometimes obscured in broad surveys of mathematical innovation. Mathematical logic investigates the formal properties of inference, while set theory studies collections and their foundational relationships. Proof theory asks what may be derived in a formal calculus and how the complexity of derivations can be analysed. Model theory considers interpretations of languages and the correspondence between syntactic expression and mathematical structure. Theoretical computer science overlaps with these areas whenever algorithms, verification and computational complexity depend upon formal reasoning. Fuzzy logic constitutes a particular family of many-valued approaches rather than a synonym for every form of uncertain reasoning. These subjects share techniques, yet their research questions and criteria of success are not interchangeable. A national history should therefore identify particular schools, authors and publications rather than presenting all mathematical formalisation as a single innovation. This distinction is especially important when discussing the work of Petr Hájek and the researchers who subsequently extended aspects of his programme. It also prevents the practical popularity of fuzzy systems from being mistaken for evidence of a specific theorem in mathematical logic.

One of the most influential landmarks at the beginning of the period was the 1993 monograph Metamathematics of First-Order Arithmetic by Petr Hájek and Pavel Pudlák. The book developed a systematic account of formal arithmetic, its fragments and the logical strength of their deductive principles. It examined the relationship between definability, induction, incompleteness and the extent of reasoning available in restricted formal theories. A central problem in the subject concerns the difference between a mathematical statement being true in an intended structure and being provable within a selected axiomatic system. Another concerns the extent to which computational complexity can illuminate the limitations of short proofs. By discussing bounded arithmetic, the authors situated foundational questions in direct contact with computational reasoning. Their contribution was not the invention of first-order arithmetic, whose foundations were much older, but a comprehensive organisation and development of its metamathematical theory. The volume provided a reference point for researchers studying arithmetic theories, proof complexity and the limits of formal derivation. Its publication in 1993 also illustrates why the intellectual chronology of mathematics does not coincide neatly with the political chronology of the Czech Republic.

Hájek's subsequent work became particularly visible through a second major research programme concerning mathematical fuzzy logic. In ordinary language, vagueness is often described through examples such as predicates whose application admits borderline cases. A mathematical treatment, however, must specify a language, semantic values, operations and formal rules of consequence. The use of more than two truth values does not by itself guarantee a satisfactory deductive calculus. Researchers must establish which formulas are valid, which inferences are sound and whether relevant completeness theorems can be proved. Algebraic semantics supplies structures in which logical connectives are interpreted by operations obeying precise laws. These mathematical questions distinguish logical foundations from the engineering practice of assigning graded values to a controller. Hájek helped develop a programme in which degrees of truth could be studied with the same formal seriousness as traditional two-valued logics. This approach allowed philosophical questions about vague

expressions to be placed within rigorous algebraic and proof-theoretic frameworks. Its implications would become particularly important for Czech research from the late 1990s onwards.

The publication of Hájek's Metamathematics of Fuzzy Logic in 1998 was a defining milestone of this programme. The monograph gave a systematic treatment of several real-valued propositional and predicate logics and their deductive properties. Rather than treating fuzzy inference as a collection of informal numerical conventions, it investigated logical systems whose semantics and proof rules could be compared precisely. Basic fuzzy logic and its related calculi supplied a framework within which different truth functions could be analysed. Completeness questions connected deductive systems with suitable classes of algebraic models. The book also clarified why differing formal interpretations of conjunction, implication and negation lead to distinct logical systems. Its enduring significance lies in making the metatheory itself a central object of mathematical research. It consequently became a point of reference beyond the Czech Republic for logicians concerned with many-valued reasoning. The work exemplifies an innovation in mathematical knowledge whose influence cannot be measured by laboratory equipment or industrial patents.

Algebraic methods were essential to the conceptual progress of fuzzy and other non-classical logics. An abstract algebra can represent how formulas combine without depending upon the everyday meaning of their component propositions. Lattices, ordered structures and residuated operations became important tools for investigating consequence relations. In suitable algebraic settings, implication and conjunction may be connected through a residuation principle. Such connections help explain why particular deductive rules are valid under specific semantic assumptions. The challenge is to determine which properties arise from the general algebraic architecture and which depend on a chosen logic. Czech-based researchers contributed to this wider effort by studying algebraic semantics and the boundaries between families of many-valued systems. The resulting work created channels of communication between universal algebra, mathematical logic and theoretical computing. Algebraic semantics therefore functioned as more than a convenient notation, because it supplied proofs and classifications unavailable from examples alone. The history of this subject illustrates the value of mathematical abstraction in unifying apparently different formal theories.

The continuation of this research tradition after 2000 can be seen in the work of Petr Cintula and his collaborators. Their investigations examined fuzzy logics as part of the broader landscape of algebraizable and non-classical consequence relations. An important methodological question was how to generalise familiar theorems without assuming that every logic behaves like classical propositional calculus. The study of implication became particularly significant because formal consequence depends upon the relationship between premises and conclusions. Researchers considered axiomatic systems, algebraic companions, completeness results and the structural properties of deduction. Work at Czech Academy institutions also involved international partners, demonstrating that a research school need not be confined to a single nationality. The resulting papers and monographs reached readers in logic, mathematics and philosophy. This intellectual succession should not be interpreted as the simple preservation of a teacher's

completed framework. Instead, it involved reformulating earlier questions and developing more general theories capable of accommodating different logical systems.

A visible expression of the field's maturity appeared in the multi-volume Handbook of Mathematical Fuzzy Logic edited by Petr Cintula, Petr Hájek and Carles Noguera. The first two volumes appeared in 2011, bringing together systematic treatments of the foundations and principal lines of investigation. Later editorial work extended the reference framework for researchers dealing with non-classical inference. Such handbooks are significant in the history of a discipline because they organise accumulated results and identify a vocabulary for subsequent investigations. They do not replace original articles, but they make complex research traditions more accessible to new generations. The collection demonstrated that mathematical fuzzy logic had become a substantial international field with distinct technical problems. Czech participation was visible in both the authorship and the organisation of this scholarly enterprise. The presence of contributors from different countries also cautions against describing the entire field as a national invention. Its relevance to the Czech experience lies in the country's sustained role in developing and consolidating the underlying theory. The handbook thus marked both the internationalisation and the institutional continuity of a recognisable research direction.

Set theory formed a related but analytically distinct component of Czech foundational mathematics. Its principal questions concern mathematical collections, infinite processes, cardinality, order and the independence of particular statements from accepted axioms. The field has close historical links with topology, descriptive set theory and the logical foundations of analysis. Czech traditions associated with mathematicians such as Bohuslav Balcar and Petr Simon provided an intellectual background for later work. Some of these contributions originated before 1993 and must not be retrospectively assigned to the independent Czech Republic. Their continuing relevance, however, shaped seminars, doctoral training and the transmission of specialised methods. Research on infinite combinatorics and topological constructions illustrates the interaction between set-theoretic techniques and other mathematical areas. The strongest historical account distinguishes the persistence of a school from the publication date of each theorem. This distinction makes it possible to discuss continuity without exaggerating the novelty of the 1990s.

The role of set theory also emerges in descriptive problems where mathematical objects must be classified according to definability or complexity. Researchers may ask whether a set belongs to a specified hierarchy and what regularity properties follow from that classification. Questions of this kind appear in real analysis, probability, topology and the theory of functions. The point is not to claim that every modern analytic result derives from set theory. Rather, foundational techniques offer languages for stating and resolving problems that arise elsewhere. Connections with infinitary combinatorics and the structure of function spaces were especially natural within the broader Czech mathematical environment. When the relevant results are attributed to researchers at universities or Academy institutes, their publication history must be checked individually. Institutional proximity should not be confused with a shared theorem or identical research agenda. The value of a nationally focused account lies in tracing documented

connections between people, institutions and mathematical questions. The development of these connections helped sustain the diversity of Czech foundational studies.

Proof theory provides another perspective on the transformation of mathematical reasoning into an object of rigorous investigation. A proof is not merely a convincing explanation, because in a formal setting its steps are governed by explicitly specified rules. Theories may differ in the induction principles, quantifier rules or axioms they permit. These differences affect what can be established inside the system and the resources required for a derivation. Research related to bounded arithmetic is especially revealing because restricted formal theories can be compared with classes of feasible computation. Such comparisons establish a bridge between foundational mathematics and computational complexity. The writings of Hájek and Pudlák provided a substantial point of departure for this analysis during the period under review. Subsequent international research continued to investigate proof length, definability and the expressive power of logical calculi. Czech foundational mathematics therefore participated in discussions that are simultaneously philosophical, mathematical and computational.

Formal incompleteness also shaped the intellectual setting in which Czech logicians worked. The classical incompleteness theorems demonstrated limitations of sufficiently expressive, consistent formal systems under specified mathematical assumptions. Their importance does not imply that mathematics is generally unreliable or that proofs are merely subjective conventions. On the contrary, identifying such limitations requires exact definitions of consistency, expressibility and derivability. The study of arithmetical theories examines how these properties change when different axioms or induction schemes are adopted. Metamathematics thus operates at a second level by analysing mathematical theories as structured objects. This perspective was developed in foundational textbooks, advanced seminars and specialised research publications. It also influenced questions of formal verification, where a computer-checked proof remains dependent upon the assumptions of the underlying calculus. The Czech contribution is best recognised through identifiable mathematical studies rather than through a general claim that the country discovered foundational limitations. Precision in attribution is essential when writing the history of international mathematical ideas.

The interaction of logic with computer science became increasingly significant as the Czech research system developed after 1993. Formal languages and logical calculi supplied techniques for expressing properties of programs, databases and computational processes. Decision problems then asked whether a machine could determine the validity or satisfiability of a formula under clearly stated conditions. Even when decidability is established, complexity analysis is needed to understand how demanding an algorithm may be. Non-classical logics introduce further questions because richer truth structures can change the computational properties of inference. Czech researchers worked within international communities where these issues connected mathematical logic to theoretical informatics. The Institute of Computer Science of the Czech Academy of Sciences became an important institutional setting for related research. Collaboration with universities and the Institute of Information Theory and Automation further strengthened the proximity of formal foundations and computational methods. These

developments should be understood as a convergence of scientific questions rather than a wholesale replacement of pure logic by application.

Knowledge representation offers a particularly instructive interface between formal logic and intelligent systems. A computational system requires some method of expressing propositions, relationships and restrictions on permissible conclusions. Logical formalisms make it possible to specify the conditions under which new conclusions follow from recorded knowledge. Classical methods often employ binary truth values, while other systems allow additional distinctions. Mathematical fuzzy logic can represent forms of gradual predication, although it does not automatically solve every problem involving incomplete or uncertain data. Probability, non-monotonic reasoning and paraconsistent approaches answer different kinds of questions and must not be conflated. Czech contributions to the logical treatment of vagueness helped clarify the theoretical options available to researchers designing formal representations. An important gain was the ability to evaluate whether a proposed inference procedure has a defensible semantic basis. This is a conceptual contribution to artificial intelligence rather than evidence that every Czech artificial intelligence system uses fuzzy logic. This distinction remains relevant when interpreting the period as a whole.

The analysis of vague predicates also reveals the philosophical dimension of mathematical research. Everyday terms often lack sharp boundaries, even when speakers use them competently in ordinary conversation. A formal model of vagueness must decide what counts as a truth value and how propositions composed from vague expressions should behave. Different many-valued logics embody different answers, so selecting a system is not a neutral technical step. Hájek's work helped demonstrate that the resulting choices can be subjected to precise mathematical scrutiny. The central achievement was not to make ordinary language entirely exact, but to formulate rigorous questions about possible logics of graded truth. International philosophical interest provided an additional audience for research conducted within Czech mathematical institutions. This cross-disciplinary visibility was supported by English-language publication and participation in international conferences. The field thus illustrates how abstract mathematical research can influence debates beyond mathematics without surrendering its formal character.

Institutional continuity played a decisive role in sustaining these research traditions. The Czech Academy of Sciences maintained specialised institutes after the dissolution of Czechoslovakia, while universities preserved departments and graduate programmes devoted to mathematics and informatics. The Institute of Computer Science supported investigations in logic and theoretical computer science that benefited from international cooperation. The Mathematical Institute provided a wider environment for foundational and neighbouring mathematical disciplines. Charles University linked logical research with programmes in mathematics and theoretical informatics, creating opportunities for student participation. Institutional descriptions must nevertheless be treated carefully because official names, structures and research groups changed over the decades. A scholar's affiliation at the time of one publication cannot automatically be assigned to every stage of a career. The historical importance of research institutes resides in their ability to sustain seminars, doctoral supervision and publication programmes over long periods. Such forms of continuity helped mathematical logic

retain depth amid changing funding conditions. This distinction remains relevant when interpreting the period as a whole.

The development of postgraduate education provided another important means of reproducing specialised knowledge. Mathematical logic demands sustained familiarity with formal languages, metatheorems and techniques that are rarely acquired through introductory courses alone. Doctoral students need to learn both the classical literature and the unresolved problems shaping contemporary research. Seminar discussions are especially useful because incomplete arguments and competing definitions can be examined before publication. International visits expose early-career researchers to other styles of proof and to new questions. The post-1993 opening of Czech universities made such exchanges increasingly ordinary components of academic life. A successful research group also depends on the possibility that young mathematicians will eventually establish independent directions. The work of later scholars in non-classical logic demonstrates how intellectual continuity may coexist with changes in problem selection and methodology. Training therefore belongs in the history of research achievement rather than being treated as merely an administrative activity.

International publication became essential to the visibility of Czech mathematical logic. Results in formal systems are normally presented through exact definitions, propositions, theorems and proofs. Peer review tests whether arguments are sound, whether assumptions are explicit and whether the advance over existing literature has been adequately established. Publishing in recognised mathematical and logical journals situates a result within an international conversation. Monographs perform a complementary role by developing a coherent theory across a larger body of work. The books associated with Hájek and his collaborators were particularly important because they offered extensive systematic accounts rather than isolated findings. International editorial collaboration further strengthened the movement of ideas between Czech institutions and research centres abroad. Nevertheless, a publication's international circulation must not be mistaken for proof that every claim within it is universally accepted. Responsible historical analysis distinguishes documented influence from unmeasured assertions of scientific dominance. This discipline of attribution is especially important in research fields whose achievements are primarily theoretical.

The period after 2010 brought further diversification of non-classical logic research. Petr Cintula and Carles Noguera investigated the general algebraic study of consequence and implication across different logical families. Their 2021 monograph Logic and Implication presented an extensive account of these topics and their relationships. Questions about completeness, first-order systems and algebraic semantics remained central to the programme. Related institutional publications also documented work on structural completeness and other properties of many-valued logics. The significance of these investigations is that they move beyond a single preferred calculus towards the comparison of broader classes of systems. Abstract results can show which theorems follow from a shared set of assumptions and where additional conditions are required. This enables greater conceptual economy and more reliable transfer of proof techniques between theories. Czech research environments provided part of the institutional basis for this work, while the scholarship itself remained international in personnel and readership.

Mathematical logic increasingly intersected with the study of computational reasoning under inconsistency and incomplete information. Inconsistent databases or conflicting statements may require formal systems that avoid the unrestricted consequences of classical contradiction. Paraconsistent logics offer one response by changing how consequence behaves in the presence of conflict. Other systems investigate modalities, uncertainty or gradations of logical strength. The existence of these research directions does not mean that they all solve the same problem. An appropriate calculus must be chosen according to the properties a representation is intended to capture. Publications associated with Czech research institutions have addressed such issues within broader programmes on non-classical inference. These studies demonstrate the continuing role of exact deductive analysis in discussions often framed in technological language. The value of such work lies in making assumptions visible and establishing what kinds of reasoning they permit. Its applications must be assessed separately from the correctness of the mathematical theory.

A noteworthy feature of foundational research is its potential to connect subjects that appear distant at first sight. Set-theoretic constructions may illuminate questions about functions and topological spaces. Algebraic semantics can transform a problem in logic into a problem about ordered algebraic structures. Proof theory can connect the expressive limits of arithmetic with the resources needed for computation. Formal verification brings deductive principles into contact with the reliability of software and hardware. These bridges do not erase disciplinary differences, because each field retains specialised problems and standards. The Czech mathematical environment provided opportunities for such connections through proximity among universities, Academy institutes and computational research groups. International collaborations extended these networks beyond national borders. The resulting pattern was not a simple progression from abstract theory to marketable technology.

Evaluation of Czech achievements in this domain must consider the distinctive timescale of theoretical mathematics. A fundamental monograph may shape a field for decades even when it produces no immediate industrial outcome. The establishment of a new completeness theorem may matter because it changes the set of systems mathematicians can understand rigorously. An algebraic representation theorem may reveal an unexpected equivalence between seemingly unrelated constructions. Such results are difficult to assess through infrastructure statistics or patent counts. Their significance is better studied through subsequent mathematical publications, the development of research programmes and their integration into scholarly teaching. Hájek's 1998 monograph provides a clear example of a work whose enduring role concerns theoretical foundations. Later handbooks and general algebraic studies demonstrate that subsequent scholars continued to develop related questions. At the same time, citation counts and institutional prestige alone cannot determine the full mathematical importance of a result. A careful history must combine bibliographic evidence with substantive explanation of the underlying mathematical problems.

The geographical structure of Czech mathematical research also complicates any account centred on one institution. Prague has been a major centre because it brings together Charles University and several institutes of the Czech Academy of Sciences. Brno possesses longstanding mathematical traditions and university research in algebra, geometry and

mathematical analysis. Other regional universities contribute to mathematics and theoretical informatics through their own specialisations. Logical research may occur within a mathematics department, a computer science institute or an interdisciplinary laboratory. The identity of a scientific school consequently cannot always be derived from an administrative title. Personal networks, doctoral lineages and shared research problems frequently cross institutional boundaries. The field's international character further weakens the assumption that scholarly collaboration follows political borders. Czech mathematics after 1993 is therefore best understood as a set of interconnected research communities rather than a collection of isolated national laboratories.

The years 2020 to 2025 brought renewed attention to the foundations of intelligent computational systems. Advances in machine learning encouraged wider discussion of explainability, reliability and the representation of complex information. Mathematical logic offered relevant languages for specifying constraints and investigating formal inference. Nevertheless, contemporary machine learning is not simply an application of traditional proof theory, and many systems rely on statistical optimisation rather than deductive search. Distinguishing these approaches is necessary to avoid overstating the direct technological effects of Czech foundational research. Work in mathematical fuzzy logic remained important as part of the wider investigation of graded predicates and algebraic consequence. Research on non-classical systems likewise contributed formal tools for reasoning about incompatible or incomplete information. These theoretical developments belong to the broader intellectual background of artificial intelligence, but particular applications need their own evidence. The strongest account therefore presents logic as a continuing source of conceptual and methodological resources rather than as a universal explanation for technological change. This distinction remains relevant when interpreting the period as a whole.

The specific contribution of Petr Hájek deserves a balanced historical assessment. His work on arithmetic with Pavel Pudlák connected formal reasoning to important questions of incompleteness and computational complexity. His later account of fuzzy logic developed a systematic setting for understanding many-valued deduction. Together, these books reveal a mathematician concerned with the general possibilities and limitations of formal systems. They also illustrate how one research career can bridge subjects that are usually separated in introductory classifications. The importance of his research should not be reduced to a claim that he personally invented fuzzy logic, which had significant earlier international traditions. Rather, his achievement was to develop a particularly influential mathematical treatment of its deductive foundations. Later scholars extended these ideas, tested their scope and placed them in more general algebraic frameworks. The history of Czech logic is consequently a history of collective intellectual development in which exceptional individual contributions remain identifiable.

The Czech experience between 1993 and 2025 shows that innovation in mathematics often consists in improving the conceptual architecture of knowledge. A new system of axioms can alter what is formally expressible and provable. A completeness result can connect proofs with models and thereby establish the reliability of a deductive calculus. An algebraic generalisation can reveal the structural assumptions shared by apparently different logics. Research on

arithmetic can expose the relationship between formal derivation and computational resources. Set-theoretic and topological techniques can support further investigations in analysis and mathematical structure. Institutions contribute to these developments by preserving intellectual traditions while allowing new questions to emerge. The work of Hájek, Pudlák, Cintula and their collaborators provides documented examples of this cumulative transformation. These contributions are best assessed within the international scientific environment that shaped their formulation, criticism and reception. Mathematical logic in contemporary Czechia thus demonstrates how rigorous abstract reasoning can remain a vital part of a modern research ecosystem.

Combinatorics, Graph Theory, and Discrete Mathematical Structures

The development of combinatorics in the Czech Republic between 1993 and 2025 is best understood as an intellectual transformation grounded in much older mathematical traditions. Its central concerns included the organisation of finite structures, the existence of regular patterns, and the behaviour of mappings between discrete objects. These questions acquired particular force as computer science made the precise structure of finite systems increasingly important. Czech researchers did not merely apply established graph theory to new computational problems, but participated in constructing its theoretical language. The work associated with Jaroslav Nešetřil at Charles University became internationally influential because it connected combinatorial arguments with abstract structural reasoning. At the same time, distinct research communities in Prague, Brno, and other centres explored algorithms, extremal problems, and related mathematical structures. The period therefore presents an example of continuity through conceptual enlargement rather than replacement of classical mathematical subjects. A historical assessment must distinguish achievements originating in Czech research groups from the contributions of their international collaborators. Such distinctions illuminate the reciprocal character of modern mathematical scholarship rather than diminish the importance of collaborative discovery.

Czech discrete mathematics entered the independent-state period with scholarly connections inherited from Czechoslovak university departments and longstanding international research relationships. Earlier work in combinatorial theory had already established problems and methods upon which later investigations could build. The political transformation after 1989 facilitated broader circulation of researchers, publications, and mathematical ideas. Independence in 1993 changed administrative arrangements but did not sever the intellectual continuity of the mathematical community. The Faculty of Mathematics and Physics at Charles University offered an institutional setting in which young researchers could combine rigorous training with advanced investigation. Its Department of Applied Mathematics contributed particularly to the interaction of combinatorics and theoretical computer science. Research seminars and collaborative projects helped turn individual expertise into a more durable scientific school. This process depended less on specialised equipment than on the

concentration of intellectual talent and sustained opportunities for scholarly exchange. The distinction is important because infrastructure in pure mathematics often takes the form of libraries, graduate programmes, research visits, and international networks. Such resources proved essential for maintaining mathematical creativity during the institutional adjustments of the 1990s.

Combinatorics studies finite arrangements, yet its most powerful conclusions frequently concern entire infinite families of possible structures. Graph theory provides one of its principal languages by representing relationships through vertices and edges. A graph may model a network, but its mathematical significance does not depend upon a particular physical interpretation. Questions about colouring, connectivity, paths, embeddings, and forbidden configurations expose general patterns beneath very different examples. The Czech school contributed to an increasingly structural understanding of these questions by investigating classes of graphs rather than isolated instances. This perspective asks which properties persist under specified operations and which features constrain the behaviour of every member of a class. The shift from single objects to structured families often permits stronger classification theorems and more informative connections with logic. Such work also clarifies why computational difficulty varies dramatically between apparently similar discrete problems. The resulting theoretical framework has significance independent of practical applications, even when those applications subsequently become substantial.

Jaroslav Nešetřil occupies a distinctive place in this history as both a mathematician and an organiser of sustained international collaboration. His research traversed Ramsey theory, graph homomorphisms, structural combinatorics, and the mathematical foundations of theoretical computer science. Rather than treating these fields as unrelated specialisms, his work repeatedly identified transformations and correspondences linking them. This intellectual approach helped make Czech combinatorics visible through ideas that could be developed by researchers in several countries. The research environment at Charles University enabled continued interaction between senior specialists, doctoral students, and visiting scholars. Mathematical progress in this context relied upon precise conjectures, conceptual tools, and the critical testing of difficult examples. International publications strengthened the circulation of results while providing demanding scrutiny from independent specialists. The scientific significance of the resulting school cannot be measured solely through honours or institutional reputation. It is more convincingly established by the enduring use of its concepts within subsequent research on graphs, relational structures, and algorithms. The development of this school thus illustrates how intellectual leadership can shape the direction of a distributed international research community.

Ramsey theory investigates conditions under which order must appear within sufficiently large or richly organised systems. Its foundational intuition is that unavoidable configurations can emerge even when individual arrangements are chosen arbitrarily. In graph-theoretic settings, the questions often concern coloured edges, substructures, and embeddings preserving specified relationships. Classical numerical Ramsey results are only one part of this much larger mathematical landscape. Structural Ramsey theory shifts attention towards collections of finite objects equipped with relations, orderings, or additional operations. Within this setting,

researchers ask when sufficiently large structures necessarily contain prescribed monochromatic copies of smaller configurations. Such arguments are technically demanding because the types of admissible embeddings depend upon the structural language being employed. Czech research was important in developing a broad classification-oriented approach to these problems. This connection between combinatorics and the theory of mathematical structures became particularly productive after 1993.

The earlier Nešetřil–Rödl tradition provided an essential historical foundation for later structural Ramsey investigations, although its origins preceded the period considered here. Distinguishing this inheritance from post-1993 innovations is necessary for an accurate account of scientific development. A productive line of work examined how ordered expansions can make classes of finite structures possess Ramsey properties that are absent without the additional order. Ordering is therefore not merely decorative information, but can change what general combinatorial statements become possible. Research in the following decades further connected these questions with homogeneous structures, model theory, and categorisation problems. In a 2005 article, Nešetřil discussed Ramsey classes and homogeneous structures through a programme combining combinatorics with model-theoretic classification. That programme treated graphs, tournaments, and partially ordered sets as examples of a wider structural phenomenon. It also drew attention to the careful choice of admissible orderings and the relationship between properties of classes and properties of individual members. The analysis demonstrated how an established mathematical tradition could generate new research questions through interaction with neighbouring disciplines. Its significance lies in the systematic perspective rather than in a simple extension of a single classical theorem.

Structural Ramsey theory became increasingly connected with the general study of finite relational structures as researchers sought results applicable beyond familiar graph classes. An individual relational structure may encode several kinds of adjacency, ordering, or incidence simultaneously. The challenge is to identify principles that survive this broader descriptive freedom without becoming mathematically empty. Jan Hubička and Jaroslav Nešetřil developed an influential framework for Ramsey classes involving closures and forbidden homomorphisms. Their 2019 work in Advances in Mathematics unified and extended several earlier results and supplied examples involving ordered metric structures and finite models. The authors formulated broad sufficient conditions rather than presenting a separate ad hoc proof for every individual structure class. This emphasis on reusable methods is one reason the work is significant within contemporary structural combinatorics. It also shows the importance of younger collaborators in extending established Czech research traditions. The mathematical achievement is therefore both technical and methodological, bringing previously separated constructions into a coherent setting.

Graph homomorphisms constitute a second major strand of the Czech contribution to modern discrete mathematics. A homomorphism maps the vertices of one graph to those of another while preserving adjacency in the prescribed direction. This relatively simple definition allows many apparently unrelated graph properties to be reformulated as questions about mappings. Ordinary graph colouring, for example, can be expressed through the existence of a homomorphism to a suitable complete graph. The conceptual advantage is that a large variety

of structures can be compared using one common mathematical relationship. Homomorphisms are generally less restrictive than isomorphisms, which makes them useful for describing approximation, reduction, and structural complexity. Their composition also introduces an order-like organisation among graphs and related relational systems. Czech work helped develop the algebraic and combinatorial foundations needed to study these relationships systematically. This broad perspective proved especially valuable as theoretical computer science adopted structural mappings as a language for constraint problems. The resulting field is an instructive example of abstract mathematics generating practical explanatory power without sacrificing formal precision.

The 2004 monograph Graphs and Homomorphisms by Pavol Hell and Jaroslav Nešetřil marked a major synthesis of this research programme. Published by Oxford University Press, it assembled the mathematical theory of graph mappings alongside their implications for colourings and constraint problems. The book examined graph products, retracts, partial orders, and the combinatorial structure of homomorphism existence. It also connected the subject to complexity theory and the mathematical analysis of finite relational systems. Such a synthesis performs a distinct scientific function from the publication of a new theorem. By organising results, definitions, examples, and open problems, it establishes a common vocabulary for subsequent researchers. The collaboration itself demonstrates that Czech mathematical influence often operated through international intellectual partnerships. Its importance should consequently be attributed to the joint authorship rather than presented as an exclusively national achievement. The work remains particularly relevant for understanding the close relationship between combinatorial structure and algorithmic tractability.

An important consequence of the homomorphism perspective is its interaction with constraint satisfaction problems. These problems ask whether a collection of variables can be assigned values satisfying a specified family of restrictions. Many such questions can be translated into the existence of a homomorphism between finite relational structures. The translation gives a precise framework for connecting the algebra of relations with computational complexity. It also allows distinctions between easy and difficult computational tasks to be investigated through mathematical properties of the underlying templates. Czech researchers contributed both the graph-theoretic language and later algebraic developments informing this research. Nevertheless, the field's major classification achievements resulted from extensive international work and cannot be assigned to one institution. The historical role of the Czech school is better described as the creation and refinement of influential structural tools. These tools permitted deeper questions about algorithms to be expressed in mathematically transparent terms. In this way, a theory originating in abstract combinatorics became a foundation for important parts of modern computer science.

Colouring remains one of the most accessible examples through which the subtleties of graph structure can be understood. The chromatic number measures how many colours are required when adjacent vertices must receive different colours. Yet colouring theory extends far beyond the calculation of this single numerical invariant. Researchers also study list colourings, circular colourings, constrained mappings, and interactions between colouring and structural decomposition. The homomorphism formulation permits these variants to be related to broader

properties of graph classes. In the Czech tradition, such relationships helped unify classical combinatorial questions with more abstract mapping-based methods. The resulting explanations can reveal obstructions that are invisible when one treats each colouring problem independently. They also suggest new conjectures by moving between equivalent or closely related descriptions of a problem. Consequently, modern graph colouring is best understood here as a component of structural combinatorics rather than a disconnected collection of puzzles.

Extremal combinatorics asks how large, dense, or complicated a discrete structure may become while avoiding specified forbidden configurations. For graphs, a typical problem concerns the maximum number of edges possible without containing a particular subgraph. Structural approaches investigate how such numerical restrictions interact with decomposition, homomorphisms, and classes defined by excluded patterns. The distinction between an individual forbidden graph and an entire hereditary family is methodologically important. Questions about unavoidable patterns often connect extremal methods with Ramsey theory, even when the relevant statements have different logical forms. Czech combinatorial scholarship contributed to this broader conversation through investigations of structural restrictions and admissible embeddings. The conceptual links help explain why apparently elementary counting questions can require substantial abstract machinery. They also demonstrate that finite combinatorics is not isolated from questions of classification and mathematical logic. The pursuit of precise bounds remains valuable, but understanding the structural reasons behind those bounds is equally significant. This methodological orientation is central to the subsequent emergence of sophisticated theories of graph sparsity.

The study of sparse graphs became one of the most influential international developments associated with Nešetřil and Patrice Ossona de Mendez. A sparse graph informally possesses relatively few edges, but modern structural sparsity requires more refined definitions than simple edge counts. A class may have controlled density in its members while exhibiting unexpectedly complicated behaviour under local transformations. Researchers therefore developed concepts that measure structural complexity across whole classes of graphs. The theory distinguishes properties such as bounded expansion and nowhere denseness, each expressing a different level of restriction. These notions connect combinatorial decompositions with algorithmic behaviour and with definability in mathematical logic. Their formulation created a framework for asking which large graph classes retain enough order to permit efficient general methods. The Czech contribution was particularly visible because the underlying theory was developed through sustained international co-authorship. This is a clear instance in which mathematical innovation involved identifying a new conceptual landscape rather than solving one previously isolated problem.

The 2012 monograph Sparsity: Graphs, Structures, and Algorithms provided an extensive account of this emerging field. Written by Jaroslav Nešetřil and Patrice Ossona de Mendez, the book was published in Springer's Algorithms and Combinatorics series. Its organisation brought together measurements of sparsity, structural classifications, decomposition methods, and applications to algorithms. The treatment explained why controlling shallow minors and related constructions can reveal information not captured by ordinary edge density. It also drew

connections with homomorphism dualities, logic, and property testing. The value of the monograph lies partly in its ability to make several parallel research developments intelligible as one programme. This synthesis helped establish shared terminology for specialists in graph theory and theoretical computer science. The framework has been developed further through numerous independent investigations beyond the original authors' institutions. Its historical importance within Czech mathematics therefore reflects agenda-setting scholarship conducted in international cooperation. The book also illustrates how a mature research school can influence graduate education by organising a technically demanding subject.

Bounded expansion describes a robust form of sparsity that restricts the densities of certain shallow minors uniformly across a graph class. The restriction is stronger than a statement that every graph in the class happens to contain few edges. It controls what becomes visible when local regions are contracted at a bounded depth. This viewpoint is productive because algorithms often depend on the complexity of local neighbourhoods rather than on overall graph size alone. Classes with bounded expansion consequently support useful decomposition and colouring techniques. Such methods can make structural properties accessible through efficiently computable intermediate representations. The theory does not assert that every problem on every sparse graph becomes easy. Instead, it identifies specific structural conditions that permit particular algorithmic or logical results. The precision of these qualifications is essential in distinguishing rigorous mathematical statements from general claims about computational efficiency.

Nowhere dense graph classes provide a wider and more flexible framework for studying structural sparsity. Their definition rules out the appearance of arbitrarily large complete graphs as shallow minors at any fixed depth. The condition admits more varied classes than bounded expansion while preserving significant structural restrictions. Nešetřil and Ossona de Mendez investigated how these restrictions interact with combinatorics, model theory, and algorithms. Their 2016 survey on structural sparsity described the relationship between nowhere-dense and somewhere-dense behaviour. The discussion connected graph decomposition with concepts such as independence, neighbourhood covers, and first-order model checking. These connections were significant because they allowed techniques from distinct mathematical areas to inform one another. The resulting theory is not simply a new definition of a familiar property, but an organising principle for a broad class of research problems. It demonstrates how a concept developed in graph theory can also influence the understanding of logical expressibility. For Czech mathematics, this work exemplified internationally visible contributions to the theoretical foundations of modern discrete structures.

The intersection of graph theory with model theory became increasingly important as researchers investigated first-order properties of finite structures. First-order logic permits statements using quantifiers over vertices and relations without requiring arbitrary set quantification. Many natural graph questions can be expressed within this language, although not all relevant properties have such a formulation. The complexity of deciding a logical property may depend sharply upon the structural class to which the graph belongs. In 2014, Nešetřil and Ossona de Mendez published work on first-order properties of nowhere dense structures. Their investigation illustrates how concepts of scattered sets and neighbourhood separation enter

logical questions about finite graphs. The mathematical interest lies in identifying which combinatorial constraints enable general preservation or decision phenomena. The resulting connections expanded the audience for Czech structural graph theory beyond traditional combinatorics. They also encouraged a more careful dialogue between abstract descriptions of finite objects and computational procedures for analysing them.

Limits of graphs raise a different question from the finite classification problems traditionally associated with combinatorics. Researchers ask whether an increasingly large sequence of graphs can converge in a meaningful mathematical sense. Such convergence requires specifying what quantities or logical properties are observed and how they behave along the sequence. Dense graph limit theory provided one influential starting point, but sparse graph sequences create additional difficulties. Nešetřil and Ossona de Mendez pursued approaches combining structural methods, logic, and measurable limit objects. Their 2016 work on modelling limits in hereditary classes examined reductions and applications to families of trees. The topic illustrates how finite mathematics can meet ideas usually associated with analysis and measure theory. It also shows why mathematical innovation may consist of discovering the right notion of convergence rather than merely computing a limit. Through these developments, discrete mathematics entered a broader conversation about the large-scale organisation of complex systems. The relevant results are methodological as much as substantive because they change how entire sequences of structures can be studied.

Structural graph theory also interacts fruitfully with algorithm design, particularly when general computational problems remain difficult on unrestricted inputs. A graph class with controlled sparsity may admit decompositions that expose regularities unavailable in arbitrary graphs. Such decompositions can reduce a complex computation to operations on smaller or better organised components. The practical value depends on careful assumptions concerning graph representation, parameter values, and the problem being solved. Czech-led theoretical research contributed classifications that later became useful in the study of parameterised algorithms and logical model checking. This relationship should not be interpreted as a direct technological application of every structural theorem. Instead, the mathematical framework supplies conditions under which subsequent algorithm designers can establish rigorous guarantees. The distinction between conceptual foundations and implemented software is important for evaluating scientific achievements responsibly. The wider lesson is that abstract structural results can acquire computational relevance through several stages of independent research.

Graph decompositions help researchers translate global complexity into local relationships that are easier to analyse. Tree-depth, tree-width, and related parameters describe different aspects of a graph's structural organisation. They should not be treated as interchangeable measurements, because each is sensitive to different configurations. The theory of sparsity makes substantial use of bounded-height trees and low tree-depth decompositions. These ideas provide bridges between classification results and algorithms that exploit restricted structural complexity. Within international research associated with Charles University, the systematic comparison of such tools enriched modern graph theory. The resulting methodology encourages proofs that describe whole families of graphs through a limited set of structural operations. Such proofs frequently reveal not only whether a theorem holds, but why the relevant restrictions are

sufficient. The integration of decomposition methods into a coherent theory marked an important change in the ambitions of discrete mathematics. It demonstrates the movement from isolated invariants towards a network of connected conceptual techniques.

The relationship between structural combinatorics and category-like reasoning deserves separate attention because mappings are central to both perspectives. Graph homomorphisms permit mathematical objects to be organised by the existence and composition of structure-preserving transformations. This orientation makes it possible to compare graphs through relational properties rather than solely through numerical invariants. The resulting order of homomorphism types can possess rich and sometimes surprising structure. Czech work on homomorphisms connected these questions with dualities, obstructions, and constrained representation problems. The significance extends to theoretical computer science because many constraint systems are likewise specified by permissible mappings. Nonetheless, the categorical language should not obscure the concrete combinatorial content of the problems being investigated. Researchers continually move between abstract formulations and examples that test the sharpness of general assertions. This balance between formal generality and finite constructions is characteristic of the Czech contribution to the field.

International collaboration was a defining organisational feature of the development described in this section. The partnership between Nešetřil and Ossona de Mendez is particularly visible in books and articles concerning sparsity and structural limits. The earlier collaboration between Hell and Nešetřil offers a parallel example involving homomorphisms and graph colouring. Connections with researchers in neighbouring European countries also supported the circulation of Ramsey-theoretic and algorithmic ideas. These exchanges cannot be adequately represented as the transfer of finished discoveries from one nation to another. Mathematical concepts evolve through conversations, criticism, joint proofs, and the reformulation of difficult problems. Charles University provided a stable base from which Czech mathematicians could participate in this distributed scientific process. At the same time, the resulting publications properly attribute results to all their contributors regardless of national affiliation. This distinction avoids exaggerating the national ownership of research that was collaborative from its conception. It also clarifies how a relatively compact mathematical community can achieve exceptional international intellectual reach.

The training of younger researchers helped sustain the Czech school across the period covered by this book. Graph theory and combinatorics require a distinctive combination of creative construction, rigorous proof, and familiarity with abstract mathematical language. Doctoral research offers an environment in which these abilities develop through sustained engagement with unresolved questions. Seminars and international workshops facilitate the critical examination of proof strategies before they appear in formal publication. The movement of students and postdoctoral researchers between Prague and foreign mathematical centres broadened the intellectual community. Collaborative projects offered access to complementary expertise in logic, algorithms, and other parts of discrete mathematics. The work of Jan Hubička on structural Ramsey theory illustrates how subsequent generations extended research programmes associated with established senior scholars. Such continuity should not be confused with intellectual dependence because successful scientific schools create space for

independent contributions. The lasting strength of a mathematical institution lies partly in its capacity to renew research questions through successive generations.

The growth of mathematical computing did not make the theoretical work of graph combinatorics redundant. Computational experiments can reveal counterexamples, test conjectures on finite instances, and suggest patterns that merit formal investigation. Nevertheless, any finite collection of numerical checks remains distinct from a general mathematical proof. The Czech school developed its reputation predominantly through definitions, theorems, and structural methods whose validity does not depend on particular hardware. At the same time, theoretical computer science offered new motivations for investigating discrete mathematical objects. Algorithms and complexity analysis helped turn structural graph properties into questions with consequences for computational reasoning. This interaction is especially visible in the study of homomorphisms and structurally restricted graph classes. The scientific relationship is reciprocal because theoretical advances clarify what algorithms can achieve, while algorithmic questions motivate new abstract mathematics. The period therefore witnessed an expanding interface between classical proof-based combinatorics and computational perspectives. Neither side can be understood adequately if the other is reduced to a merely instrumental role.

The international recognition of Czech discrete mathematics reflects its participation in several distinct kinds of scientific achievement. Some advances were theorem-centred, establishing Ramsey properties or precise results about classes of structures. Others were conceptual, introducing definitions and classifications that reorganised existing mathematical knowledge. Still others involved synthesis through major books that made broad research programmes accessible to new generations. These achievements should not be combined into an undifferentiated account of national prestige. A mathematical monograph, a joint research article, and a foundational classification framework have different forms of influence. Evaluation consequently requires attention to the content of publications, their historical context, and their later mathematical use. The case of Nešetřil is particularly illuminating because his career encompasses collaborative books, structural theories, and long-term institutional leadership. His contributions also demonstrate why mature research programmes cannot be assessed solely by counting individual publications.

Several unresolved questions continued to shape graph theory and structural combinatorics by the end of 2025. Theoretical frameworks such as sparsity create new problems even while resolving difficulties inherited from earlier mathematics. Researchers continue to investigate how structural restrictions influence logic, graph limits, algorithmic complexity, and the existence of prescribed configurations. The boundaries between graph theory, finite model theory, and theoretical computer science remain intellectually productive. Generalisations also demand care because statements valid for one graph class can fail when the assumptions are weakened. For this reason, precise definitions and explicitly stated hypotheses remain central to modern research practice. The Czech school demonstrated that lasting advances often come from identifying exactly which assumptions make a powerful general theorem possible. Its historical importance lies in sustaining this style of inquiry across successive conceptual developments. The period 1993–2025 therefore represents an ongoing research trajectory rather

than the completion of a closed mathematical chapter. Contemporary results are best viewed as foundations for further investigation instead of as final statements about the structure of all discrete systems.

The Czech contribution to international combinatorics cannot be described adequately through the reputation of a single mathematician, however prominent. It encompasses institutional continuity, graduate education, international co-authorship, methodological innovation, and the development of theories subsequently used beyond their original setting. Ramsey theory demonstrated the power of structural embeddings and ordered expansions in classifying finite mathematical objects. Homomorphism theory established a common language for graph mappings, colourings, and many computational constraint problems. The theory of sparsity revealed how whole graph classes could be organised according to subtle measures of structural complexity. Research on logical properties and graph limits extended these ideas into areas traditionally associated with model theory and analysis. Together, the developments illustrate how mathematical disciplines evolve through contact with neighbouring forms of reasoning. They also show that innovation in pure mathematics may involve a new perspective on familiar objects rather than a novel technological instrument. The decisive achievement was the creation of a connected research programme capable of supporting diverse questions and methods.

Between 1993 and 2025, Czech discrete mathematics developed from a strong inherited scholarly tradition into a particularly visible component of international structural combinatorics. The historical process involved no abrupt abandonment of classical graph theory or Ramsey-theoretic ideas. Instead, established questions were reinterpreted through homomorphisms, relational structures, sophisticated classifications, and notions of sparsity. Jaroslav Nešetřil and his collaborators played a major role in formulating and communicating these developments. The mathematical significance of their work lies in the durable concepts and theorems that enabled other researchers to construct new approaches. Charles University and its international networks provided an organisational environment in which the necessary long-term scholarship could flourish. At the same time, collaboration across national boundaries remained intrinsic to the scientific achievements examined here. The experience demonstrates that a country's influence in mathematics depends more on the originality and usefulness of its ideas than on the material size of its research infrastructure. Czech combinatorics consequently offers an instructive case of scientific continuity transformed into methodological leadership within contemporary mathematical science. Its history provides a natural transition to subsequent discussions of algorithms, logic, and the wider mathematical foundations of computation.

Theoretical Computer Science and Algorithmic Mathematics

The development of theoretical computer science in the Czech Republic between 1993 and 2025 illustrates how abstract mathematics can shape practical approaches to computation. The discipline was never confined to faster programming, because its central questions concern what can be computed, which resources are necessary, and how mathematical structure determines efficiency. Czech researchers inherited established traditions in discrete mathematics, mathematical logic, optimisation and the mathematical foundations of computing. The establishment of an independent republic changed their institutional setting without erasing these intellectual connections. At Charles University in Prague, mathematical departments and computer science groups increasingly treated algorithms as objects of mathematical proof rather than merely instruments of implementation. The resulting research landscape connected combinatorial existence theorems with constructive procedures and quantitative bounds. This relationship became particularly important as increasingly large datasets made inefficient procedures practically unusable. The period therefore has to be understood through both original theoretical results and the institutions that allowed specialists to collaborate internationally. Its most characteristic development was a gradual convergence of algebra, logic, probability and combinatorics around computational questions.

A foundational problem of algorithmic mathematics is the definition of a computational model precise enough to support rigorous comparisons. Time complexity measures the growth of operations required by an algorithm, whereas space complexity concerns the information it must retain. Neither measurement is independent of the representation of the input or the assumptions made about elementary computation. Czech work in complexity theory has consequently addressed the relationship between formal models, lower bounds and the structure of difficult problems. Polynomial-time solvability provides one useful dividing line, although problems admitting such algorithms can still be demanding on realistic instances. Conversely, an intractable problem in its unrestricted form may admit efficient algorithms when its instances possess additional structure. Theoretical investigations must therefore distinguish worst-case complexity from restrictions that can be exploited systematically. This conceptual distinction has linked Czech mathematical research with international discussions about efficient computation and provable limitations. By 2025 the relationship between structural assumptions and complexity had become central to several leading algorithmic programmes. Such results are scientifically valuable even when they prove that an apparently attractive computational strategy cannot succeed.

Pavel Pudlák provides a particularly important connection between mathematical logic and theoretical computer science in Czech research. His investigations of proof complexity ask how long a formal demonstration must be when expressed in a specified deductive system. Unlike ordinary mathematical reasoning, this question makes the size of a proof itself an object of quantitative analysis. Lower bounds can reveal limitations of whole families of proof systems and illuminate the difficulty of automated reasoning. Pudlák also examined circuits, coding problems and connections between bounded arithmetic and computational complexity. His 2013 monograph Logical Foundations of Mathematics and Computational Complexity brought these topics together within a broad mathematical interpretation. The work illustrates the

importance of moving carefully between philosophical questions about proof and precise combinatorial statements. Czech contributions to this field should therefore not be reduced to software verification or to the engineering of automated theorem provers. They include foundational results concerning the resources needed for formal deduction and computation.

The design of an algorithm begins with a mathematical description of its input, admissible operations and intended output. Correctness must then be established independently of favourable experimental examples, ideally through an invariant or a structural theorem. The complexity analysis explains how the required resources change when the size or geometry of the instance grows. For graph problems, such reasoning frequently relies on connectivity, decomposition, colouring or the existence of particular substructures. Czech mathematical groups have developed this style of research in close proximity to the country's strong tradition of combinatorics. An algorithm can also become the constructive content of a theorem by turning an existence argument into an explicit method. Conversely, an impossibility result may show why a particular computational task resists improvement under established assumptions. The most productive work therefore alternates between structural mathematics and procedural construction. This interplay helps explain why theoretical computer science occupies a natural place in a history of Czech mathematical thought. The distinction between an elegant algorithm and a reliable one is often settled by the precision of its proof.

The Centre for Discrete Mathematics, Theoretical Computer Science and Applications, known as DIMATIA, represents a concrete institutional expression of this connection. It was established at the Faculty of Mathematics and Physics of Charles University as a joint initiative involving academic and research partners. Its remit connected theoretical computer science with discrete structures and their applications rather than treating these subjects as separate domains. The centre encouraged exchanges between mathematical specialists and researchers addressing algorithmic problems. Such arrangements mattered because international collaboration in theory often depends more on sustained intellectual contact than on costly experimental apparatus. Workshops, visiting researchers and shared projects helped integrate Prague into wider European mathematical conversations. The institutional model also demonstrated how an established university could develop specialised programmes without abandoning a broader curriculum. The research themes emerging from this environment included graph algorithms, structural combinatorics and algorithmically relevant discrete mathematics. DIMATIA thus offers a useful case of internationalisation grounded in a recognisable national mathematical tradition.

Graph theory provides one of the clearest bridges between combinatorial reasoning and algorithm design. A graph can represent a communication network, a collection of constraints or a system of pairwise relationships while preserving an abstract mathematical structure. Algorithms on graphs may seek paths, colourings, matchings, decompositions or decisions about the presence of prescribed patterns. Their efficiency often depends on whether the input graphs belong to a structurally restricted class. The Czech school associated with Jaroslav Nešetřil contributed influential work on homomorphisms and sparse graph classes. In this setting, a homomorphism preserves adjacency and provides a flexible language for analysing relational structures. An abstract structural classification can therefore lead to a

computationally meaningful distinction between different families of instances. The connection is not automatic, because additional algorithmic arguments may be necessary to transform structural insight into a tractable procedure. Nevertheless, graph structure became an organising principle for theoretical investigations with wide international significance. The resulting research shows how problems that look unrelated in applications may share the same mathematical core.

Graph sparsity is not simply a statement that an input has relatively few edges. Modern structural theories analyse how density behaves under subgraphs, minors and other transformations. The monograph Sparsity by Jaroslav Nešetřil and Patrice Ossona de Mendez, published in 2012, synthesised a far-reaching programme connecting graph structure with logic and algorithms. Its treatment of bounded expansion and related classes provides a framework for identifying recurring forms of structural simplicity. Such classes can support more effective treatment of problems that are difficult on unrestricted graphs. This insight is especially important because real networks may be extremely large yet exhibit exploitable patterns of local interaction. The mathematical challenge is to specify these patterns rigorously rather than to assume that every practical graph is easy. The Prague contribution lies in developing reusable structural concepts that other researchers can apply to different computational questions. Sparsity consequently illustrates the passage from pure combinatorial classification towards algorithmic mathematics.

Parameterized complexity refines the traditional distinction between efficient and inefficient computational problems. It considers a parameter that captures a relevant aspect of the input and studies whether the remaining dependence on input size can be controlled. Fixed-parameter tractability identifies algorithms whose potentially expensive component is confined to a function of that parameter. This framework is especially useful for problems involving small solutions, restricted widths or structured constraint systems. Czech researchers have contributed to parameterised methods through work on graph problems and mathematical optimisation. Martin Koutecký and collaborators, for example, investigated algorithms for structured integer programmes. The significance of these results is not that every integer programme suddenly becomes easy, but that certain structural measures permit stronger guarantees. Parameterized analysis also provides a disciplined language for comparing algorithms designed for different kinds of practical instances. It has therefore helped extend discrete mathematics towards nuanced assessments of algorithmic feasibility. The parameter must be justified from the problem itself rather than selected merely because it makes a theorem easier to state.

Integer programming formulates decisions through variables constrained to take integral values. The framework is powerful because scheduling, allocation and combinatorial selection can frequently be represented by linear constraints. In general, integrality introduces substantial computational difficulty even where the corresponding continuous relaxation is manageable. Theoretical progress therefore depends on identifying matrix structure, decomposition or numerical conditions that allow improved algorithms. Work by Martin Koutecký and collaborators examines such restrictions through parameterised complexity. Their study of mixed-integer linear programmes with small treedepth demonstrates how a structural property

of the constraint matrix can affect tractability. The result does not eliminate the difficulty of general mixed-integer optimisation, and its assumptions are essential to interpreting its meaning. Nevertheless, it provides a precise example of abstract mathematics contributing to the methodology of computational decision-making. Such work connects theoretical computer science directly with operations research while maintaining rigorous complexity guarantees.

When exact optimisation is computationally prohibitive, approximation algorithms seek solutions with provable relationships to the optimum. The guarantee may be expressed as an approximation ratio or another mathematically defined measure of quality. This shifts attention from absolute optimality towards what can be achieved predictably within available resources. Approximation research requires careful distinctions between minimisation and maximisation and between worst-case and empirical performance. Charles University identifies approximation algorithms among the recognised strengths of its theoretical computer science groups. Work in this area draws on combinatorics, probability, mathematical programming and the design of efficient procedures. A good approximation result may be more valuable than an exact method that cannot finish on the instance sizes of interest. At the same time, a numerical experiment alone does not establish the theoretical guarantee claimed by an approximation scheme. The Czech research environment has supported the reciprocal development of mathematical bounds and implementable algorithmic ideas. The conceptual advance is often a new proof of how close a computed answer must be to the best possible one.

Online algorithms must act before all relevant information has become available. They therefore address a different challenge from traditional optimisation, which assumes the entire instance is known in advance. A scheduling decision, for example, can influence later opportunities even when future jobs or demands remain uncertain. Competitive analysis compares an online procedure with an idealised method possessing complete information. This produces a mathematical framework for studying uncertainty without requiring a probability distribution over future inputs. Jiří Sgall and other Prague researchers have contributed to algorithmic research involving scheduling and related optimisation problems. Such investigations are important because practical computing systems constantly allocate limited resources under evolving conditions. An algorithmic result must specify its information assumptions carefully, since small differences can alter the best achievable guarantee. Online computation therefore links abstract mathematical reasoning to the temporal structure of real decisions.

Randomised algorithms incorporate controlled random choices into a formally specified computational procedure. Their guarantees may concern expected running time, error probability or the likelihood of producing a suitable solution. The approach is closely related to the probabilistic method in combinatorics, which can establish existence without exhibiting a single deterministic example. Probability can make some algorithmic analyses simpler or permit performance that would otherwise be difficult to obtain. However, expected performance and a guarantee for every execution are fundamentally different mathematical claims. Czech work in algorithms and discrete structures forms part of an international environment where such distinctions are central. Randomisation also connects counting problems, sampling methods and complexity-theoretic classifications. The design of a rigorous probabilistic algorithm therefore requires both a precise model of randomness and an explicit error analysis. This

combination demonstrates the interdisciplinary character of modern algorithmic mathematics. Statistical evidence from experiments may complement but cannot replace a proof of probabilistic correctness.

Data structures determine how information is organised so that recurring computational operations can be performed efficiently. Their study goes beyond programming conventions because mathematically different representations may have sharply different complexity properties. Trees, heaps, dictionaries and graph representations are analysed through the operations they support. Dynamic settings introduce additional difficulty because insertions and deletions change the stored object over time. Theoretical computer science at Charles University explicitly encompasses data structures and their mathematical analysis. This field connects information theory with combinatorics and computational models of memory. Questions about lower bounds are important because they reveal whether an apparently slow operation can be improved in principle. The organisation of data can thus become a mathematical innovation even when the underlying information remains unchanged. The Czech research landscape includes this subject as an integral component of algorithmic foundations.

Formal logic allows algorithms to be studied as objects satisfying mathematically precise specifications. In verification, a programme or system is judged against a declared property rather than against a limited collection of tests. Decision procedures aim to determine whether statements within a restricted formal language are satisfiable or logically valid. The computational complexity of such procedures varies greatly with the expressive power of that language. Czech mathematical logic supplied intellectual resources for this domain through research on proof systems and formal foundations. At the same time, theoretical computer science developed practical motivations for investigating efficient symbolic reasoning. The interaction is reciprocal because new computational questions can prompt developments in logic itself. Neither formal verification nor automated reasoning should be confused with an unrestricted ability to prove arbitrary mathematical truths. Their achievements are most accurately described through the classes of systems and formulae to which particular methods apply. This discipline of explicit scope distinguishes robust formal methods from broad technological promises.

Constraint satisfaction problems ask whether a collection of variables can be assigned values obeying specified relations. Their formulation captures problems from graph colouring to scheduling, databases and formal reasoning. A major theoretical question is why some families of constraints admit polynomial-time algorithms while others remain computationally hard. The algebraic approach studies operations preserving the relevant relations and connects algorithmic complexity with universal algebra. Libor Barto contributed important results to this international programme, including work on algebraic formulations of complexity classifications. Research with Michael Pinsker investigated algebraic criteria for constraint satisfaction over infinite domains. The importance of these results lies in translating algorithmic questions into structural statements about operations and relational systems. Once a suitable algebraic condition is identified, it can guide the search for algorithms or hardness arguments. This example demonstrates a distinctive Czech contribution at the boundary of algebra, logic and theoretical computing.

Graph homomorphisms provide a natural mathematical representation of many constraint satisfaction problems. A mapping between relational structures must preserve prescribed relationships, turning a computational question into a problem about structure-preserving functions. Pavol Hell and Jaroslav Nešetřil developed influential work on graph homomorphisms that became foundational for subsequent research. Their 2004 monograph Graphs and Homomorphisms brought together a large body of structural theory with algorithmic implications. Patterns of tractability and hardness emerge only after the class of target structures has been specified with care. This illustrates why the same general computational task may behave very differently across restricted families of inputs. The mathematical language of homomorphisms also creates connections with algebraic and categorical viewpoints. For Czech research, the area exemplifies an unusually close interaction between discrete mathematics and theoretical computer science. The literature has continued to use these ideas in broader investigations of constraint problems and finite structures. The relevant achievement is the creation of a transferable mathematical framework rather than the solution of a single isolated application.

Mathematical programming translates a complicated optimisation task into an objective function and a feasible region. Linear programming provides a particularly important setting because geometric and duality methods yield rigorous optimality criteria. Integer and combinatorial variants introduce further challenges connected with discreteness and non-convexity. Algorithmic researchers use these formulations as tools for deriving approximation bounds, relaxations and decomposition methods. Charles University lists linear and semidefinite programming among the methodologies supporting its algorithmic research. These approaches illustrate how geometry, algebra and optimisation can converge in a computational problem. Theoretical analysis is essential because the quality of a relaxation depends on mathematically identifiable properties. An efficient numerical solver is valuable, but understanding what it computes remains a distinct intellectual task. Czech mathematical work contributes to this conversation through the interaction of optimisation theory and discrete algorithms.

Large input size changes not only the practical cost of algorithms but also the questions researchers consider worthwhile. A procedure with an attractive asymptotic bound may still be unsuitable if its hidden constants or memory requirements are excessive. Conversely, a specialised algorithm may outperform a general one by exploiting predictable characteristics of a dataset. Theoretical computer science therefore studies fine-grained complexity and structural restrictions alongside classical complexity classes. It also asks which operations can be parallelised and which aspects of a problem create unavoidable sequential dependencies. The growth of large-scale computing in Czechia increased the practical importance of these questions. Nevertheless, a faster machine cannot by itself resolve a mathematical obstruction associated with an intractable problem. The role of algorithmic research is to make the limits and opportunities of computation intelligible before implementation. This perspective connects theoretical results with the expanding capacity of scientific computing infrastructure. It guards against the mistaken assumption that improved hardware automatically supplies fundamentally better algorithms.

Parallel computation distributes work among several processors or computational units. Its effectiveness depends on whether a problem can be decomposed without introducing overwhelming coordination costs. Mathematical analyses of parallel algorithms distinguish total work, depth and communication requirements. These criteria differ from ordinary sequential running time and can expose bottlenecks hidden by a simple operation count. Czech computational infrastructures created opportunities for applying sophisticated numerical and algorithmic techniques at larger scales. The theoretical foundations nevertheless remained essential for deciding which methods could benefit from additional processors. Algorithms for graph processing and discrete optimisation may pose different parallelisation challenges from matrix-based numerical methods. The most successful approaches therefore combine formal analysis of structure with attention to the organisation of computational resources. The interaction between mathematics and computing architecture became increasingly visible during the period under examination.

Theoretical computer science developed in Czechia through exchanges with a much wider international research community. Co-authored articles, seminars and specialised workshops enabled relatively small groups to contribute to rapidly developing subjects. Such collaboration is particularly productive where a problem benefits from complementary expertise in algebra, logic, optimisation and combinatorics. Prague institutions provided an important base, while partnerships extended through European and wider international networks. Mathematical results circulated through peer-reviewed journals and international conference proceedings. Conference publication has special significance in theoretical computer science, although its role must not be confused with that of informal announcements. International assessment relied increasingly on the originality and validity of proofs rather than on institutional reputation alone. The visibility of Czech research consequently depended on active participation in shared scientific problems. This process helped maintain continuity between national mathematical schools and globally evolving computational theory. The intellectual contribution of a researcher should be traced to identifiable papers and theorems rather than inferred from institutional affiliation alone.

The vitality of algorithmic mathematics depends on education capable of joining abstract reasoning with computational intuition. University curricula must introduce discrete structures, algorithms, logic and probability without treating them as disconnected topics. Advanced training then adds complexity theory, approximation methods, optimisation and formal proof techniques. Charles University’s theoretical computer science programme illustrates the presence of these subjects within specialised higher education. Doctoral researchers acquire skills through reading primary literature and participating in problems whose solutions are not already known. Theoretical supervision also requires attention to what counts as a complete, reproducible mathematical argument. International mobility exposes students to alternative proof techniques and research traditions. As research themes evolve, new courses can reflect developments in parameterised algorithms and automated reasoning. The educational system therefore contributes to the reproduction of scientific schools as well as the circulation of specific methods.

Prague occupied a major position in Czech theoretical computing through Charles University and academy-based mathematical research. This concentration should not be mistaken for a complete absence of work elsewhere in the country. Brno and other centres maintained substantial programmes in informatics, mathematics and related technical disciplines. Regional specialisation developed through different combinations of mathematical theory, software research and engineering applications. The organisation of science also changed as university departments gained greater research responsibilities and international connections. Specialised centres could pursue focused themes while remaining integrated into broader teaching institutions. Because theoretical research often requires modest physical infrastructure, sustained staffing and intellectual community are especially important. The geographic distribution of expertise therefore cannot be inferred simply from large buildings or supercomputing facilities. An accurate institutional history must account for both the concentration of nationally visible schools and the contribution of regional groups. Institutional diversity provides alternative routes through which mathematical innovation can emerge.

Practical relevance in theoretical computer science rarely takes the form of an immediately marketable product. A theorem may identify an algorithmic possibility that becomes technologically useful only after substantial further research. A complexity lower bound may be valuable precisely because it discourages the pursuit of an impossible approach. An approximation guarantee can influence future software even if the original mathematical paper contains no deployable implementation. Czech contributions should consequently be assessed through conceptual influence as well as direct application. This distinction is especially important when national innovation policies emphasise short-term measurable outcomes. Theoretical inquiry often proceeds on longer timescales than commercial development and depends upon continuity of expertise. Applications in scheduling, communications and data analysis may nevertheless motivate productive mathematical questions. The interplay between long-term discovery and applied demand helps explain the diversity of Czech algorithmic research.

Evaluating theoretical computer science requires methods appropriate to a field centred on proofs, algorithms and formal models. Publication counts alone do not show whether a result created a new method or resolved a difficult longstanding question. Citation patterns are informative but can be influenced by the breadth and age of a research community. Major monographs may consolidate an entire research programme and shape subsequent work for many years. Conference papers sometimes constitute central publications, provided their review and archival status are established. Collaborative achievements must also be described without assigning all credit to a single institution or national tradition. The Czech example illustrates the need to distinguish mathematical originality from participation in internationally established topics. Historical assessment therefore benefits from careful reading of primary literature and corroboration by institutional records. This approach protects the interpretation of scientific achievement from both exaggeration and undue modesty. It also makes the national narrative more useful to researchers seeking transferable lessons from Czech experience.

Theoretical computer science intersects with mathematical logic, combinatorics, operations research and information theory. These connections mean that the classification of a result as

mathematics or computer science is often partly institutional. An algebraic proof concerning constraint problems may have direct consequences for the complexity of algorithms. A graph-theoretic classification may provide the framework for a new family of efficient procedures. A lower bound for formal proofs may illuminate automated reasoning and foundational mathematics simultaneously. For this reason, the present account treats algorithmic mathematics as a coherent intellectual domain rather than an isolated departmental category. Its Czech development is most visible when work across university and academy boundaries is considered together. Such an approach also avoids double-counting closely related achievements in separate discussions of logic and combinatorics. The decisive question is what a particular result contributed to understanding computation.

By the early 2020s, Czech theoretical computer science operated within a substantially more international research environment than in 1993. Advances in parameterised complexity, combinatorial optimisation and algebraic methods continued to demonstrate the importance of mathematical structure. The expanding use of automated reasoning and data-intensive computation raised new questions about verification, efficiency and mathematical guarantees. However, the conceptual foundations of these fields remained distinct from the empirical performance of contemporary artificial intelligence systems. A computational method capable of impressive practical results does not automatically provide a proof of correctness or generality. Theoretical research therefore retained a critical role in distinguishing demonstrated mathematical properties from observed behaviour. The increasing prominence of algorithmic techniques also created opportunities for collaboration with applied mathematics and other sciences. Czech research groups approached these developments from established strengths in logic, discrete structures and mathematical optimisation. The period 2021–2025 should accordingly be interpreted as an expansion of existing intellectual trajectories as well as a response to new technologies. Such continuity is a recurring feature of the wider history of Czech mathematical sciences.

The history of theoretical computer science and algorithmic mathematics in Czechia from 1993 to 2025 reveals a sustained interaction between structural insight and computational ambition. Research on proof complexity clarified the relationship between deduction and computational resources. Graph homomorphisms, sparsity and discrete structures supplied a mathematical language for identifying algorithmically significant restrictions. Parameterized methods and structured integer programming demonstrated how assumptions about problem instances can lead to sharper computational guarantees. Work on approximation, scheduling and optimisation linked mathematical bounds to questions arising in practical decision-making. Institutional networks centred on universities and research institutes supported this development through international collaboration and advanced training. The importance of these achievements cannot be measured solely by the speed of a programme or the construction of computing facilities. It lies equally in the capacity to establish what is possible, what is difficult and which structural properties make a problem tractable. Viewed in this way, algorithmic mathematics became one of the clearest examples of how the Czech mathematical tradition contributed to a changing scientific understanding of computation.

Probability Theory and Stochastic Processes: Mathematical Models of Randomness and Uncertainty

Probability theory entered the independent Czech Republic in 1993 with a substantial intellectual inheritance rather than as a newly constituted discipline. Its foundations had been established through university teaching, Academy research, statistical methodology, and investigations of random dynamical phenomena in Czechoslovakia. The political transition after 1989 made international exchanges easier, yet it did not erase established mathematical questions or research communities. The work of Petr Mandl on Markov processes and diffusion theory provided one important historical point of reference. Zbyněk Šidák had likewise contributed to the spectral and probabilistic analysis of Markov chains and to mathematical statistics. Ivo Vrkoč investigated stochastic differential equations alongside the qualitative theory of deterministic differential systems. These contributions furnished concepts and methods that younger researchers could reinterpret under new conditions of scientific cooperation. After independence, the challenge was to sustain demanding theoretical work while adopting competitive funding and international publication practices. The ensuing development of Czech probability thus belongs to a history of continuity combined with expanding methodological ambition.

The distinction between randomness as a mathematical object and uncertainty as a practical condition is fundamental to understanding this history. Probability theory defines measures, distributions, dependence structures, and limiting behaviour within precise logical frameworks. Stochastic-process theory extends that apparatus to phenomena whose evolution is indexed by time, space, or another parameter. In applications, uncertain observations acquire meaning only after assumptions about mechanisms and data quality have been made explicit. A stochastic model can therefore be mathematically rigorous without being empirically adequate for every proposed use. Czech mathematicians working in probability, analysis, and cybernetics confronted this distinction in studies ranging from diffusion processes to decision systems. Their contributions are best assessed through the properties established for mathematical models rather than through broad claims about technological transformation. Existence, uniqueness, regularity, invariant measures, convergence, and stability each answer a different question about probabilistic behaviour. Those questions also determine whether computational procedures can be trusted when models are transferred to engineering, economics, or biology. The evolution of the discipline after 1993 increasingly connected these theoretical distinctions with more complex systems and richer sources of data.

The institutional landscape supporting probabilistic research was distributed across several rather than one exclusively specialised centre. The Institute of Mathematics of the Czech Academy of Sciences preserved a setting for fundamental research in analysis and related mathematical fields. The Institute of Information Theory and Automation combined mathematical methods with problems of estimation, prediction, control, and decision-making. Charles University provided an especially important route through which students encountered stochastic analysis and probability at advanced levels. Its relationship with Academy institutes allowed teaching and research to reinforce one another without dissolving their institutional

differences. Research programmes in Brno and other university centres contributed further diversity to the country's probabilistic and statistical environment. The resulting scientific geography cannot be reduced to administrative boundaries because collaboration frequently crossed departmental and national lines. For a comparatively small mathematical community, intellectual specialisation and access to international collaborators were particularly valuable. The strength of the emerging system lay in connecting highly technical theory with multiple institutional settings.

An especially revealing research line concerned stochastic equations in spaces of infinitely many dimensions. Such equations arise when the evolving state is a function rather than a finite vector of measured quantities. Noise may act through random forcing, uncertain boundary conditions, or fluctuations distributed throughout a physical system. The mathematical difficulty is then not simply to add randomness to a familiar differential equation. Researchers must establish that the equation defines a meaningful solution under assumptions compatible with functional analysis and probability. Bohdan Maslowski and Jan Seidler developed studies of stochastic partial differential equations and associated transition mechanisms. Their published investigations addressed questions of invariant probability measures, uniqueness, and stability in non-linear settings. The corresponding results demonstrate why stochastic analysis cannot be separated from operator theory and the geometry of function spaces. They also illustrate a recognisably Czech contribution to a wider international programme on infinite-dimensional random dynamics. Rather than serving merely as computational approximations, these theories provide conditions under which simulations have a defensible mathematical basis.

The study of invariant measures supplies a particularly clear example of the relationship between transient behaviour and long-term uncertainty. A probability distribution is invariant when evolution under the random dynamics leaves that distribution unchanged. Such a distribution need not describe the behaviour of every trajectory, nor does its existence automatically imply uniqueness. Researchers therefore ask whether solutions approach a common statistical regime from different initial conditions. For dissipative systems, a balance between random forcing and stabilising mechanisms may create the possibility of such convergence. Maslowski and collaborators examined ergodicity and stability in stochastic systems for which direct deterministic arguments are inadequate. Work on uniform exponential ergodicity, including research with Beniamin Goldys, supplied rigorous criteria for certain dissipative models. The implications extend beyond any particular physical application because long-run averages underpin many statistical and numerical procedures. By distinguishing stationarity from convergence towards stationarity, these results sharpened the interpretation of uncertain dynamic behaviour.

Another significant direction dealt with the regularising effects of randomness in stochastic evolution. In a deterministic system, an initially irregular distribution of states may retain features inherited from its starting position. Random perturbations can sometimes alter continuity properties of transition probabilities, though the effect depends on the model. The strong Feller property expresses an important form of smoothing by the associated Markov transition operator. Maslowski and Seidler investigated probabilistic approaches to this property, including the use of changes of probability measure. Their 2000 work in Probability Theory and Related

Fields developed arguments relevant to both finite- and infinite-dimensional diffusions. Such investigations required careful control of the drift, admissible noise, and the meaning of solutions. They offered methods for relating probabilistic identities to regularity properties that would otherwise be difficult to establish. The broader significance is methodological: a carefully chosen probabilistic technique can yield information traditionally pursued through analytical estimates. This interaction of disciplines became increasingly characteristic of advanced stochastic research after the turn of the century.

Stochastic convolution became an essential technical object in the development of this research. It represents the accumulated effect of random disturbances propagated by an underlying evolution operator. The resulting object may be function-valued, making its regularity difficult to measure with elementary probabilistic inequalities. Moment estimates and exponential-integrability properties help establish whether exceptional fluctuations are sufficiently controlled. Jan Seidler's research on the exponential integrability of stochastic convolutions addressed this problem within a rigorous analytical setting. Related international collaborations examined stochastic convolutions driven by different classes of random noise. The theoretical importance lies in obtaining estimates that remain meaningful despite the infinite-dimensional state space. Such results can enter existence proofs and support the analysis of numerical approximation or long-run stability. They also help explain why modern stochastic analysis developed in close contact with sophisticated Banach-space techniques.

Not every random evolution is adequately represented by idealised Brownian forcing. Some systems experience discontinuous jumps, sudden shocks, or disturbances whose effects are concentrated in time. Others display dependence persisting over longer intervals rather than the short-memory structure of standard models. These alternatives motivate stochastic integration against jump processes and forms of fractional or correlated noise. In Czech research, attention to fractional noise connected abstract stochastic evolution equations with questions of temporal dependence. A 2018 study by Maslowski and Jana Šnupárková addressed stochastic affine evolution equations with multiplicative fractional noise. The mathematical issue was not merely to replace one random input with another but to retain coherent solution concepts. Non-standard forcing can change the calculus available and the stability mechanisms that may legitimately be inferred. Such work exemplifies the diversification of probabilistic methods during the second decade of the twenty-first century. It also cautions against assuming that results for classical diffusion models automatically extend to more complex random environments.

Stochastic partial differential equations acquired growing relevance because many natural processes are spatially distributed. Diffusion of substances, transport of heat, fluid motion, and changing biological concentrations all possess spatial structure. A deterministic partial differential equation can capture some mechanisms while omitting environmental and observational variability. Stochastic formulations introduce uncertainty without discarding the analytical laws governing the underlying process. Researchers must nevertheless distinguish modelling fluctuations in physical mechanisms from representing imperfect knowledge of parameters. The existence of weak, mild, or variational solutions may depend on quite different hypotheses. Uniqueness can be delicate where non-linearity interacts with irregular spatial

forcing. Czech mathematical contributions to such questions depended on sustained connections between probability, functional analysis, and the theory of differential equations. Their significance is the establishment of well-posed frameworks in which substantive interpretations can be developed without concealing unresolved analytical difficulties.

The relationship between stochastic analysis and mathematical mechanics illustrates how abstract theory can inform physical research. Mechanical systems may be subject to random loading, fluctuating material properties, or unresolved external disturbances. The addition of such effects complicates energy inequalities that ordinarily guide deterministic stability arguments. Researchers therefore examine Lyapunov functions and probabilistic estimates suited to the modified dynamics. A 2005 article by Zdzisław Brzeźniak, Bohdan Maslowski, and Jan Seidler studied stochastic non-linear beam equations. The work explored global mild solutions and asymptotic stability under a stochastic forcing model. Its importance lies in showing how probabilistic methods can be integrated with a mathematically specific mechanical equation. It does not imply that every real beam or structure behaves according to the same stochastic assumptions. Instead, the study demonstrates how mathematical conditions determine what can be concluded about a physically motivated random system. This distinction between rigorous results and potential application remained central to the development of Czech applied probability.

Markov processes continued to connect the country's earlier mathematical traditions with new theoretical research. A Markov model specifies how the distribution of a future state depends on the current state under an explicit memory assumption. Its mathematical analysis can concern transition operators, recurrence, invariant measures, and the distribution of passage times. Earlier Czech and Czechoslovak contributions by Mandl and Šidák had established a valuable legacy in this field. Their pre-1993 results should be recognised as foundations rather than inaccurately recast as discoveries of the independent republic. Later research could build upon this heritage while addressing infinite-dimensional evolutions and less conventional sources of noise. The continuity is visible in shared analytical questions even when new mathematical methods change their precise formulation. Markovian modelling also supports a range of applications, but its suitability must be tested rather than presumed. A process with genuine long-range memory, for example, may require a richer state description before the Markov property becomes appropriate.

The transformation of uncertainty into decisions formed another distinctive branch of Czech mathematical development. In decision theory, a probability distribution represents information relevant to choices whose consequences are not fully known. The central problem involves both learning about the system and specifying how competing outcomes should be valued. Bayesian methods permit information to be updated as evidence accumulates, subject to declared modelling assumptions. Miroslav Kárný and colleagues at the Institute of Information Theory and Automation developed theoretical and algorithmic research on adaptive systems. Kárný's work has emphasised Bayesian identification and the fully probabilistic design of decision strategies. These investigations connect statistical estimation with the problem of choosing actions over time. They therefore differ from stochastic-equation research while sharing a commitment to formal descriptions of uncertainty. Their practical significance appears in systems where

sensing, prediction, and intervention are interdependent. At the same time, decision quality depends on preferences and model adequacy, not solely on the mathematical consistency of probability calculations.

An adaptive system must revise its description of the world when newly observed data conflict with previous expectations. The mathematical process often combines prior distributions, likelihood functions, and measures of uncertainty about unknown parameters. In dynamic settings, information arrives sequentially and earlier decisions can influence which later observations become available. This creates a feedback relationship between identification and control that static statistical inference does not fully express. Czech work in adaptive Bayesian systems addressed this difficulty through algorithms and probabilistically formulated decision rules. Kárný's research also examined the integration of information originating from different sources or participants. Such integration requires attention to dependence because treating correlated evidence as independent may exaggerate confidence. Theoretical models can clarify these hazards and prescribe coherent strategies under a stated set of assumptions. As digital sensing expanded after 2000, these questions became increasingly relevant to automation and information-intensive engineering.

The development of probabilistic information fusion illustrates a further expansion of the discipline's application domain. Multiple sources may supply measurements that are incomplete, inconsistent, or obtained at unequal intervals. A mathematical framework must distinguish uncertainty inherent in the underlying process from disagreement caused by the observation system. Combining distributions can provide an operational summary, although different pooling rules encode different assumptions. Research on Bayesian merging and probabilistic decision design at Czech institutions placed these choices within explicit mathematical constructions. Such constructions are useful only when the meaning of each supplied distribution is understood. The same principle applies when different agents possess unequal knowledge or pursue partially conflicting objectives. A quantitative framework cannot remove the need for judgement about whose information is reliable and how decisions should be evaluated. Its contribution is to make those judgements more transparent, analysable, and potentially open to systematic improvement. This kind of conceptual innovation deserves attention alongside new theorems about diffusion and stochastic stability.

The application of probability to economic processes grew through the broader international development of quantitative finance and risk analysis. Asset returns, insurance claims, demand fluctuations, and economic transitions are commonly modelled as uncertain variables or processes. Models of these phenomena must balance tractability against irregular behaviour and changes in the underlying economic environment. Czech mathematicians and statisticians participated in a research landscape increasingly shaped by stochastic modelling and optimisation. Probabilistic methods helped formulate questions about forecasting, estimation, scenario analysis, and the distribution of potential losses. Nevertheless, a mathematical model of financial uncertainty should not be mistaken for a guaranteed prediction of market events. Model errors, structural changes, and exceptional shocks can overwhelm estimates derived from a narrowly calibrated distribution. The academic value of probability lies partly in providing systematic tools for identifying and communicating such limits. Connections with decision

theory further explain how uncertain forecasts may inform, but cannot themselves determine, economic choices.

Biological applications offer a different motivation for probabilistic modelling because variability is often an intrinsic feature of living systems. Populations fluctuate through births, deaths, migration, and environmental disturbances even when average tendencies are relatively stable. At smaller scales, molecular interactions and gene-expression events may also exhibit random behaviour. Stochastic models can represent distributions of outcomes where deterministic equations describe only an expected trajectory. Methods from Markov processes, stochastic differential equations, and random branching systems offer several possible frameworks. The choice among them depends on the biological scale, observational evidence, and the mechanisms believed to operate. Czech contributions to mathematical probability formed part of the methodological foundation available to interdisciplinary biological researchers. It would be misleading, however, to attribute particular biological discoveries to probability specialists without evidence of an identifiable collaboration. The more defensible observation is that mathematically developed tools made complex sources of biological uncertainty increasingly accessible to analysis. This relationship underscores the importance of separating disciplinary contributions from the practical settings in which their methods may later be used.

Engineering applications demand particular attention to the reliability of probabilistic conclusions under imperfect data. Sensors measure real processes with noise, and engineered systems encounter loads and disturbances that cannot be specified exactly in advance. Stochastic models allow engineers to examine how uncertain inputs may affect performance or the probability of threshold exceedance. They can also support filtering, state estimation, fault detection, and adaptive control. Yet performance guarantees often depend on assumptions about the magnitude and temporal structure of the disturbances. Theoretical research in stochastic stability and probabilistic identification is therefore relevant to engineering for reasons that extend beyond simulation. It explains which assertions remain valid under specified types of uncertainty and which require additional restrictions. Academy research linking probability to automation provided an institutional route between abstract mathematics and technical problems. Such interaction became more practical as numerical computing and data acquisition improved during the period under examination.

High-performance computing broadened the range of stochastic models that researchers could investigate numerically. Monte Carlo methods approximate quantities by repeated random sampling from a specified mathematical distribution. Their appeal lies in flexibility, particularly where deterministic quadrature or exact analytical integration is infeasible. Computational cost can nevertheless become substantial for rare events, high-dimensional problems, and systems requiring expensive simulations. Variance reduction, parallel sampling, and careful error analysis therefore matter as much as raw processing speed. Czech investment in scientific computing during the 2010s created a wider infrastructure for computational work across disciplines. Such resources could support probabilistic simulation, although the existence of a supercomputer should not be presented as proof of any single research breakthrough. Numerical probability remains constrained by approximation bias, sampling variability, and imperfections in the mathematical model itself. The maturation of scientific computing thus

strengthened the relationship between probabilistic theory and large-scale empirical investigation. It did not remove the need for rigorous mathematical arguments about convergence or the interpretation of simulated results.

The assessment of uncertainty also requires a distinction between parameter variability and uncertainty about model structure. Two models may fit an observed sample similarly while implying substantially different probabilities for unobserved events. Bayesian inference can express uncertainty about parameters and sometimes about competing models, but conclusions depend on prior choices. Frequentist procedures instead evaluate estimators or tests through their repeated-sampling behaviour under specified assumptions. These approaches answer overlapping questions without being interchangeable in every practical setting. Czech research at the boundary of statistics, stochastic systems, and information theory engaged with several aspects of this methodological diversity. The intellectual development of probability after 1993 should consequently not be written as a simple replacement of classical methods by Bayesian techniques. More accurately, the period saw a multiplication of modelling strategies adapted to different scientific aims. Their common requirement was a transparent account of how uncertainty enters the analysis and how the validity of results is established.

International collaboration became an essential feature of advanced probabilistic research because the field crossed national disciplinary boundaries. Studies of stochastic partial differential equations routinely combined expertise in functional analysis, dynamical systems, and modern probability. Researchers based in Czech institutions cooperated with colleagues working in other European mathematical centres. The jointly authored publications of Maslowski, Seidler, Brzeźniak, Goldys, and other collaborators illustrate this scientific geography. Such publications represent joint achievements and should not be assigned exclusively to any one national mathematical school. At the same time, continuing Czech institutional affiliations contributed to the persistence of domestic research expertise and training. International collaboration could accelerate access to new methods while also exposing local work to demanding peer review. The Czech contribution is therefore recognisable through identifiable researchers, institutions, and published mathematical results. Its historical meaning emerges from sustained participation rather than the misleading assumption that national science develops in isolation. This more careful account also reflects the cooperative character of mathematical knowledge production after European integration.

Teaching was another crucial mechanism through which probabilistic research became part of a durable national scientific culture. Advanced training in probability presupposes measure theory, analysis, functional spaces, and mathematical reasoning beyond elementary statistics. Students must learn not only computational techniques but also the assumptions under which stochastic constructions are defined. At Charles University, Jan Seidler's teaching of stochastic differential equations, Markov processes, and ergodic theory exemplified the link between research and postgraduate education. The presence of active researchers in teaching creates opportunities to encounter unresolved questions as well as established methods. Doctoral mobility further allows specialised training to be complemented by experience in foreign research groups. Such exchange is particularly important in technical subjects where a small number of researchers may possess highly specialised expertise. It can also lead to joint

publications and subsequent collaboration between graduates and established institutions. The renewal of probabilistic research therefore depended on intellectual transmission as much as on grants or computational equipment.

The publication record of Czech and Czech-affiliated probabilists reveals both disciplinary continuity and thematic diversification. Work appeared in international outlets including Probability Theory and Related Fields and the Journal of the London Mathematical Society. Czech journals such as Czechoslovak Mathematical Journal, Applications of Mathematics, and Kybernetika also continued to host relevant research. The Czech Digital Mathematics Library preserves bibliographic access to both historical foundations and later contributions. Its records make it possible to distinguish research published during 1993–2025 from work that shaped the field decades earlier. That distinction is indispensable for an accurate historical evaluation of scientific innovation in the independent Czech Republic. A journal title alone does not establish the importance of an article, nor does citation frequency directly measure conceptual originality. More convincing assessment considers the theorem proved, the mathematical method introduced, and its subsequent role in research. This evidence-based approach avoids treating publication counts as a substitute for intellectual history. It also respects the diversity of contributions ranging from foundational theory to methods designed for specific applications.

Despite these advances, stochastic modelling continued to face limitations intrinsic to uncertainty rather than merely to available technology. Real observations often cover a short period, contain missing values, or fail to represent rare but consequential events. Even exact solutions of a probabilistic model cannot compensate for assumptions that misrepresent the mechanism under study. For infinite-dimensional systems, existence and uniqueness of solutions may remain open under physically desirable hypotheses. For numerical simulations, increasing computational resolution may not eliminate approximation errors without a valid convergence argument. Decision models introduce additional difficulties because utilities, preferences, and social consequences are not determined by probability laws. Researchers therefore need to explain both what their methods establish and the conditions under which these results cease to apply. Such qualifications do not weaken the contribution of probability theory but define the boundaries of responsible mathematical inference. The Czech experience offers a case in which theoretical rigour and applied relevance developed through continuing interaction rather than simple substitution.

The period from 2020 to 2025 brought intensified interest in uncertainty within data-driven modelling and automated decision systems. Machine learning systems often generate predictive scores that must not automatically be interpreted as calibrated probabilities. Reliable probabilistic forecasts require evaluation under the intended data-generating conditions and attention to distributional changes. Stochastic modelling can contribute tools for filtering, regularisation, uncertainty quantification, and principled sequential decisions. It can also help explain why algorithms trained on historical observations may fail when the environment changes. Research traditions in Bayesian identification and stochastic dynamics therefore remained relevant to contemporary computational intelligence. Nevertheless, it is important not to describe all artificial-intelligence research in Czechia as an extension of its probability school. Different mathematical methods address distinct problems, and the institutional relations

among them must be supported by evidence. The historically defensible claim is that probability supplied a durable set of concepts repeatedly useful in emerging computational contexts. These questions continued to shape the methodological development of Czech probability research.

A useful perspective on the whole period is provided by the growing variety of mathematical meanings assigned to uncertainty. Diffusion models investigate irregular trajectories, while invariant measures describe statistical patterns that may persist through time. Bayesian identification expresses what can be learned about unknown systems from accumulating observations. Decision theory asks how uncertain knowledge can inform actions under explicitly represented preferences. Monte Carlo computation estimates otherwise inaccessible quantities but introduces its own measurable sources of error. These are related activities, yet they cannot be collapsed into a single generic account of stochastic innovation. Czech researchers contributed through rigorous investigations of selected problems in several of these areas. The connecting feature was the ambition to replace vague descriptions of unpredictability with analysable mathematical structures. Where assumptions were unverified, its theory also supplied grounds for caution and methodological restraint.

By 2025, Czech research in probability and stochastic processes had become part of a mature European mathematical environment. Its inherited traditions in Markov processes, stability, and diffusion remained relevant without limiting the field to historical methods. The work of Maslowski and Seidler contributed to the theory of stochastic evolution and infinite-dimensional dynamics. Research associated with Kárný developed probabilistically formulated methods for adaptive decision-making and the integration of information. Connections with engineering and the natural sciences gave these mathematical directions potential practical significance, though not identical empirical outcomes. Computational advances enlarged the range of models that could be explored and tested numerically. Academic institutions and international collaborations supported both the creation of results and the preparation of younger researchers. Persistent challenges concerned model validity, incomplete data, long-run stability, and the responsible interpretation of probabilistic outputs. The distinctive achievement of the period was therefore neither a single discovery nor the arrival of one dominant modelling paradigm. It was the sustained development of a sophisticated mathematical language through which randomness and uncertainty could be studied, compared, and acted upon with greater precision.

Questions about extreme events sharpened the distinction between ordinary variability and consequences that occur infrequently. A model calibrated around typical observations can underestimate the probability or severity of unusual disturbances. In reliability analysis and risk assessment, such failures may be more consequential than small errors in an average forecast. Probability theory provides tools for considering tails, passage times, concentration inequalities, and dependence among exceptional events. Yet their application demands careful specification of the relevant sample space and adequate empirical justification for chosen distributions. Theoretical estimates remain valuable because they identify what follows from mathematical assumptions even when rare-event data are scarce. Computational approaches can complement these estimates through simulation, provided the sampling strategy captures the events of interest. In this way, probabilistic reasoning connects mathematical precision with an

explicit account of residual uncertainty. That connection remained important to the expanding use of stochastic methods in technical and societal risk analysis.

Mathematical Statistics and Statistical Inference: Theory, Methods, and Data-Driven Research

Between 1993 and 2025, mathematical statistics in the Czech Republic developed through the interaction of an established intellectual tradition and a newly international research environment. The foundation of an independent state did not create a statistical discipline where none had existed, because sophisticated work on probability, inference and data analysis had already been pursued in Czechoslovak universities and academic institutes. Rather, political independence altered the administrative institutions, funding arrangements and international conditions under which that inherited expertise could be sustained. Mathematical statisticians faced questions that remained recognisably theoretical, including the validity of estimators, the interpretation of dependence and the reliability of tests under imperfect assumptions. At the same time, more extensive computing facilities gradually enabled methods that could not easily be explored through closed-form calculations alone. Research therefore evolved along two connected trajectories, one concerned with rigorous mathematical properties and the other with computationally feasible methods for real observations. The Czech contribution is best understood through particular researchers, publications and research groups rather than a single national programme directed towards one preferred methodology. This combined historical and analytical perspective reveals a community in which classical statistical reasoning remained relevant as the informational environment changed. The period is consequently best examined as a sequence of overlapping intellectual developments rather than an abrupt replacement of one statistical paradigm by another.

Charles University in Prague provided a major institutional setting for research in mathematical statistics, particularly through its Faculty of Mathematics and Physics and its specialist statistical teaching. The Czech Academy of Sciences maintained complementary research capacities through institutes concerned with mathematical theory, information processing and computation. Among these, the Institute of Information Theory and Automation connected statistical reasoning with probabilistic modelling, decision systems and signal analysis. The Institute of Computer Science supported mathematical work that linked statistical theory with classification, numerical computation and interdisciplinary data problems. Such institutions differed in mission and methodological emphasis, yet their activities could reinforce one another through jointly supervised research and shared scientific networks. University departments contributed sustained doctoral education, while Academy laboratories offered specialised research communities and projects extending beyond individual academic courses. International contacts expanded the scientific audience for work undertaken in Prague and other centres, without removing the importance of local seminar traditions. The resulting institutional

landscape is therefore better described as a network of overlapping mathematical competences than as a rigid separation between statistics and informatics. This institutional plurality helped preserve specialist mathematical expertise while enabling work to move across disciplinary boundaries. Researchers consequently operated within a research system whose boundaries were institutional but whose mathematical questions were shared.

Estimation theory remained a central concern because quantitative conclusions depend upon the relationship between observations, mathematical models and unknown parameters. Classical estimators often obtain attractive properties under idealised assumptions, such as independent errors or suitably specified probability distributions. Research becomes more demanding when those assumptions are uncertain, when the sample is modest or when a small number of observations have disproportionate influence. Czech statisticians investigated procedures whose theoretical behaviour could be described even as the statistical model became more complicated. Consistency, asymptotic distributions, efficiency and robustness provided distinct criteria for evaluating an estimator rather than interchangeable claims of quality. The balance among these criteria was important because an estimator that performs well under one model may be unreliable under another. Analytic results were consequently complemented by computational comparisons that made differences visible under finite-sample conditions. This concern with reliability linked rigorous mathematical inference to the increasingly heterogeneous datasets encountered in empirical research. A careful historical account must therefore assess the assumptions and results of particular contributions rather than merely record the appearance of new procedures.

Robust statistics formed one of the most identifiable strands of the Czech research tradition in the period under examination. Jana Jurečková helped establish an internationally visible programme addressing rank procedures, nonparametric inference and statistical methods resistant to departures from standard assumptions. Her work with Pranab Kumar Sen and later collaborators explored mathematical behaviour that could not be captured by ordinary least-squares reasoning alone. The term robustness refers to a carefully defined form of stability under specified changes in a model or its observations, not to an unconditional guarantee of correctness. Rank procedures can reduce dependence on the precise magnitude of extreme measurements, although their performance still depends on the design and assumptions of the study. Analysing asymptotic properties remains necessary because numerical demonstrations cannot establish the general validity of a statistical procedure. The practical importance of these investigations became more apparent as researchers encountered heterogeneous measurements and datasets with outliers. Czech work in this field linked mathematical proof, methodological comparison and the training of statistical specialists. The strength of the tradition lay in its willingness to examine failure conditions as carefully as successful calculations. This insistence on explicit conditions remains crucial whenever statistical methods are transported to unfamiliar fields.

The publication of major methodological books made these research traditions visible beyond their original institutional settings. Jurečková and Sen's 1996 volume on robust statistical procedures provided an extensive treatment of asymptotics and relationships among inferential methods. The 2000 book Adaptive Regression, written by Yves Dodge and Jurečková, developed

another connection between theoretical statistics and the analysis of regression models. Jurečková and Jan Picek subsequently published Robust Statistical Methods with R in 2006, combining theoretical argument with implementation in a statistical computing environment. These books should not be interpreted as evidence that all of their methods originated in Czechia, since statistical theory develops through international contributions. Their significance lies instead in the sustained Czech participation in constructing, clarifying and disseminating a coherent methodological programme. The presence of reproducible computational procedures further strengthened the relationship between advanced theory and the practical assessment of data. Such publications also provided foundations for doctoral study and international collaboration throughout the following decade. This connection between theoretical exposition and practical reproducibility became an enduring feature of the field.

Jan Ámos Víšek contributed to the theoretical examination of robust regression estimators, including the least trimmed squares procedure. Trimming reduces the influence of selected large residuals, but the inferential properties of the resulting estimator are not automatic. Víšek's three-part study published in Kybernetika in 2006 addressed consistency, the rate of convergence and asymptotic normality. The division of the investigation into these questions illustrates why robust estimation is a mathematical research problem rather than merely a computational technique. Establishing consistency concerns convergence towards an appropriate target under stated conditions. Demonstrating a rate of convergence then clarifies how estimation error diminishes with growing information, while asymptotic normality can support further inferential approximations. These distinctions matter when an estimator is proposed for observations whose distribution or error structure differs from textbook models. The research exemplifies the Czech contribution to the rigorous foundations required before robust tools can be used with confidence. The three investigations together clarify how several different mathematical claims are needed to justify one estimation procedure. The example also shows why statistical advances often emerge through a sequence of linked publications rather than a single isolated result.

Weighted statistical procedures offered another means of controlling the influence of observations without simply removing them from the analysis. A weighting rule may give less influence to cases regarded as incompatible with the principal structure of the data, but its justification depends on precisely defined statistical objectives. Czech research on least weighted squares explored how implicit weights could produce regression methods with useful resistance to outliers. Víšek's contributions and subsequent investigations by Jan Kalina connected these questions with asymptotic reasoning and computational implementation. Heteroscedasticity posed a particularly important difficulty because the variability of errors can change with the explanatory variables. A method designed only for equal error variance may therefore provide misleading assessments when used without suitable diagnostics. Investigations of weighting, scale estimation and model sensitivity strengthened the link between robust statistics and econometrics. They also illustrate how the development of an apparently specialised estimator can influence broader debates about reliable inference. Research on weighting consequently exemplifies the productive tension between sensitivity to evidence and protection against contamination.

Multivariate analysis became increasingly important as observations began to describe many characteristics of the same phenomenon simultaneously. Covariance matrices, dependence patterns and the geometry of high-dimensional data create problems that are not resolved by analysing each variable separately. Jan Kalina's 2012 study of multivariate methods in robust econometrics examined implicitly weighted procedures, dimension reduction and robust classification. Among the methods discussed was a weighted covariance-determinant approach intended to improve resistance to atypical observations. The distinction between constructing a method and establishing its practical superiority must remain clear, because comparative performance depends on the data-generating process. High dimensionality can make covariance estimation unstable and can obscure the boundary between informative structure and random variation. Czech contributions were particularly relevant where mathematical properties needed to be considered alongside numerical feasibility and empirical diagnostics. This combination reflects the growing methodological convergence of statistics, econometrics and computational data analysis. The mathematics of multivariate estimation thereby became central to both scientific interpretation and algorithmic design. It was this interaction between geometry, probability and robustness that made multivariate inference a continuing research frontier.

Rank-based inference provided a bridge between classical nonparametric statistics and robust methods for complex observations. When a procedure depends upon the ordering of values rather than their precise distances, its behaviour may be less sensitive to certain measurement distortions. That advantage does not mean that ranks preserve every kind of relevant information or automatically handle arbitrary dependence. The mathematical task is to identify conditions under which tests and estimators retain reliable distributions and useful efficiency. Jurečková's research helped sustain international attention to such questions and to their connections with regression. Work on adaptive procedures explored how statistical methods could respond to information about the underlying distribution without assuming that its shape was already known. The resulting techniques are useful in experimental research where unusually strong assumptions may be difficult to defend. Within Czech mathematics, this line of enquiry reinforced the continuing relevance of rigorous inference even as algorithmic analysis became increasingly prominent. Its continuing relevance reflects the fact that modern datasets do not eliminate the need for distributional reasoning.

An important form of statistical inference concerns the possibility that a process changes its behaviour during observation. A sequence that appears approximately stable may contain an abrupt alteration in location, scale, dependence or model parameters. Detecting such a change requires more than noticing a visually unusual segment, because ordinary random fluctuation can create misleading patterns. Jaromír Antoch and Marie Hušková studied permutation tests for change-point analysis in a 2001 paper in Statistics and Probability Letters. Their results connected formal testing principles with simulated performance under explicit model conditions. Permutation methods can derive reference behaviour by rearranging observations, although their validity still depends upon appropriate assumptions or asymptotic arguments. The problem was relevant to monitoring systems, economic series and laboratory measurements where structural stability could not be assumed indefinitely. This research

demonstrates how theoretical statistics can directly address the interpretation of evolving observations. The resulting framework distinguished a structural change supported by evidence from one suggested only by visual inspection. The distinction between an estimated location of change and its uncertainty is especially significant when decisions have practical consequences.

The analysis of time series becomes more difficult when neighbouring observations are statistically dependent. A bootstrap method that simply treats successive measurements as independent may fail to reproduce essential features of the process. Marie Hušková and Claudia Kirch examined bootstrap confidence intervals for a change point in time series in a 2008 study. Their approach used resampling methods suited to dependent errors under specified mixing conditions. Confidence intervals are particularly important because an estimated change date should not be presented as an exact historical event when substantial sampling uncertainty remains. The study illustrates how resampling can support inference while retaining a connection to asymptotic theory. Further collaborative investigations examined autoregressive models whose parameters might change during the observation period. Such work connected Czech mathematical statistics with an international literature on structural breaks and dependent stochastic data. These investigations contributed to more cautious interpretation of apparently precise change-point estimates.

Resampling changed the practical repertoire of statistical inference by allowing researchers to approximate uncertainty through repeated computational experiments. Bootstrap procedures, permutation tests and related methods do not remove the need for mathematical assumptions, even when calculations are straightforward to perform. Their validity must be evaluated with reference to the dependence structure, the target parameter and the behaviour of the chosen statistic. Czech investigations of change points provide a particularly clear example of this requirement. A simulated sampling distribution is useful only to the extent that its construction reproduces relevant aspects of the underlying stochastic process. Finite-sample comparisons can reveal practical advantages that are not obvious from limiting theory alone. Yet the opposite danger also exists when attractive simulations are taken as substitutes for proofs or broader sensitivity analysis. The Czech experience therefore illustrates the complementarity of computational experimentation and mathematical justification. Properly designed resampling thus offers a complement to, rather than an escape from, statistical theory. The growing availability of computing power made such experiments cheaper, but it did not make their interpretation automatic.

Probabilistic graphical models offered a different route towards understanding relationships among many random variables. Directed and undirected graphs can encode particular forms of dependence and conditional independence, making complex models easier to investigate. However, the meaning of a graphical representation depends on its statistical interpretation and on the relationships that the graph is authorised to express. Milan Studený developed a significant programme on the mathematical foundations of conditional independence structures. His 2005 monograph Probabilistic Conditional Independence Structures provided an algebraic treatment that went beyond purely graphical description. Structural imsets and supermodular functions furnished methods for investigating independence relationships and

their formal properties. Such developments matter because different graphs may express equivalent models, while a visually appealing graph need not correspond to every desired independence statement. The Czech contribution was therefore conceptual as well as computational, clarifying the mathematical language used in probabilistic knowledge systems. A central achievement was to give abstract independence statements a rigorous language that could survive changes of graphical representation.

The theory of imsets demonstrates how statistical modelling can create research problems in discrete mathematics and algebra. Conditional independence statements form intricate systems of relationships whose logical implications are not always transparent from a single graphical representation. Studený investigated algebraic objects capable of representing such structures in a form suitable for theoretical comparison. This approach connects questions of statistical equivalence with convex geometry and combinatorial reasoning. It can assist in distinguishing structural features of a probability model from the particular notation used to describe them. A graphical model may be useful for communication, while an algebraic representation may support different forms of proof and computation. Czech work in this direction consequently helped build bridges between mathematical statistics, information theory and artificial intelligence. These bridges became increasingly valuable as learning algorithms sought to reconstruct dependence relationships from data rather than assume them in advance. The algebraic perspective further revealed why apparently simple independence assertions can carry subtle mathematical consequences. Formal descriptions were especially important where the same independence content could appear in apparently different models.

Bayesian networks became a major point of contact between statistical inference and knowledge representation in computing. Their practical attraction lies in the possibility of expressing a joint probability model through local conditional relationships. Yet the learning of a network from observational data is complicated by equivalent structures, modelling choices and the need for reliable probabilistic estimation. Research at the Institute of Information Theory and Automation included mathematical investigation of these questions. Milan Studený and Jiří Vomlel, for example, contributed to discussions of graphical and algebraic representations of conditional independence models. Such work should be distinguished from generic applications of Bayesian software, because the research concerns the formal meaning and limitations of the models themselves. Computational procedures gain interpretative value when their assumptions and equivalence classes are understood. This emphasis on foundations illustrates an important characteristic of Czech work at the boundary between mathematical statistics and computer science. Such investigations created common problems for statisticians, logicians and computer scientists.

The relationship between probability and information theory deepened the conceptual foundations of many statistical methods. Entropy, divergence and conditional independence can measure or characterise different aspects of the information contained in a probability model. These concepts are particularly useful when competing models are compared or when observations update prior assumptions. Czech researchers working in information theory and automation investigated problems in which such mathematical ideas were central. Their work did not collapse statistical inference into a single information measure, because different

measures encode different properties of uncertainty. Rather, information-theoretic reasoning broadened the mathematical instruments available for model representation and decision-making. It also encouraged connections with discrete structures and optimisation, especially where complex dependence relationships could be expressed through algebraic constraints. Such theoretical developments made the boundary between statistics and informatics increasingly productive. These shared concepts facilitated collaboration without dissolving the distinct purposes of inference and information processing. Information measures offered a shared language, although meaningful application continued to require statistical interpretation.

Bayesian estimation offered another prominent approach to inference by expressing uncertainty through probability distributions over unknown quantities. A prior distribution, a likelihood function and the resulting posterior must all be interpreted in the context of a specified model. Researchers at the Institute of Information Theory and Automation, including Miroslav Kárný, investigated Bayesian methods for adaptive learning and decision processes. Their projects linked probability updating with situations where observations arrive sequentially and actions may affect subsequent information. The mathematical challenge concerns not only computation of an estimate but also the coherent combination of evidence, uncertainty and decisions. Model misspecification, dependence and the availability of limited data can complicate Bayesian procedures as much as frequentist ones. Comparative assessment therefore remains essential rather than assuming the superiority of a philosophical framework. Czech work in this area connected statistical mathematics with control theory, automation and information fusion. The work illustrates how statistical theory can remain relevant inside technological systems that operate continuously.

Statistical decision theory places estimation and prediction within a wider question about choosing actions under uncertainty. A probability model alone cannot determine the best decision without specifying the consequences of different outcomes. Loss functions and optimisation criteria therefore connect mathematical statistics with economics and engineering. Czech work in adaptive and Bayesian decision-making treated this connection as a research problem rather than as an incidental application. A decision system may need to learn from new information while retaining sufficient stability to avoid reacting excessively to random noise. The quality of its decisions depends simultaneously on the adequacy of its statistical model and the relevance of its objective function. Mathematical investigation is especially important when decisions are repeated over time and errors can accumulate. This illustrates how statistical inference developed alongside the broader Czech tradition of systems science. The analytical challenge is to ensure that probabilistic uncertainty is not silently replaced by unjustified confidence in a decision rule. Questions of utility and loss therefore made mathematical modelling of preferences inseparable from probabilistic inference.

Classification became an increasingly visible statistical problem as scientific and technological applications produced labelled and unlabelled datasets. Discriminant methods attempt to relate observed variables to classes, but their reliability depends on assumptions about distributions and the quality of training data. Robust classification research addressed the possibility that atypical observations might distort estimated class boundaries. Czech investigations by Jan Kalina examined multivariate approaches designed to reduce such

sensitivity. At the Institute of Information Theory and Automation, statistical pattern recognition also involved mixture models, feature selection and probabilistic representations of images. These approaches combined ideas from mathematical statistics with computational techniques for organising complex information. Assessment required attention to prediction error, training-sample composition and the stability of estimated parameters. The resulting methodological overlap helps explain why statistical research increasingly interacted with machine learning during the period under consideration. Classification therefore became a field in which theoretical guarantees and empirical validation had to inform each other.

Finite mixture models provide a probabilistic description of populations that may contain several latent components. They can represent heterogeneity without requiring every observation to arise from one homogeneous distribution. Estimation becomes difficult when components overlap, when parameters are poorly identified or when the number of components is uncertain. Czech pattern-recognition researchers investigated such models in connection with classification, image analysis and visual scene interpretation. Markov random fields provided related tools for representing dependence among neighbouring elements of an image or spatial system. Although these applications are computational, their mathematical interpretation rests on probability models and inferential criteria. Practical success must be tested against alternative explanations, rather than inferred solely from an algorithm's ability to fit observed data. This convergence of statistical modelling and computer vision was an important part of the national interdisciplinary research environment. The quality of these models ultimately depends upon both their probabilistic coherence and their usefulness outside the training sample. The flexibility of mixture models brought with it substantial responsibility for checking identification and model complexity.

High-dimensional data posed substantial difficulties for both established estimators and emerging machine-learning procedures. When the number of variables approaches or exceeds the number of observations, apparently conventional covariance or regression calculations can become unstable. Regularisation introduces additional structure or penalties to control estimation, but the resulting bias must be understood alongside any reduction in variance. Robustness presents a separate challenge because even a regularised estimator may respond poorly to contaminated observations. Czech contributions to multivariate and robust methods addressed these connected problems through mathematical analysis and computational comparison. Such research was relevant to econometrics, biological measurements and classification systems containing many correlated characteristics. Numerical experimentation helped illuminate difficult finite-sample behaviour, while theoretical results specified the circumstances under which desirable properties could be expected. This sustained attention to assumptions distinguished rigorous statistics from the mere deployment of increasingly complex algorithms. High-dimensional research reinforced the need to distinguish numerical convenience from genuine inferential reliability.

Econometrics offered an important environment for applying and testing statistical methodology developed within Czech mathematics. Economic observations commonly exhibit serial dependence, heteroscedasticity, missing information and sensitivity to exceptional events. Robust regression and multivariate techniques therefore addressed problems of immediate

relevance to quantitative economic analysis. Kalina's 2012 work explicitly connected such methods with econometric applications and with the computational aspects of estimation. Change-point analysis also provided a framework for examining whether relationships remained stable during periods of economic transformation. These applications should not be confused with proofs that particular methods were universally optimal in economic research. Instead, they show how mathematical procedures can improve the clarity of assumptions and the scrutiny of uncertainty. The resulting interaction between statistics and economics strengthened the practical significance of research on estimation, diagnostics and model comparison. In this sense, applications informed theoretical priorities without determining them completely. Inference in economic datasets thus continued to require critical scrutiny of structural assumptions and historical context.

Biomedical studies illustrate a different set of challenges for mathematical statistics, frequently involving heterogeneous populations and imperfect measurements. Statistical models may need to distinguish a genuine association from the influence of confounding variables or from chance variation. Robust methods can be valuable when isolated measurements strongly influence model coefficients, but they cannot correct a fundamentally unsuitable study design. Multivariate techniques help organise complex combinations of biomarkers and clinical characteristics. Classification algorithms may support exploration, while valid conclusions about performance require external validation and an appropriate evaluation protocol. Czech statistical research provided methods and trained specialists capable of engaging with such problems, often through interdisciplinary collaboration. The relevance of mathematical statistics thus arose from its capacity to specify what empirical evidence could justify. This approach preserves the difference between an informative statistical association and a stronger claim of causal explanation. Such collaborations required statistical caution precisely because the scientific consequences of overinterpretation could be substantial.

Engineering applications required statistical methods that could accommodate measurement noise, dynamic behaviour and uncertainty in operating conditions. Automated systems may generate streams of observations whose characteristics change as components age or external circumstances alter. Change-point methods can indicate possible changes in system behaviour, while probabilistic models can help quantify the remaining uncertainty. Bayesian and adaptive approaches also offer mathematical frameworks for updating a model as new evidence becomes available. Researchers working across statistics and systems science in Czech institutions considered these kinds of connections. Their work depended on cooperation with specialists in optimisation, numerical analysis, information processing and control. Statistical conclusions are particularly consequential when they influence a physical decision rather than merely describe historical observations. The growth of these applications therefore reinforced the need for interpretable inference supported by sound mathematical assumptions. Statistical mathematics thus contributed to engineering reliability through explicit evaluation of uncertainty and change. The same distinction matters in the design of dependable monitoring and automatic decision systems.

The international publication culture of Czech statistics remained diverse, encompassing specialist journals, major monographs and interdisciplinary outlets. Research on robust

methods, time-series change points and conditional independence appeared in publications addressed to different but overlapping scientific communities. Kybernetika provided an important journal environment for work joining statistical mathematics with cybernetics and information sciences. The Czech Digital Mathematics Library helped preserve access to substantial parts of this publication record. International books by Jurečková and Studený made distinctive theoretical programmes available to audiences extending beyond Czech academic networks. The value of these outputs cannot be reduced to numerical publication counts because a mathematical theorem, methodological monograph and applied case study serve different functions. Scientific assessment must distinguish original theoretical advances from exposition, implementation and participation in collaborative projects. Such distinctions are necessary for a historically accurate interpretation of Czech statistical achievement. The publication record reveals intellectual continuity while documenting the gradual broadening of disciplinary connections.

Education and doctoral supervision were essential to the reproduction of Czech expertise in mathematical statistics across the three decades. University courses in probability, mathematical statistics and econometrics linked formal mathematical training with problems arising in empirical disciplines. Students increasingly encountered statistical computing as part of their professional preparation, alongside asymptotic theory and probability. The work of established researchers became visible in dissertations on robust regression, classification, heteroscedasticity and related topics. Collaboration between Charles University and institutes of the Czech Academy of Sciences facilitated contact with active research questions. International seminars and conferences provided additional opportunities to compare mathematical methods and to develop independent research networks. The transmission of expertise therefore involved not merely teaching a catalogue of procedures but learning how to formulate and test their assumptions. A sustainable scientific community depends on this capacity for critical methodological judgement across generations. Future progress depended as much on methodological education as on access to increasingly powerful computational infrastructure. Research training thereby preserved the link between mathematical originality and responsible empirical interpretation.

By 2025, Czech mathematical statistics combined several mutually reinforcing research traditions rather than a single dominant theoretical programme. Robust and nonparametric inference retained an identifiable intellectual profile, while change-point methods addressed dependence and evolving data-generating processes. Probabilistic graphical models connected statistical reasoning with combinatorics and the formal representation of knowledge. Bayesian estimation and adaptive decision-making linked uncertainty quantification to systems science and practical control problems. Multivariate analysis and statistical pattern recognition responded to the increasing complexity of computational data. Across these areas, the most consequential achievements were not merely new computer programmes but mathematical accounts of when procedures could be trusted. Persistent challenges included model sensitivity, high dimensionality, reproducibility and the interpretation of algorithmic results. The Czech experience demonstrates that the advance of data-driven research depends on the continued development of rigorous and critically examined inferential foundations. Taken

together, these developments show mathematical statistics functioning as a foundational science of reliable inference in an increasingly data-intensive world.

Optimisation, Operations Research, and Mathematical Decision-Making

The significance of optimisation in Czech mathematics after 1993 lies in the continuing exchange between rigorous mathematical reasoning and problems arising in engineering, industry, economics, and public administration. The independent republic inherited a substantial tradition of applied analysis, cybernetics, numerical mathematics, and decision theory from the Czechoslovak research system. Political and institutional restructuring altered the conditions under which these traditions were financed, evaluated, and linked to practical users. Yet the central mathematical questions remained demanding: whether a feasible solution exists, whether an optimum can be characterised, and whether an implementable algorithm can find it reliably. In many applications, the relevant variables are constrained simultaneously by physical laws, scarce resources, and the actions of other decision-makers. Consequently, optimisation cannot be reduced to an exercise in minimising a formula without examining the model that produced it. Czech research in this field developed through several complementary communities rather than a single national methodological school. Institutes of the Czech Academy of Sciences, universities in Prague and other cities, and internationally connected researchers supplied distinct approaches to theoretical and applied problems. The period from 1993 to 2025 therefore offers a revealing account of how mathematical optimisation became an increasingly important bridge between abstract analysis and complex decision-making. This perspective is essential when assessing innovation over several decades.

Linear programming provided an essential foundation for the practical application of mathematical optimisation during the transformation of the Czech economy. Its distinctive feature is the representation of objectives and restrictions by linear functions, allowing many large allocation problems to be handled systematically. Production planning, blending, transport allocation, and capacity management could be analysed through models that made assumptions and trade-offs explicit. Such formulations were useful precisely because they transformed vague managerial preferences into a set of mathematically examinable alternatives. Nevertheless, real production processes frequently introduced integrality requirements, uncertain demand, or nonlinear technical relationships that exceeded the reach of a simple linear model. Mixed-integer programming addressed some of these difficulties by representing discrete choices such as whether a facility should open or a vehicle should be assigned. The existence of mature general-purpose solvers did not remove the need for research into stronger formulations, relaxations, and problem decomposition. Czech universities consequently had reason to teach linear optimisation both as a classical mathematical subject and as the entry point to more sophisticated operations research. The resulting distinction between a model, its numerical solution, and the managerial decision remained central to responsible applied mathematics.

Convex analysis occupies a pivotal position because geometric structure can convert a potentially difficult optimisation problem into one with powerful existence and optimality results. Convex feasible sets contain the segment joining any two of their points, while convex objectives constrain the geometry of local minima. Under appropriate hypotheses, first-order conditions provide global rather than merely local information about an optimum. Duality then offers another perspective, relating constrained problems to multipliers and bounds that can have practical economic interpretations. This conceptual framework is particularly valuable in mathematical programming, statistical estimation, energy systems, and engineering design. Czech contributions to optimisation have drawn upon these ideas while investigating situations where classical smoothness or convexity assumptions do not apply. Understanding a convex relaxation may reveal useful bounds even when the underlying application contains combinatorial decisions or equilibrium constraints. The distinction between mathematical guarantees and empirical numerical performance is especially important in such cases. A numerically convincing answer is not automatically a proof that the global optimum has been found. Its relevance extends beyond any individual software implementation.

Nonlinear programming broadened the scope of Czech applied mathematics because realistic constraints often involve nonlinear responses, feedback effects, or geometric relations. Engineers may need to optimise an object whose deformation changes the loads it experiences, while economists may investigate a market where prices depend on strategic behaviour. These settings expose the limitations of algorithms built exclusively for smooth and unconstrained objectives. Even differentiable nonlinear programmes may possess multiple stationary points, with no simple rule identifying the best one. Researchers therefore examine constraint qualifications, second-order optimality conditions, local convergence, and sensitivity to perturbations. Such theoretical issues influence whether numerical answers can be trusted under changing experimental or organisational conditions. The work of Jiří Outrata and his collaborators illustrates the importance of treating difficult nonsmooth structures directly rather than hiding them within unrealistic approximations. Their research linked mathematical analysis to optimisation problems in which equilibrium relationships occur among the constraints. This direction offered a distinctive way to connect theoretical optimisation with mechanics, economics, and design.

A particularly significant achievement of the post-1993 period was the systematic treatment of mathematical programmes with equilibrium constraints. In these problems, a decision-maker optimises an objective while requiring another system to satisfy an equilibrium condition. The subordinate equilibrium may represent contact forces in mechanics, competing agents in a market, or a network response to imposed controls. Standard nonlinear programming conditions can become inadequate because the feasible set often has exceptional nonsmooth or degenerate features. Outrata, Michal Kočvara, and Jochem Zowe addressed these difficulties in their 1998 monograph on nonsmooth approaches to equilibrium-constrained optimisation. The book developed theoretical tools and numerical procedures while examining contact, elasticity, and economic examples. Its importance lies not merely in extending a catalogue of algorithms but in clarifying which kinds of mathematical information remain available under implicit constraints. The collaboration also exemplified the international character of Czech

mathematical research during the first decade of independence. It connected expertise based in Prague with researchers working elsewhere in Europe. The scholarly value of the result rests upon its carefully stated assumptions.

Variational inequalities provide an alternative language for describing equilibrium and constrained optimisation. Instead of demanding that a gradient vanish at a solution, a variational inequality expresses a one-sided relation between an operator and admissible perturbations. The formulation is well suited to contact mechanics, constrained dynamics, competitive equilibrium, and several classes of network models. However, the mapping that defines a variational inequality need not be monotone, and the admissible region need not possess the regularity assumed in elementary theory. Stability analysis therefore becomes essential when inputs, loads, prices, or institutional rules change. Czech investigations of generalized equations and implicit multifunctions have advanced the mathematical framework for addressing such perturbations. Research by Outrata and collaborators examined how solution mappings respond to disturbances and when Newton-type methods can be adapted to their structure. These results illuminate the difference between finding one feasible equilibrium and understanding whether that equilibrium is robust. For applied decision-making, the latter question may be as important as the computed solution itself.

Nonsmooth optimisation is especially relevant when objectives involve absolute values, maximum functions, switching regimes, or complementarity relationships. Such expressions occur naturally in robust estimation, transaction costs, mechanical contact, and systems with thresholds. Classical derivatives are then unavailable at precisely the points where an optimum may occur. Generalised derivative concepts and variational analysis provide tools for formulating optimality conditions without pretending that the problem is smooth. The mathematical issue is not simply how to run a familiar algorithm on difficult data but how to define the local geometry of an irregular feasible set. Research associated with Czech optimisation groups contributed to this broader international methodological debate. In 2021, Helmut Gfrerer and Jiří Outrata published a semismooth Newton approach for solving generalised equations. Such methods exploit carefully formulated derivative information while confronting the irregularity of the original model. Their development shows why rigorous analysis remains indispensable in computationally oriented optimisation. Careful validation remains necessary before a numerical method is deployed.

Semidefinite programming expanded the possibilities of mathematical optimisation by replacing some scalar inequalities with positive-semidefiniteness conditions on matrices. These constraints arise in control theory, structural design, polynomial optimisation, and relaxations of difficult discrete problems. Although the resulting programmes may be convex, their matrix structure creates substantial computational challenges when models become large. Sparse data, efficient linear algebra, and reliable treatment of nonlinear constraints consequently determine practical success. Michal Kočvara and Michael Stingl presented PENNON in a 2003 article describing an algorithmic approach to convex nonlinear and semidefinite programming. Their method used an augmented-Lagrangian framework and was designed to handle combinations of different constraint classes. The publication provided mathematical explanation alongside numerical comparisons rather than offering software without analysis.

PENNON illustrates how a theoretical framework can be made operational through careful algorithm engineering. Its development is particularly relevant to the relationship between Czech mathematical expertise and international numerical optimisation research.

A semidefinite model is not merely a more complicated version of linear programming; it expresses a different geometry of feasible solutions. Positive-semidefinite matrices encode conditions involving eigenvalues, covariance structures, and quadratic forms. These properties make matrix constraints attractive for control systems and the design of reliable physical structures. Yet the computational cost of matrix factorisations can quickly become decisive as the dimension increases. Theoretical understanding must therefore be combined with algorithmic attention to conditioning, sparsity, and stopping criteria. When such issues are ignored, a formally elegant convex formulation may be impractical for a large engineering problem. Czech work on semidefinite methods exemplifies the value of viewing mathematical theory and numerical implementation as complementary pursuits. The most useful research does not treat computational performance as a substitute for mathematical justification. Instead, it establishes which numerical approximations preserve the guarantees promised by the model. Numerical scalability is never independent of mathematical structure.

Optimal control investigates decisions that evolve over time and influence the subsequent state of a dynamic system. Its mathematical formulation commonly involves a state equation, admissible controls, an objective functional, and conditions describing the beginning or end of a process. Industrial applications may concern energy consumption, process regulation, vehicle motion, or the management of physical resources. Unlike static programming, the decision at one time can alter what becomes feasible later. This interdependence makes dynamic programming, variational methods, and differential-equation techniques particularly important. Czech traditions in control theory continued after independence through research institutes and technical universities. The Institute of Information Theory and Automation maintained a profile linking systems science, applied mathematics, and decision-making. The substantive contribution of such work is to connect optimality with feedback, adaptation, and uncertain observations. A control policy must function while the underlying system changes, not only in an idealised calculation made in advance.

Miroslav Kárný and Tatiana V. Guy offered a mathematically distinctive approach to stochastic control in their 2006 article on fully probabilistic control design. The framework describes both actual and desired closed-loop behaviour through probability distributions. It then chooses a controller by comparing these distributions with an information-theoretic divergence. This method shifts attention from a single deterministic target value towards the probabilistic character of system behaviour. Such a shift is meaningful when measurements are uncertain, external influences are variable, and control decisions must be repeatedly updated. The authors showed how a unified probabilistic formulation could address discrete-time state-space settings. Their work belongs to an intellectual tradition linking statistical inference, information theory, and control rather than treating these subjects as unrelated specialisms. It also exemplifies a broader Czech interest in decision-making under incomplete knowledge. The mathematically appropriate decision is then one that reflects uncertainty explicitly instead of

concealing it behind an arbitrary point estimate. Information quality is consequently part of the mathematical problem.

Decision theory extends optimisation beyond the mechanical selection of the smallest numerical value. A decision-maker must define what counts as a desirable outcome and how competing outcomes should be compared. In public administration and management, objectives may include expenditure, reliability, time, environmental impact, and social acceptability. Multi-criteria methods make conflicts between these aims visible, but their results depend on the meaning and justification of the adopted preferences. Weights and rankings are therefore not neutral substitutes for policy judgement. Czech research in the theory of decision-making has examined formal procedures for handling uncertainty and interacting agents. The Department of Decision-Making Theory at the Institute of Information Theory and Automation represents an institutional setting for such work. Mathematical methods can improve transparency by showing how a recommendation changes when assumptions change. They cannot eliminate the ethical and institutional responsibility of selecting the objectives themselves.

Operations research brings several mathematical disciplines together when coordinating resources across complex organisations. Network flow, scheduling, routing, queueing models, integer programming, and simulation each address different aspects of organised activity. A transport operator may need to reconcile passenger connections with fleet capacity, operating costs, maintenance requirements, and uncertain travel times. A manufacturer may confront a comparable problem when allocating machines, workers, and materials across competing orders. The value of an operations-research model lies in representing these interactions without losing sight of their practical meaning. However, introducing every real-world detail can make exact optimisation computationally prohibitive. Analysts must therefore balance model fidelity, solution quality, and the time available for decisions. Czech technical education and applied research have provided contexts in which these compromises are investigated systematically. The discipline is best understood as a partnership between mathematical abstraction and knowledge of the processes being modelled. Problem formulation remains as important as the choice of solver.

Combinatorial optimisation becomes unavoidable when decisions concern indivisible objects or mutually exclusive alternatives. Vehicle assignments, timetable slots, depot locations, and production sequences cannot generally be represented by continuous variables alone. The discrete nature of such choices may make a superficially simple problem computationally intractable at scale. Exact algorithms respond with bounding, branching, cutting planes, or decompositions that exploit the mathematical structure. Heuristics and metaheuristics may instead deliver useful solutions within a strict operational deadline. Neither approach should be treated as universally superior because their appropriateness depends on the application and the quality required. Czech work at the intersection of operations research and theoretical computer science has helped preserve awareness of algorithmic complexity. Practical models also require careful verification that the solution satisfies all essential constraints. Efficiency gains have little value if they depend on violating a safety rule or ignoring a critical resource limitation.

Transport optimisation offers a particularly clear example of the relationship between mathematics and public infrastructure. Planning a route network involves the geography of demand, the location of transfer points, vehicle availability, and time-dependent travel conditions. Timetable design introduces additional interactions because changes on one line can propagate to connecting services. Mathematical models can compare alternative operating schemes more consistently than intuition alone. Nevertheless, a model constructed from average journey times may fail when congestion or disruptions become prominent. Robust planning therefore requires scenarios, sensitivity analysis, and explicit attention to uncertainty. Research and teaching at the Czech Technical University in Prague reflect the practical relevance of network algorithms, scheduling, and transport decision-making. Individual projects should be assessed on their documented methods and outcomes rather than assumed to have transformed an entire national system. The broader scientific contribution is the development of transferable techniques for making large interconnected transport decisions more intelligible. The resulting plans must be assessed under realistic operating conditions.

Uncertainty changed the mathematical character of many Czech optimisation applications during the twenty-first century. Conventional deterministic programmes assume that the parameters entering the model are known, even when they represent future costs or demand. Stochastic programming replaces some of these quantities with random variables or scenarios. Chance constraints restrict the probability that a prescribed operational condition will be violated. Their appeal is obvious in financial risk, electricity markets, logistical scheduling, and production planning. Their mathematical difficulty is equally significant because probability constraints can be nonconvex or hard to evaluate. Researchers therefore investigate approximation, decomposition, and the regularity of the resulting feasible sets. Czech-associated work in stochastic optimisation has contributed to this international agenda through collaborations crossing institutional and national boundaries. Such research shows that a realistic decision problem often requires explicit statements about what is unknown as well as what is controllable.

Martin Branda and collaborators provide concrete examples of the development of stochastic optimisation within Czech mathematical research. A 2020 article with Lukáš Adam, Holger Heitsch, and René Henrion investigated joint chance-constrained problems using regularisation and Benders decomposition. The combination is important because several probabilistic restrictions must be satisfied together, rather than each being treated independently. The authors investigated algorithmic approaches to difficult problems where uncertainty and feasibility are closely intertwined. Related research examined distributionally robust scheduling when task completion times are uncertain. These contributions illustrate the connection between sophisticated probability models and the practical need to allocate resources. They also reveal why problem decomposition matters when the complete model is too costly to solve directly. International co-authorship in these studies should be understood as a strength of Czech scientific participation, not as grounds for exclusive national attribution. The resulting methods belong to the wider shared development of modern operations research. The interaction of uncertainty and discrete choices remains mathematically challenging.

Financial optimisation adds another layer of complexity because the objective itself may depend upon the decision-maker's attitude towards risk. Maximising expected returns does not generally produce the same decision as controlling the probability or magnitude of severe losses. Portfolio selection therefore involves the interaction of uncertain outcomes, regulatory restrictions, transaction costs, and diversification. Mathematical models can incorporate risk measures, stochastic dominance, or limits on the number of selected assets. Yet the results remain sensitive to the quality and stability of the assumed probability distributions. Czech publications on portfolio and risk optimisation have investigated how these features affect both feasible decisions and algorithmic behaviour. Work by Branda and co-authors illustrates research connecting continuous optimisation with cardinality restrictions and statistical uncertainty. Such connections are mathematically demanding because discrete selection decisions interrupt the smooth geometry of traditional portfolio models. The broader lesson is that a mathematically precise solution may still depend on imperfect estimates of future economic conditions.

Energy markets have created a fertile context for mathematical programming with uncertainty and strategic interaction. Producers choose quantities and prices while responding to uncertain demand, regulation, network restrictions, and the anticipated behaviour of competitors. A model of one producer's optimal response may consequently be embedded in a wider equilibrium problem. These features connect stochastic optimisation, variational analysis, and economic decision theory. In 2023, Martin Branda, René Henrion, and Miroslav Pištěk published a value-at-risk approach to a producer's best response under uncertain electricity demand. The study illustrates the practical relevance of examining adverse outcomes rather than assuming a single forecast will materialise. Such models can clarify which assumptions are driving an apparently optimal generation or trading strategy. They should not, however, be confused with predictions that automatically remain valid under unexpected regulatory or geopolitical changes. Their greatest scientific value lies in making the structure of uncertain strategic decisions amenable to analysis. Changes in policy can alter the feasible set as well as the objective.

Industrial optimisation often begins with a physical model rather than an accounting spreadsheet. The design of a mechanical component may be restricted by equilibrium equations, material properties, loading conditions, and geometric manufacturability. Shape and topology optimisation seek improvements while maintaining these technical constraints. Contact problems introduce additional inequalities because surfaces may press against each other without interpenetrating. These relationships link optimisation directly to variational inequalities and nonsmooth mechanics. The Czech-associated work of Outrata and Kočvara provides a foundation for understanding such equilibrium-constrained designs. Semidefinite and nonlinear programming methods can also become relevant when structural requirements are formulated through matrix or quadratic constraints. The challenge is to make mathematical improvements that remain meaningful once numerical approximations and engineering tolerances are considered. This requirement explains why collaboration between mathematical analysts and engineers has been important throughout the period.

Optimality conditions are essential because a numerical algorithm can terminate without proving that its answer is globally best. For unconstrained smooth problems, stationarity is a

familiar necessary condition, but constraints substantially alter the picture. Lagrange multipliers encode the relationship between feasible perturbations and the objective near an optimum. Complementarity conditions express the fact that some restrictions bind while others remain inactive. In degenerate or nonsmooth problems, however, standard multiplier theory may fail or require a revised formulation. Czech work in variational analysis has addressed precisely these limitations through generalised derivatives and stability conditions. Such theoretical developments can appear remote from applications until an apparently plausible numerical solution behaves unpredictably under small changes. Establishing appropriate qualifications and sensitivity results then becomes crucial for interpreting the computation. Mathematical rigour is therefore not an ornamental addition to optimisation software but one of its safeguards. The meaning of a computed stationary point depends on the relevant hypotheses.

Sensitivity analysis studies how an optimal solution or objective value changes when the data of the model are perturbed. A transport schedule may be efficient under ordinary conditions but highly vulnerable to modest increases in travel time. A financial portfolio can appear optimal while being exceptionally sensitive to estimated correlations. An industrial design may satisfy all nominal constraints yet fail when a material coefficient is slightly different. These examples demonstrate that practical usefulness requires more than a single calculated optimum. Dual information and parametric analysis can identify influential constraints and reveal the cost of additional resources. For nonlinear or equilibrium-constrained systems, the corresponding mathematics becomes considerably more subtle. Work on solution multifunctions and generalised equations provides one way to formalise such changes. The ability to distinguish structural robustness from accidental numerical success is a defining feature of mature optimisation research.

Multi-objective optimisation addresses situations where competing requirements cannot be combined without making value judgements. A production system may seek lower cost, improved reliability, reduced emissions, and greater worker safety simultaneously. A transport agency may wish to reduce expenditure while improving accessibility and service frequency. Solutions can then be compared through Pareto efficiency, which rejects alternatives that are worse in every objective. Yet Pareto efficiency alone rarely identifies one uniquely preferred decision. Additional preference information, compromise methods, or institutional negotiation remains necessary. Czech mathematical work in decision theory and applications to engineering provides a setting for investigating these methodological choices. The distinction between optimisation and decision-making is particularly important when several legitimate stakeholders are affected. Mathematics can clarify the available trade-offs without claiming authority to choose society’s priorities. Public decision-making demands additional attention to procedural fairness.

The expansion of computational capacity altered the kinds of optimisation questions that could be investigated in Czech research organisations. Large sparse models, complex simulations, and scenario-based decisions can require substantial memory and parallel processing. Access to powerful machines facilitates experiments with formulations that would previously have been computationally impractical. However, increasing processor speed does not change the mathematical complexity class of an algorithm. Poor conditioning, inadequate modelling

assumptions, and badly structured constraints may continue to undermine a calculation regardless of hardware. The growth of high-performance computing in the Czech Republic therefore strengthened the relevance of algorithmic mathematics rather than diminishing it. Researchers must determine how decomposition, parallelisation, and efficient linear algebra interact with the particular problem structure. Benchmarking requires reproducible data and transparent reporting of numerical tolerances. A credible computational achievement is one whose assumptions and accuracy can be examined independently.

Data-driven optimisation introduces a further methodological question: how should a decision be made when the model is learned from incomplete observations? Estimated demand, inferred risk, and predicted equipment failures are not fixed mathematical facts. They carry sampling uncertainty, modelling error, and possible bias from changes in the environment. Integrating predictive models with optimisation can improve responsiveness, but it also propagates statistical limitations into the recommended decision. Robust and distributionally robust formulations attempt to guard against some of these vulnerabilities. Their effectiveness depends on whether the assumed uncertainty sets are themselves justifiable. Czech research linking applied statistics, information theory, and decision science offers a natural basis for such investigations. The connection has become increasingly relevant with the expansion of machine learning and automated management systems. Nevertheless, predictive accuracy and decision quality are distinct performance criteria and should be evaluated separately. The data-generating process can change after the model has been fitted.

The institutional geography of Czech optimisation is more diverse than any single list of individual discoveries might suggest. The Institute of Information Theory and Automation connects applied mathematics with system science, probabilistic reasoning, and decision-making. Charles University contributes through mathematical statistics, stochastic programming, and related theoretical disciplines. The Czech Technical University in Prague provides engineering and transportation contexts where optimisation models can be evaluated against operational requirements. Other universities and internationally affiliated researchers contribute through computational mathematics, mechanics, discrete algorithms, and applied economic modelling. This distributed structure has encouraged the exchange of methods across traditional disciplinary boundaries. At the same time, projects vary widely in scale and scientific purpose, making simple institutional rankings unreliable. The contributions of internationally mobile mathematicians also resist classification by a single national affiliation. A historically responsible account therefore follows identifiable research results and collaborations rather than assuming every development belongs exclusively to one institution.

The period from 1993 to 2025 also changed expectations concerning the practical accountability of mathematical models. In economically significant applications, optimisation results may influence employment, public expenditure, logistics, and access to services. Model developers therefore need to disclose the assumed objectives, the limits of available data, and the circumstances under which a solution may no longer be reliable. Optimisation can produce a mathematically correct answer to an improperly formulated question. The distinction becomes especially consequential when automated systems translate numerical recommendations into institutional action. Sensitivity testing and independent validation can reveal weaknesses that a

successful solver run leaves hidden. Czech research in mathematical programming, stochastic control, and decision science offers tools for examining such issues with analytical precision. Their application nevertheless requires cooperation with domain specialists and decision-makers who understand the substantive constraints. Responsible mathematical innovation consists partly in recognising where formal models should inform judgement rather than replace it. Mathematical clarity cannot by itself resolve disagreements about social priorities.

Viewed as a whole, Czech optimisation research during 1993–2025 developed through the interaction of deep mathematical theory, international collaboration, and increasingly demanding applications. Nonsmooth and equilibrium-constrained programming demonstrated the value of studying irregular structures that do not fit elementary optimisation assumptions. Semidefinite methods and algorithmic software showed how new mathematical formulations could be translated into computational tools. Research on stochastic programming, probabilistic control, and decision theory addressed the central difficulty of acting under uncertainty. Transport, industrial design, energy markets, and financial planning supplied settings in which methodological choices acquired practical consequences. No single method can cover all these settings because their feasible sets, uncertainty structures, and time horizons differ fundamentally. The most durable advances therefore concern mathematical insight into particular problem classes and reliable procedures for solving them. Czech contributions are best understood as part of an international research conversation in which theories, algorithms, and applications develop reciprocally. This trajectory illustrates how optimisation became not only a technical instrument but also a rigorous language for analysing complex choices in contemporary society.

Mathematical Physics and Spectral Theory: Mathematics at the Foundations of Physical Science

The history of mathematical physics in the Czech Republic after 1993 illustrates how a relatively small research community could influence the conceptual foundations of quantum theory through sustained mathematical specialisation. Its central achievements arose less from the ownership of large experimental apparatus than from the construction of rigorous models for phenomena that experimental physics had made intellectually pressing. The relevant questions concerned spectra of differential operators, propagation along constrained geometries, resonance formation, and the mathematical meaning of idealised quantum networks. Such questions required a productive exchange between functional analysis, differential geometry, partial differential equations, and physical intuition. The constitutional establishment of the independent republic did not originate these traditions, which had developed within Czechoslovak universities and Academy institutes. It nevertheless altered the funding, mobility, and collaborative conditions in which established groups could pursue them. Prague became an important meeting place for researchers who approached quantum physics as a mathematically

exact study of operators and boundary conditions. The work of Pavel Exner provides an unusually coherent perspective upon this intellectual transformation. His publications trace the movement from individual spectral problems towards a substantial international programme concerning quantum waveguides and graphs.

A mathematical model acquires physical significance only when its assumptions and conclusions can be interpreted without confusing idealisation with empirical reality. Spectral theory makes that distinction particularly clear because an operator spectrum encodes mathematically defined possibilities rather than an immediate picture of a physical experiment. In quantum mechanics, a Hamiltonian is commonly realised as a self-adjoint operator on a Hilbert space, and its spectral properties inform the interpretation of energy and dynamical evolution. Defining the appropriate operator requires specifying its domain, a step that may be as consequential as writing the differential expression itself. Boundary conditions distinguish reflecting, transmitting, and interacting configurations even when the formal differential operator appears unchanged. Continuous spectra, discrete eigenvalues, and resonant phenomena should therefore be analysed as distinct mathematical objects. Czech research in this area has benefited from the unusually close connection between abstract operator theory and geometric questions about thin physical structures. Mathematical innovation often occurs when that connection produces a theorem describing a previously unexplained qualitative effect. The most influential results do not simply compute a particular energy level but establish why an entire family of models exhibits stable behaviour. This gives the Czech contribution importance beyond the immediate systems that inspired the original calculations.

The programme of quantum waveguides offers an instructive starting point because it translates familiar geometric shapes into precise spectral questions. A waveguide is modelled by a region or network that constrains quantum propagation, with the geometry of the region determining the admissible states. An ideal straight strip may support transport above appropriate energy thresholds, whereas bending and local perturbations can change the spectral picture. The central issue is whether geometry alone can create eigenstates localised in a portion of the structure. A historically important investigation by Exner and Petr Šeba, published in 1989, established bound-state results for curved quantum waveguides and thus predates the independent Czech state. This earlier paper must be understood as a foundation of the post-1993 research programme rather than incorrectly presented as a discovery made during the period of this volume. Subsequent studies extended the mathematical treatment to more complicated geometries, dimensions, boundary conditions, and perturbations. The line of enquiry also made visible the delicate distinction between intuitive confinement and a rigorous existence proof. Its enduring significance lies in demonstrating that geometry can act as an effective source of spectral localisation.

A curved guide poses analytical difficulties that do not arise in the elementary separation of variables for a straight domain. Curvilinear coordinates introduce metric coefficients and additional terms that encode the effects of bending. These terms may be interpreted through effective Hamiltonians, although their precise form depends upon the asymptotic regime and boundary conditions. A rigorous result must control the resulting operator rather than rely on a suggestive effective potential alone. Variational methods can establish the existence of

spectrum below a continuum threshold by constructing appropriate trial functions. Comparison arguments and operator inequalities provide further tools for excluding spurious conclusions or bounding spectral quantities. Research associated with Prague explored how these analytical mechanisms survive changes in geometry and external fields. This interplay became especially fertile because the same formal methods could be adapted to strips, tubes, layers, and network-like configurations. The mathematics thereby joined differential geometry to functional analysis without reducing either discipline to an auxiliary role. In the Czech case, the resulting theoretical school sustained a recognisable identity across several decades of international collaboration.

The evolution from straight and curved strips towards tubular structures enlarged both the mathematical difficulty and the potential applications of the theory. A tube can possess curvature and torsion, and its cross-section may vary along the longitudinal direction. These geometric features affect transverse modes, thresholds of continuous spectra, and the coupling between different propagation channels. Deriving a simplified one-dimensional description requires careful attention to the smallness of the transverse scale. Not every apparent one-dimensional limit gives the same effective operator or interaction law. Mathematical physicists consequently distinguish convergence of eigenvalues from stronger forms of operator convergence that also control resolvents. Such distinctions are indispensable when a model is intended to describe transport rather than merely isolated bound states. Czech-led research contributed to this broader theoretical movement through both original papers and later systematic synthesis. It helped establish quantum waveguides as a domain in which geometric intuition generates questions answerable through exact spectral methods.

The 2015 monograph Quantum Waveguides by Pavel Exner and Hynek Kovařík provides a particularly valuable marker of the maturity of this research direction. Published within Springer's Theoretical and Mathematical Physics series, the book presents a comprehensive treatment of geometrically induced bound states and transport in constrained quantum systems. Its chapters also address spectral properties of layers, local perturbations, weak coupling, external fields, thin-network limits, periodic structures, and leaky waveguides. These topics form a unified field because they examine how operator spectra change under geometric or physical modification. The monograph does not claim that every result originated in Czechia, and its significance rests partly on the systematic incorporation of international achievements. It nevertheless demonstrates the intellectual leadership of a Czech mathematical physicist in defining the organisation and principal problems of an international speciality. Books of this kind constitute scientific achievements in their own right because they stabilise terminology, methods, and standards of proof. They also help doctoral researchers recognise which conjectures have been resolved and which difficulties remain open. The volume's detailed bibliography makes it a useful bridge between historical reconstruction and technical mathematical research. Its publication illustrates how the research trajectory established before 1993 continued to generate internationally important scholarship decades later.

Quantum graphs provide a second major direction in which Czech mathematical physics became internationally visible. A quantum graph consists of edges equipped with differential operators and vertices carrying matching or coupling conditions. Although the underlying combinatorial object is discrete, the quantum state varies continuously along the edges. This

hybrid nature makes quantum graphs especially useful for studying networks in which propagation is essentially one-dimensional. A graph Hamiltonian cannot be specified by connectivity alone because conditions at the junctions determine its self-adjoint realisation. Kirchhoff-type conditions model one familiar form of continuity and flux balance, while other self-adjoint couplings encode different interactions. Such models allow researchers to investigate transport, scattering, localisation, and spectral gaps within a mathematically tractable framework. The Czech contribution involved both the analysis of particular couplings and the deeper question of how these models arise as limits of physical structures. Their value is therefore methodological as well as descriptive.

The passage from a thin network to a quantum graph is considerably subtler than a drawing of narrow tubes might suggest. An actual network has nonzero thickness and junction regions with geometries that may differ substantially from its edges. Shrinking those dimensions can lead to distinct limiting boundary conditions, depending upon the relative rates at which different parts contract. It would therefore be mathematically misleading to assume that every thin network converges to a single universal graph operator. One must specify the family of domains, the relevant Laplacians, and the sense in which convergence is claimed. The geometry of vertex neighbourhoods can preserve interactions that disappear under a different scaling regime. This sensitivity makes the limiting problem both physically interesting and technically demanding. Czech researchers participated in resolving these questions using precise operator-theoretic and geometric arguments. The resulting theorems clarify when a convenient graph approximation is justified rather than merely plausible. This feature illustrates the sustained exchange between mathematical innovation and the interpretation of physical systems.

A decisive example is the article Convergence of spectra of graph-like thin manifolds by Pavel Exner and Olaf Post, published in the Journal of Geometry and Physics in 2005. Their analysis considers compact manifold structures becoming thinner until their geometry approaches that of a graph. Under specified scaling assumptions, they establish convergence towards the spectrum of a graph Laplacian with Kirchhoff conditions at vertices. When vertex regions contract sufficiently differently from the edges, the limit instead produces decoupled edges with Dirichlet endpoint conditions. A borderline scaling regime yields a further possibility with nontrivial vertex coupling. The result is important precisely because it rejects the intuitive expectation that all geometrically thin networks behave in the same way. It reveals a mathematical mechanism through which microscopic junction geometry affects effective macroscopic connectivity. The work also provided an analytical foundation for later investigations of quantum transport in complex networks. Its collaborative character illustrates the internationalisation of Czech mathematical physics during the early twenty-first century.

Spectral convergence must be interpreted with caution because a sequence of changing domains does not automatically supply a fixed underlying Hilbert space. Operators describing thin manifolds and those describing graph edges may naturally act in different function spaces. Establishing a relation between these spaces is therefore part of the proof rather than a purely notational detail. Appropriate identification mappings allow the comparison of eigenfunctions, resolvents, and associated spectral information. Norm-resolvent convergence can sometimes provide stronger control than convergence of selected eigenvalues alone. However, the precise

conclusions depend on whether the problem involves compact or noncompact structures and on the relevant spectral region. Later mathematical literature extended techniques for analysing such singular geometric limits. The Exner–Post collaboration occupies an important position in that development because it made the scaling dependence of vertex conditions explicit. Its value illustrates a characteristic form of innovation in pure mathematics: clarifying the hypotheses that determine which limiting theory is valid. The distinction also helps clarify why this mathematical work acquired importance beyond its immediate physical motivation.

Vertex coupling itself became a productive object of research within the quantum-graph literature. Different conditions at a junction can alter reflection, transmission, spectral degeneracy, and the presence of bound states. Some couplings are represented by delta-type interactions, while more singular conditions require careful approximation procedures. In 2004, Taksu Cheon and Pavel Exner examined an approximation to delta-prime couplings on graphs. Their work addressed the possibility of obtaining less elementary effective junction rules from more accessible interaction models. Such constructions help distinguish mathematically admissible boundary conditions from conditions that can be approximated by particular physical arrangements. A crucial question is whether the approximating operators converge in a mathematically controlled sense. The larger significance is that interactions can sometimes be encoded geometrically or through concentrated potentials rather than introduced as unexplained formal assumptions. This research strengthened the connection between extension theory and concrete modelling of quantum networks.

Scattering theory gives the study of quantum graphs and waveguides a natural dynamical dimension. Rather than investigating only bound energy levels, scattering analysis asks how incoming waves are redistributed by geometry and interaction. The scattering matrix records the amplitudes linking incoming and outgoing channels under suitable assumptions. Its poles and analytic continuation may reveal resonances associated with states that are temporarily localised but not permanently bound. Mathematical care is essential because resonances are not simply ordinary eigenvalues of a self-adjoint Hamiltonian. Their formal description can depend upon complex scaling, resolvent continuation, or equivalent frameworks adapted to a particular problem. Czech mathematical physicists have investigated the relation between graph connectivity, coupling conditions, and resonant transport. This line of work makes clear why spectral theory and scattering theory should be treated together without being conflated. The resulting methods contribute to a rigorous account of how networks respond to perturbations. Its consequences deserve attention when assessing the international development of Czech mathematical physics.

A resonance often emerges when an apparently isolated state is coupled weakly to a continuum of propagating states. Its mathematical signature may appear as a pole displaced from the real axis in an analytically continued object. The imaginary component is then associated, under appropriate conventions and assumptions, with the width or lifetime of a metastable configuration. Such language is physically suggestive but requires a precise analytic setting to acquire rigorous meaning. In networks, seemingly minor changes to vertex coupling or edge lengths may strongly affect resonance positions. The phenomenon highlights the sensitivity of transport to geometry, symmetry, and interference. Analysing this sensitivity demands tools

from complex analysis and operator perturbation theory. Mathematical research centred in Prague contributed to the broader international effort to understand these relationships. Its importance resides partly in providing mathematically defensible explanations of effects that simplified physical diagrams can obscure.

Spectral perturbation theory provides a common language for many of the problems already considered. A perturbation may change a potential, domain, boundary condition, magnetic field, or geometry without altering the basic conceptual role of the Hamiltonian. Mathematicians seek to determine which spectral features are stable and which can move or disappear under that modification. Isolated eigenvalues are often susceptible to established perturbation techniques, whereas thresholds and embedded eigenvalues can behave far less predictably. The spectrum near a threshold is particularly delicate because bound and propagating behaviour meet there. Weak-coupling analysis asks whether a small disturbance creates a bound state and how its energy depends on the coupling strength. These questions recur in curved guides, quantum layers, and leaky structures. Czech contributions developed in dialogue with international techniques rather than as an isolated national methodology. The resulting research exemplifies how localised geometric questions can stimulate general advances in spectral analysis. This feature illustrates the sustained exchange between mathematical innovation and the interpretation of physical systems.

Leaky quantum wires and surfaces form another distinctive group of models studied in the waveguide literature. Instead of confining a particle by imposing a hard boundary, these models localise interactions along curves or surfaces within a larger ambient space. Mathematically, the interaction is represented by a suitably defined singular or concentrated potential. Curvature and the strength of the interaction can generate discrete spectral effects that resemble, but are not identical to, those found in rigid guides. A central challenge is to specify the associated self-adjoint operator and justify any limiting procedure used to obtain it. Singular interactions reveal how physical idealisation can lead naturally to sophisticated questions about quadratic forms and operator extensions. Their analysis also connects with potential theory and geometric measure considerations. The work collected in Exner and Kovařík's monograph illustrates how these models became part of a coherent mathematical research landscape. Such developments broadened Czech mathematical physics beyond the geometry of bounded tubes.

Magnetic fields introduce further complexity into spectral problems because they modify the Hamiltonian through a vector potential and associated covariant derivatives. The physical magnetic field and the mathematical choice of gauge must be distinguished when interpreting the operator. Changes in magnetic flux can alter interference effects and transport around closed loops of a network. On graphs, topological cycles make these effects especially transparent because the geometry supports paths with different phases. For waveguides, magnetic perturbations may influence confinement and threshold behaviour in ways not predicted by curvature alone. Rigorous treatment requires careful control of domains, boundary conditions, and the behaviour of magnetic Schrödinger operators. The connection between magnetic phenomena and spectral theory is therefore methodological rather than simply an additional physical application. Czech-related research addressed such questions within the expanding international literature on quantum guides and networks. It helped reveal the range of

quantum effects that could be understood through operator methods. The resulting scholarship demonstrates how mathematical physics can organise apparently diverse physical influences within a shared analytical framework.

Periodic quantum structures shift the focus from localised defects towards collective spectral organisation. When geometric or interaction patterns repeat, Floquet-type methods can reduce an infinite system to families of problems indexed by quasiperiodic parameters. The resulting band structure provides a mathematical description of allowed and forbidden energy regions. Defects inserted into periodic systems may then create localised states or resonances within or near those bands. Random perturbations introduce a contrasting problem by asking how disorder changes transport and localisation. These themes relate quantum graph analysis to the wider spectral theory of periodic and random operators. The appropriate statements depend on regularity, dimensionality, and the statistical assumptions placed on the disorder. Studies surveyed in the quantum-waveguide literature include periodic and random configurations alongside geometric perturbations. Such breadth helped make the Czech-associated research programme relevant to questions concerning complex materials and engineered networks.

Mathematical physics also requires the study of time evolution, not merely the stationary spectral equation. For a self-adjoint Hamiltonian, unitary evolution is provided by functional calculus and represents the mathematical framework for closed-system quantum dynamics. Yet questions about repeated measurements, approximations, and restricted evolution demand additional limiting arguments. Pavel Exner and Takashi Ichinose investigated a product formula related to quantum Zeno dynamics in a 2005 publication. Product formulae connect repeated operator compositions with limiting evolution and thereby clarify when a proposed dynamical constraint can be justified. Such results depend on assumptions about operators and projections, rather than on unrestricted informal analogies with observation. This topic shows the reach of Czech mathematical physics beyond waveguide geometry into the foundations of quantum evolution. It also demonstrates the usefulness of semigroup methods and functional analysis for problems motivated by quantum theory. The mathematical innovation lies in establishing valid limiting constructions under stated conditions. Their wider relevance is that they help distinguish rigorous dynamical assertions from merely intuitive descriptions of measurement.

Theoretical results in mathematical physics do not become scientific contributions simply because their authors are attached to a particular national institution. Their significance must be assessed through the mathematical problem solved, the novelty of the proof, and the subsequent ability of other researchers to use the result. This distinction is particularly important for Czech science between 1993 and 2025 because many notable papers involved collaborators working in several countries. Exner's publications with Olaf Post, Hynek Kovařík, Taksu Cheon, and Takashi Ichinose exemplify different forms of international intellectual cooperation. The relevant achievement is not exclusive national ownership of theorems but a sustained contribution by researchers based in Czech institutions. International collaboration can enlarge access to complementary expertise in geometry, analysis, and mathematical quantum theory. It can also establish shared standards concerning problem formulation and proof. The result is a

scientific school defined partly by its research programme and partly by its international network. Czech mathematical physics illustrates this mode of scientific agency particularly clearly.

The relationship between the Czech Academy of Sciences and universities is an essential institutional dimension of the story. Mathematical physics has been pursued through overlapping communities rather than through a single laboratory with an exclusive disciplinary mandate. The Nuclear Physics Institute and mathematical institutes in Prague have provided research environments in which theoretical questions can be developed over long periods. The Faculty of Nuclear Sciences and Physical Engineering at Czech Technical University in Prague has also provided an important academic setting for mathematical physics. Its combination of mathematical and physical training is well suited to operator theory, spectral analysis, and the mathematical foundations of quantum mechanics. Institutional affiliations may change across a researcher's career, so particular achievements should be associated with their documented publication affiliations rather than assumed institutional continuity. Equally, the distinction between Academy-based research and university teaching has become less rigid through doctoral supervision and joint projects. Younger researchers can encounter frontier mathematical problems in seminars before they become fully codified in graduate textbooks. This intergenerational exchange is a crucial, though less easily quantified, form of scientific development. It allows a national school to survive changes in funding conditions and research fashions.

A substantial body of mathematical physics also emerges from ordinary methodological dialogue with pure mathematics. Spectral estimates rely on functional inequalities, while singular limits often require geometric convergence arguments and delicate variational constructions. Boundary triples and extension theory provide ways of classifying or comparing self-adjoint realisations of differential operators. Differential geometry determines which effective terms arise when the coordinate system follows a curved tube or surface. Probability theory enters where disorder or random configurations become relevant to the operator. These connections show why mathematical physics should not be represented as a narrow applied branch that merely borrows finished mathematical techniques. Physical questions can expose gaps in existing analytical theory and stimulate the development of new methods. Czech research is notable for working across this boundary with sufficient rigour to be relevant to both mathematical and physical audiences. The intellectual movement is reciprocal: geometry and analysis illuminate quantum models, while those models pose new problems for geometry and analysis.

The formation of an internationally legible research programme also depends on communication through journals, conferences, and advanced monographs. Specialist journals in mathematical physics and spectral theory permit results to be evaluated by scholars who understand both their analytical content and physical motivation. International seminars make it possible to compare alternative definitions of similar models before terminological differences become entrenched. Conference proceedings and collaborative surveys can clarify which results are robust and which depend on narrow assumptions. Exner's extensive publication record demonstrates how a long sequence of articles can develop a connected set of research questions. Nevertheless, bibliographic volume by itself is not a substitute for examining individual results. The 2005

convergence theorem and the 2015 waveguide monograph illustrate two distinct forms of intellectual contribution, one producing a specific mathematical result and the other consolidating a broader field. Both forms matter to the history of mathematical innovation. A balanced account must recognise foundational research, ongoing refinement, and scholarly synthesis without collapsing them into a single measure of impact. The Czech experience makes this plurality of achievement particularly visible.

Scientific computing plays a supporting but conceptually significant role in this area of mathematical physics. Calculations can suggest spectral gaps, scattering profiles, and resonance positions, but a numerical pattern need not imply a general theorem. Discretisation of continuous operators introduces approximation errors that may be especially troublesome near spectral thresholds. A robust analysis therefore distinguishes the physical model, the operator realisation, the numerical scheme, and the resulting computed quantity. Convergence of a numerical method does not by itself establish that the underlying reduced physical model is justified. Conversely, a rigorous asymptotic theorem may explain why a simplified numerical calculation is reliable in a particular regime. Czech mathematical research has contributed to a broader scientific environment in which these analytical and computational perspectives can interact. Quantum networks are especially suitable for such dialogue because their simplified structures often admit both numerical experiments and detailed theoretical analysis. The most productive interaction occurs when computation motivates precise conjectures and proof identifies the conditions under which they hold.

The distinction between mathematical prediction and technological realisation remains important when evaluating the practical influence of spectral theory. A proof that bending induces a bound state establishes a property of an idealised Hamiltonian under explicit assumptions. It does not alone establish that a particular manufactured nanostructure will exhibit the same effect under experimental conditions. Material imperfections, interactions between particles, finite temperatures, and external fields may require additional models. Nevertheless, rigorous idealised theories can isolate mechanisms that would be difficult to identify in a fully detailed physical description. They also provide reference cases against which numerical or experimental interpretations may be tested. This is an appropriate understanding of the innovation associated with Czech mathematical physics: it expands explanatory and methodological capacity rather than claiming automatic commercial outcomes. The same caution applies to quantum graphs, whose mathematical usefulness is not confined to literal physical networks. Their structural ideas have relevance across scattering theory, inverse problems, and analytical modelling. Recognising these limits strengthens, rather than diminishes, the assessment of the field's achievements.

The period between 2020 and 2025 brought renewed international attention to quantum technologies, but established spectral research should not be retrospectively recast as a direct product of that policy agenda. Its deeper history runs through questions developed long before quantum technology became a prominent strategic label. Waveguide geometry, operator domains, network scattering, and resonance theory remained scientifically relevant because these are recurring mathematical problems across changing technological contexts. Contemporary research has continued to examine singular interactions, spectral geometry, and

graph-based models using increasingly refined analytic methods. The value of such work is not restricted to the possibility of a near-term device. Fundamental results may acquire unexpected significance when new physical platforms make previously idealised structures more accessible. Czech mathematical physics entered the early 2020s with a mature intellectual tradition and established channels of international collaboration. This placed its researchers in a position to participate in emerging discussions without abandoning foundational analytical questions. Intellectual continuity is therefore as important as topical novelty when interpreting the recent stage of the field.

Several structural features distinguish Czech mathematical physics from research branches centred on large experimental facilities. A theoretical group can sustain internationally meaningful work through specialist expertise, doctoral recruitment, research time, and international mobility rather than through a single costly machine. However, this does not make theoretical research independent of financial conditions or institutional support. Stable positions, access to scholarly literature, conference participation, and long-term grants determine whether difficult multi-year problems can be pursued. Mathematical breakthroughs are often unpredictable in timing, creating tension with evaluation systems focused on short reporting cycles. The Czech experience after 1993 shows how internationalisation can alleviate some constraints while creating stronger competition for talented researchers. Collaboration abroad may enrich domestic schools, yet the retention of younger mathematicians remains an important institutional concern. Sustained mathematical innovation consequently requires an environment that values conceptual advances alongside more immediately measurable applications. The history of spectral research provides a concrete example of why policy discussions should recognise the temporal scale of foundational mathematics. Such recognition is relevant to the future of Czech scientific excellence.

An adequate historical assessment must also acknowledge the selectivity of the examples chosen. Pavel Exner is especially prominent because his work connects many of the themes central to this section, but Czech mathematical physics cannot be reduced to his bibliography. Related research is conducted by specialists in partial differential equations, operator theory, mathematical analysis, and quantum modelling at several institutions. The intellectual boundaries of the field overlap with condensed-matter theory, geometry, computational mathematics, and applied analysis. Some important developments are collaborative results whose Czech component should be described precisely rather than magnified for narrative convenience. The aim of this section is therefore not to claim that Czech researchers invented spectral theory or quantum graphs. It is to identify where Czech scientific work made recognisable contributions to their modern development between 1993 and 2025. Primary publications and research monographs provide a firmer basis for such claims than institutional promotional summaries alone. The resulting account connects identifiable mathematical results with the scholarly and institutional environment in which they developed.

Taken together, the research directions considered here reveal a coherent transformation of Czech mathematical physics across the period 1993–2025. The field inherited strong traditions of rigorous quantum theory and operator analysis while acquiring increasingly international forms of collaboration. Investigations of curved waveguides demonstrated how geometry can

alter spectral localisation and transport. Research on thin manifold limits clarified when quantum graphs accurately represent families of more complicated physical domains. Work on vertex couplings, scattering, resonances, and singular interactions expanded the mathematical vocabulary available for describing quantum networks. Further studies connected spectral theory with magnetic effects, periodic structures, and the foundations of quantum evolution. The 2015 waveguide monograph showed how these separate research strands could be organised into a mature field with shared questions and methods. The distinctive Czech achievement lies not in a simple catalogue of applications but in the sustained production and synthesis of rigorous mathematical understanding. Within the wider history of Czech mathematics, this constitutes a powerful example of fundamental knowledge becoming an internationally recognised form of scientific innovation. It also confirms that mathematics contributes to physical science most deeply when it explains which claims a model can truly support.

Mathematical Modelling of Complex Systems: Applications in Engineering, Natural Sciences, and Economics

Mathematical modelling became an increasingly prominent feature of Czech scientific development after the establishment of the independent republic in 1993. The underlying intellectual tradition, however, was older than the new state's institutions. Applied analysts had long studied differential equations, numerical schemes and mechanical systems in cooperation with scientists from other disciplines. The post-socialist transition altered the funding arrangements and the opportunities for exchanging such methods internationally. Rather than displacing fundamental mathematics, these changes exposed its practical importance to a wider research community. Mathematical models provided a language through which physical assumptions could be translated into precise relations between variables. Their usefulness depended on the validity of those assumptions as much as on the elegance of the resulting equations. The period ending in 2025 therefore reveals a changing relationship between proof, simulation, observation and technological decision-making. Czech contributions are most accurately understood through this relationship rather than through a simple catalogue of software products.

The movement from a physical phenomenon to a mathematical model begins with deliberate simplification. A scientist must decide which processes determine the behaviour of interest and which details can be omitted without losing essential information. Conservation of mass, energy or momentum may supply basic equations, but constitutive relationships introduce further hypotheses about particular materials. Boundary conditions and initial data then connect an abstract system with an identifiable experiment or environment. Czech research in continuum mechanics repeatedly confronted the difficulty of reconciling these levels of description. A formally plausible model can still be ill posed, admit non-unique solutions or produce predictions that depend strongly on poorly determined parameters. Mathematical analysis

therefore performs a critical function before expensive numerical calculations begin. It asks whether solutions exist, whether they remain stable and whether approximations preserve meaningful properties. In applied practice, these questions help distinguish robust scientific inference from a visually convincing numerical demonstration. The Czech tradition of rigorous mathematical modelling made that distinction a recurring methodological concern.

Continuum mechanics provides one of the clearest examples of the influence of Czech mathematical analysis on applied science. Its equations represent fluids and solids as fields defined over space and time rather than as collections of individual particles. This continuous description is powerful precisely because it makes complex physical systems amenable to differential equations. Nevertheless, the nonlinear structure of these equations may prevent classical solutions from existing under realistic conditions. Researchers consequently use weak solutions, energy inequalities and compactness methods to study physically significant regimes. The Mathematical Institute of the Czech Academy of Sciences has supported important work at this interface between rigorous analysis and modelling. University groups in Prague have also connected continuum theory with thermodynamics and material science. The contribution of these communities lies not only in solving selected equations but also in clarifying which mathematical formulations are defensible. Such clarification becomes indispensable when a model is subsequently incorporated into engineering calculation or physical interpretation.

Eduard Feireisl's work on viscous compressible fluids illustrates the international relevance of Czech mathematical fluid mechanics. His monograph Dynamics of Viscous Compressible Fluids, published by Oxford University Press in 2004, developed a rigorous framework for compressible and heat-conducting flows. The central difficulties concern nonlinear coupling among density, velocity and temperature. These quantities may exhibit weak convergence even when their products appear in the governing equations. Consequently, passing from approximate solutions to genuine weak solutions requires delicate compactness and energy arguments. Feireisl's exposition addressed existence questions for barotropic fluids and the fuller Navier–Stokes–Fourier description. It exemplifies the contribution of theoretical mathematics to the credibility of fluid simulations. An existence theorem does not by itself deliver a numerical forecast or certify the accuracy of a particular engineering design. It does, however, identify a coherent mathematical setting in which subsequent approximation methods can be investigated. This distinction is central to the history of mathematical innovation in Czechia.

The study of singular limits extended this research beyond the well-posedness of an individual system. In collaboration with Antonín Novotný, Feireisl examined how equations for compressible viscous fluids can approach simpler systems under specific parameter regimes. Their book Singular Limits in Thermodynamics of Viscous Fluids appeared in 2009 and was revised in a second edition in 2017. Such limits are important because physical sciences employ models operating at different scales and levels of approximation. A low-Mach-number approximation, for example, cannot be justified merely by declaring sound waves negligible. Rigorous analysis must examine whether solutions actually converge as the relevant parameter becomes small. Boundary effects, thermal variation and the behaviour of acoustic components can complicate that convergence. This programme provides a mathematical basis for choosing

a reduced model without concealing its assumptions. It also demonstrates that theoretical progress can improve computational practice indirectly, by identifying when a less expensive set of equations is legitimate.

Research on incompressible fluids presented a related but distinct problem of constitutive diversity. The familiar Newtonian relation between stress and strain rate is insufficient for many industrial, biological and geological materials. Josef Málek and collaborators investigated classes of fluids governed by nonlinear or implicit constitutive relations. Such formulations permit a wider range of rheological behaviour than a single explicit viscosity law. In work published in 2012, Miroslav Bulíček, Piotr Gwiazda, Málek and Agnieszka Świerczewska-Gwiazda examined unsteady flows of implicitly constituted incompressible fluids. Their analysis employed monotonicity methods to accommodate models with markedly different flow responses. The value of the framework is conceptual as well as technical: multiple material behaviours can be studied within common mathematical assumptions. Later research by Bulíček, Málek and Erika Maringová extended the investigation to nonstandard boundary behaviour. These developments are relevant to polymer processing, complex fluids and other contexts where laboratory observations challenge idealised constitutive laws. They also caution against identifying one fluid equation with every physically possible flow.

Boundary behaviour is not a peripheral detail in continuum models. A solid surface may impose adhesion, slip or a threshold response that changes with the mechanical state of the material. The traditional no-slip condition is convenient, yet it is not automatically appropriate for every system. Implicit boundary relations can model transitions between sticking and sliding without forcing an artificially simple functional dependence. Their analysis requires compatibility between energy dissipation in the bulk and at the boundary. Czech mathematical studies in this area have shown how generalised constitutive relations can be brought into a rigorous existence theory. A mathematically admissible boundary law must still be connected with measurement before it becomes an empirical description of a specific material. That requirement highlights the division of labour between analysis, experimental mechanics and computational implementation. Models gain reliability when their assumptions are explicit enough to be tested and revised.

Solid mechanics introduced another set of difficulties associated with irreversible change. Elastic models describe recoverable deformation, whereas plasticity, fracture and damage involve path dependence and dissipation. Tomáš Roubíček developed influential approaches to nonlinear evolutionary problems within this broader context. His book Nonlinear Partial Differential Equations with Applications, issued in an expanded second edition in 2013, placed rigorous methods alongside problems from continuum mechanics and thermodynamics. The analysis of these systems often calls for variational inequalities, monotone operators and weak formulations. This mathematical apparatus helps describe systems in which a classical smooth trajectory cannot be expected. Constitutive modelling and analysis must proceed together because simplifying assumptions can alter the physical meaning of a result. Czech research at this intersection illustrates the increasingly porous boundary between abstract nonlinear analysis and engineering mechanics. The outcome is not a universal model of materials but a

toolkit for understanding carefully specified classes of behaviour. Its applications extend across problems involving deformation, heat and mechanical transitions.

Rate-independent systems became an especially significant point of contact between mathematical theory and material evolution. Their defining idealisation is that the evolution depends on the sequence of loading states rather than on the speed at which those states are traversed. This assumption can be appropriate for particular quasi-static phenomena, although it excludes processes in which viscosity or inertia is essential. Alexander Mielke and Tomáš Roubíček developed the subject systematically in Rate-Independent Systems: Theory and Application, published in 2015. Their treatment connected energetic formulations, functional analysis, existence results and numerical questions. It also examined applications to plasticity, contact, damage and transformations in solids. The mathematical value of this synthesis lies in making irreversible evolution accessible to a coherent analytical framework. Its practical importance depends upon identifying situations where the rate-independent approximation is physically justified. That qualified relation between mathematical elegance and material realism characterises responsible modelling.

Thermodynamic consistency provides a unifying principle across many branches of complex-system modelling. Equations constructed independently for mechanics, temperature and internal variables may contradict one another unless their energy exchanges are carefully specified. Free-energy functionals and dissipation inequalities offer means of testing their compatibility. Czech research into fluid and solid systems has frequently placed these requirements at the centre of analysis. The resulting models may be mathematically demanding because several nonlinear mechanisms are coupled. Yet ignoring thermodynamic restrictions can produce numerical solutions that violate the behaviour they are intended to represent. A rigorous derivation is therefore part of modelling quality rather than an optional theoretical decoration. In some applications, the structure of the equations also guides the construction of stable numerical schemes. The interaction between thermodynamics and mathematics exemplifies how foundational principles can shape technological modelling without prescribing its every detail. It also demonstrates why mathematical innovation is not synonymous with increasing computational complexity.

Numerical analysis supplied the bridge from abstract evolution equations to simulations that could be performed on real machines. A continuous model must generally be replaced by a finite-dimensional approximation before its predictions can be explored computationally. Finite element and related Galerkin methods provide systematic ways of approximating spatially distributed problems. Time discretisation adds further choices concerning stability, conservation and the treatment of nonlinear terms. Czech researchers such as Miloslav Feistauer and Vít Dolejší contributed to the development and exposition of numerical techniques for fluid dynamics. Their work on discontinuous Galerkin methods connected analytical principles with the approximation of conservation laws and complex flows. Such approaches are particularly useful when local conservation and geometric flexibility matter. A convincing numerical result nevertheless requires both control of approximation error and attention to the fidelity of the underlying physical model. Computation thus complements rather than supersedes the mathematical analysis of existence and stability.

The increasing availability of high-performance computing changed the range of problems accessible to Czech interdisciplinary groups. The IT4Innovations National Supercomputing Centre in Ostrava became a visible national platform for large-scale numerical experiments. Its successive computing systems enabled research across engineering, materials, environmental science and data-intensive disciplines. Mathematical models supplied much of the intellectual machinery required to exploit this infrastructure. Larger computational capacity allowed researchers to refine discretisations, examine parameter sensitivity and compare competing physical scenarios. It did not, however, remove uncertainty from model coefficients or automatically validate numerical assumptions. The distinction between computational resolution and predictive reliability became increasingly important as simulated systems grew more elaborate. The most productive use of infrastructure therefore involved close cooperation between analysts, numerical specialists and disciplinary scientists. Czech experience illustrates how computational investment can multiply the value of established mathematical expertise. It also shows why hardware achievements and mathematical discoveries should be evaluated separately.

Mathematical biology expanded the field of application beyond traditional mechanical systems. Population models use differential equations or stochastic processes to describe birth, mortality, dispersal and ecological interaction. Reaction–diffusion formulations can represent the spatial propagation of biological quantities under specified assumptions. Such models are valuable for exploring mechanisms, yet their parameters can depend strongly on local environments and observation methods. A biological interpretation therefore requires more than formal similarities between equations. Czech universities and interdisciplinary institutes increasingly provided settings for cooperation among mathematicians, biologists and statisticians. Within these collaborations, mathematical tools supported the comparison of hypotheses rather than replacing empirical work. The ability to articulate several plausible mechanisms within the same mathematical framework is an analytical advantage. It also forces researchers to confront questions of identifiability when different mechanisms produce similar observable outcomes.

Ecological systems show why scale is a persistent difficulty in mathematical modelling. Individuals, populations and ecosystems respond to influences operating over different lengths and time intervals. A model that reproduces short-term local dynamics may not describe long-term regional change. Spatial structure, delayed effects and random environmental variation further complicate the picture. Deterministic differential equations can offer transparent baseline mechanisms, while stochastic formulations admit specified sources of uncertainty. The choice between them is a scientific decision about the phenomenon rather than merely a preference for one branch of mathematics. Czech work linking quantitative ecology and computation participated in the international movement towards more explicit model-based explanation. Such research helped broaden the role of mathematics within environmental studies. Its strongest conclusions emerge when predictions are compared with independent observations. Mathematical sophistication alone cannot compensate for missing ecological data.

Epidemiological modelling became conspicuous during the coronavirus pandemic beginning in 2020. Czech public-health researchers developed models to examine transmission and short-term epidemic trajectories under rapidly changing conditions. A study of the first wave in the Czech Republic used a comparatively simple model to estimate changes in the reproduction number following government interventions. This case demonstrates that a relatively economical mathematical construction can provide information relevant to an urgent policy question. It also illustrates the importance of distinguishing an estimated association from proof that one intervention independently caused a particular change. Infection ascertainment, reporting delays and behavioural adjustments create substantial uncertainty in such analyses. Epidemiological model outputs consequently require intervals, sensitivity analysis and open discussion of assumptions. Their purpose is to support judgment under uncertainty rather than provide a deterministic account of future infections. The Czech experience revealed both the operational value and the limits of quantitative epidemic forecasting.

The reproduction number is a useful example of the difference between a mathematical parameter and a directly observable fact. It summarises aspects of transmission but usually must be inferred from imperfect epidemiological information. Changes in testing availability or reporting practices can influence the evidence from which it is estimated. Different statistical or mechanistic specifications may therefore yield somewhat different values for the same period. Models should report the uncertainty created by these choices whenever possible. Czech pandemic analysis occurred within a wider international exchange of epidemiological techniques and data practices. The work also made the connection between mathematical modelling and public communication unusually visible. A result becomes socially consequential when its limitations can be explained clearly to policymakers and the public. This responsibility is inseparable from the technical responsibility of checking calculations and assumptions. It is equally relevant to economic forecasting and environmental risk assessment.

Stochastic modelling offers a complementary perspective when deterministic descriptions conceal important variation. Random processes may represent fluctuations in populations, noisy measurements or uncertain external influences. The mathematical challenge is to specify the probability structure without treating randomness as an all-purpose substitute for unknown mechanisms. Czech work in stochastic differential equations and statistical decision theory contributed to the tools available for such problems. In applications, stochastic simulations can explore distributions of possible trajectories rather than a single forecast. The reliability of those distributions still depends on the selected assumptions and the information used for calibration. Mathematical analysis addresses existence, dependence on parameters and long-run behaviour of the resulting processes. Statistical inference tests how well the probabilistic framework corresponds to actual data. Together, these fields provide a disciplined approach to uncertainty in complex systems.

Economic modelling brought a different set of institutional relationships into the history of Czech applied mathematics. Economic systems cannot ordinarily be subjected to controlled experiments on the scale of entire national economies. Models therefore combine behavioural hypotheses, accounting identities, statistical information and policy assumptions. The Czech National Bank developed structural forecasting tools to analyse monetary transmission and

macroeconomic fluctuations. A documented milestone was the adoption of the g3 model as its core forecasting framework in July 2008. Michal Andrle, Tibor Hlédik, Ondra Kameník and Jan Vlček described the implementation in a working paper published in 2009. The model belongs to the family of dynamic stochastic general equilibrium approaches. Its significance for this book lies in the operational use of mathematically structured economic theory in an important Czech institution. Such implementation demonstrates how optimisation, dynamics and statistical estimation can enter actual policy workflows. It does not imply that the model was free from specification error or could eliminate economic uncertainty.

Structural macroeconomic models rely on a distinctive interplay between theory and estimation. Households, firms and policy authorities are represented through stylised decisions, constraints and adjustment mechanisms. Their interaction produces a system whose dynamic properties can be investigated mathematically. Calibrated or estimated parameters then connect the formal structure with observed economic data. In the Czech setting, the g3 framework helped organise forecasting and policy analysis around a transparent set of model relations. Nevertheless, the meaning of transparency is methodological rather than absolute because some assumptions remain contestable. Regime changes, exceptional shocks and revisions to economic statistics can weaken established relationships. Scenario analysis offers a way to examine how outcomes change under alternative premises. The central analytical contribution is thus disciplined comparison, not an infallible forecast.

Research in operations and decision science extends quantitative modelling from prediction to deliberate action. Transport planning, production scheduling and resource allocation often require choosing among many feasible arrangements. Optimisation expresses those choices in terms of an objective, constraints and decision variables. Mathematical theory then investigates when optimal solutions exist and how algorithms can find useful approximations. Czech researchers developed expertise in linear, nonlinear and stochastic optimisation relevant to such problems. Applications can include transport networks, industrial processes, energy systems and financial risk management. Yet an optimal answer reflects the mathematical objective supplied by the modeller, not an independently established social ideal. Real decisions may involve competing criteria that cannot be collapsed into a single uncontested number. Good applied mathematics makes these tensions explicit and helps decision-makers explore trade-offs. This is a different but equally important form of scientific contribution from proving a new existence theorem.

Transport systems illustrate the challenge of reconciling elegant optimisation models with operational constraints. A route plan may need to account for capacity, time windows, uncertain demand and changing network conditions. A purely deterministic solution can perform poorly when actual travel conditions depart from its assumptions. Stochastic and robust formulations make some of those uncertainties explicit. The resulting optimisation problems may become computationally demanding because decisions interact across many locations and times. Research at the interface of mathematical programming and transport analysis seeks manageable formulations and algorithms. In Czech engineering and applied mathematics, such work belongs to a wider tradition of quantitative planning. Its practical success depends upon

accurate operational information and the ability to revise the model as conditions change. A mathematical solution is therefore one component of a continuing decision process.

Industrial modelling similarly depends on coupling several types of mathematics rather than choosing a single universal method. Process engineers may require partial differential equations for heat or fluid transport together with optimisation for controlling operating conditions. Material scientists may combine constitutive mechanics with parameter estimation and numerical approximation. Statistical methods are needed to determine whether calculated outputs match observed behaviour within acceptable uncertainty. This combination creates opportunities for academic groups with complementary specialisations. Czech mathematical institutions benefited from a research environment in which fundamental analysis, scientific computing and engineering could interact. Interdisciplinary teams nevertheless encounter differences in terminology, validation standards and acceptable simplifications. Effective collaboration requires these disagreements to be discussed rather than hidden behind technical notation. When successful, it allows an abstract theorem or algorithm to influence the design of a concrete experimental or industrial procedure. Such pathways are often gradual, collective and difficult to attribute to any single discovery.

Inverse problems occupy a critical position in the transition from physical theory to empirical application. Forward modelling asks what a system will do when its properties and inputs are known. An inverse problem instead seeks to infer unknown parameters or internal structures from observed responses. Noise and incomplete measurements can make this inference unstable or non-unique. Regularisation introduces additional structure so that a solution can be recovered in a controlled manner. Theoretical developments in functional analysis and optimisation therefore become directly relevant to imaging, materials and environmental observation. Czech analytical and computational expertise contributed to the broader capacity for addressing problems of this kind. However, regularisation introduces choices that must be justified in relation to the scientific context. An apparently precise reconstructed value may depend strongly on those choices.

Verification and validation should be distinguished in any serious account of mathematical modelling. Verification asks whether the numerical implementation solves the selected mathematical equations with sufficient fidelity. Validation asks whether those equations adequately represent the real phenomenon for an intended purpose. A code may pass stringent numerical tests while its physical assumptions remain unrealistic. Conversely, empirical agreement may arise temporarily through compensating modelling errors. Czech work in rigorous analysis and numerical methods provided tools especially relevant to verification, including consistency and stability arguments. Interdisciplinary collaboration with experimental researchers is indispensable for validation. Uncertainty quantification must then address discretisation error, parameter uncertainty and possible defects in the model structure. This layered assessment is more demanding than comparing one simulated curve with one observed series. It is also more informative for deciding when a model should guide consequential actions.

The internationalisation of Czech science transformed the social organisation of mathematical modelling between 1993 and 2025. Collaborative research increasingly crossed borders and linked mathematicians with laboratories possessing specialised instruments or datasets.

European programmes encouraged joint projects in which analytical research supported larger scientific objectives. The Czech Academy of Sciences and research universities supplied mathematical expertise while technical centres provided complementary experimental and computational capacities. International publication strengthened the exchange of methods and enabled results to be assessed beyond national communities. These developments expanded opportunities for younger researchers to work on complex applied problems. They also increased expectations concerning reproducibility, data stewardship and the transparency of numerical procedures. A national mathematical contribution could consequently take the form of a theorem, algorithm, computational component or sustained specialist collaboration. The diversity of these contributions should be recognised without conflating them.

The growth of interdisciplinary modelling also created tensions in research evaluation. Fundamental mathematical results may require years of abstraction before their practical importance becomes apparent. Applied projects often have shorter deadlines tied to external experiments, technologies or policy requirements. Researchers working between these cultures must maintain analytical rigour while responding to demands for timely results. A successful computer simulation can be scientifically valuable without constituting a new mathematical theorem. Conversely, a difficult existence result can transform an area of modelling without immediately yielding a commercial application. Czech mathematics developed within the same international debate about how these different achievements should be recognised. Balanced evaluation requires attention to originality, reproducibility, methodological soundness and appropriate societal relevance. Reducing all innovation to patents or numerical output would misrepresent the discipline. Equally, disregarding applied work would obscure significant pathways through which mathematical ideas acquire value.

By the end of 2025, mathematical modelling in Czechia had become embedded in a network of fundamental research, numerical science and specialised applications. Fluid mechanics, nonlinear material theory, epidemiology, economic forecasting and optimisation provide distinct examples of that evolution. They should not be interpreted as a single uniform programme, because each field has different evidential requirements and practical objectives. What connects them is the effort to express mechanisms and uncertainty in a form that can be analysed, criticised and, where appropriate, computed. Czech achievements in rigorous differential equations have helped establish trustworthy foundations for complex mechanical models. Statistical and optimisation methods have supplied tools for interpreting evidence and comparing decisions. Computational infrastructure has increased the scale of experiments that can be attempted, while also making the limitations of data and model structure more visible. The historical record therefore points to mathematical innovation as an ongoing exchange between abstraction and application. Its intellectual contribution lies in making complex phenomena more intelligible without pretending that every uncertainty can be removed.

Computational Mathematics, Artificial Intelligence, and Data-Driven Scientific Innovation

The development of computational mathematics in the Czech Republic between 1993 and 2025 cannot be understood as a simple progression from slower to faster computers. It was an intellectual transformation in the relationship between mathematical reasoning, algorithmic design, empirical evidence and the scientific questions that could realistically be investigated. Czech research institutions entered independence with substantial expertise in numerical analysis, optimisation, statistics, formal logic and theoretical computer science. Their established traditions made it possible to approach new computational technologies without abandoning the mathematical principles on which reliable calculation depends. During the 1990s, access to digital resources expanded alongside the international circulation of software, publications and research methods. In the following decades, institutional partnerships and specialised computing centres enlarged the scale of investigations available to researchers. Machine learning introduced another kind of change by making the estimation of models from data a central scientific activity. By 2025, numerical simulation, statistical inference and artificial intelligence had become increasingly interconnected, although they continued to answer different kinds of questions. The Czech experience therefore illustrates how computational innovation depends as much on mathematical judgement as on the availability of powerful machinery.

A central distinction throughout this history concerns the difference between mathematical models derived from prior theory and models inferred from observations. In a conventional numerical simulation, scientists usually begin with equations expressing a proposed physical, biological or economic mechanism. An algorithm then approximates the solution subject to specified initial conditions, boundary conditions and numerical assumptions. Machine learning often reverses this order by selecting a family of functions and adapting its parameters to observed data. The resulting predictor can be accurate without providing a satisfactory explanation of the process that produced those observations. Both approaches may employ large matrix calculations, iterative optimisation and demanding computational infrastructure. Nevertheless, their evidential requirements differ because numerical convergence is not equivalent to predictive generalisation. Czech work at the intersection of scientific computing and artificial intelligence increasingly confronted this methodological distinction. The most productive developments emerged where researchers combined structural understanding with statistical assessment rather than treating mathematical computation as a substitute for either. This distinction is essential when comparing mathematical claims with practical results.

The institutional foundations of this convergence were distributed across several complementary research environments; the Institute of Computer Science of the Czech Academy of Sciences maintained activity in the mathematical foundations of computer science, computational methods and artificial intelligence. Its research agenda also embraced interdisciplinary data science, complex systems and statistical modelling. The Institute of Information Theory and Automation contributed a related tradition in optimisation, probabilistic reasoning, decision theory and automated systems. University departments in Prague, Brno and

other cities cultivated numerical mathematics, applied statistics and theoretical informatics within teaching and research programmes. The Czech Technical University in Prague developed particularly important links between optimisation, control, machine learning and engineering. In Ostrava, IT4Innovations eventually provided national-scale computing resources for researchers who could not address complex numerical workloads on ordinary departmental systems. These institutions were not parts of a single centralised artificial-intelligence laboratory. Their significance lay instead in the different mathematical perspectives they brought to common problems. The resulting network encouraged methods to move between abstract analysis, computational experiments and application-oriented research.

Mathematical optimisation supplied one of the strongest bridges between established Czech research traditions and modern machine learning. Learning an effective model normally requires the minimisation of a loss function, sometimes subject to explicit constraints. The apparently elementary formulation conceals challenging questions about the existence of solutions, local minima, conditioning and numerical scalability. Classical convex optimisation provides comparatively transparent guarantees when the feasible set and the objective have the requisite structure. Neural-network learning frequently produces non-convex objectives for which such guarantees are unavailable in their classical form. Czech specialists in numerical optimisation and variational analysis consequently possessed expertise directly relevant to emerging learning algorithms. Researchers also investigated derivative-free procedures for situations in which gradients were unavailable or expensive to evaluate. This relationship did not mean that every optimisation method became an artificial-intelligence technique. Rather, the growth of machine learning created new settings in which earlier mathematical results could be tested, refined and applied. Such limitations must be considered when assessing the broader scientific significance of the method.

The work of Ladislav Lukšan and colleagues represents a useful example of continuity in computational optimisation. Research associated with the Institute of Computer Science addressed numerical methods for solving large-scale optimisation problems and related systems of equations. Such investigations concerned convergence behaviour, computational efficiency and the practical limitations of algorithms implemented with finite precision. These questions acquired renewed importance as machine-learning systems came to involve very large parameter spaces. An optimiser that performs well on a small analytic benchmark need not behave equally well when datasets, constraints or hardware architecture change. Comparative numerical experiments therefore became inseparable from mathematical analysis. Contemporary institutional descriptions place numerical optimisation and numerical linear algebra among the algorithmic foundations of artificial-intelligence research at the Institute. The connection is substantive because learning algorithms rely repeatedly on the solution of approximated mathematical subproblems. Czech computational research thus linked a longstanding analytical discipline with a newer set of data-intensive applications.

The mathematical theory of neural networks became another distinctive field of Czech investigation. Věra Kůrková's research on approximation by neural networks contributed to understanding what classes of functions such models can represent. Approximation theory asks whether a target function can be approached sufficiently closely by a specified family of

approximants. In machine learning, this question precedes the practical issue of whether a training algorithm can identify a useful approximation from finite data. It also differs from asking whether the model will predict reliably on observations not used during training. Such distinctions became essential as claims about artificial intelligence expanded beyond the mathematical conditions supporting them. Czech contributions to the theory of approximation supplied tools for assessing representation capacity without assuming that computational success follows automatically. This intellectual tradition also connected neural methods with classical problems in functional analysis and numerical approximation. Its relevance persisted as increasingly complicated learning architectures entered interdisciplinary research. The resulting perspective encouraged more precise discussion of what learning systems can establish and what remains uncertain.

Statistical learning introduced a second conceptual foundation alongside approximation theory; a predictive model is estimated from observations that are incomplete, noisy and sometimes systematically unrepresentative. Its performance depends on the statistical assumptions governing sampling, uncertainty and possible changes in the underlying process. Czech research in mathematical statistics provided expertise in robust estimation, multivariate methods and the detection of departures from assumed data-generating mechanisms. Such work is important for artificial intelligence because impressive performance on a training dataset may conceal substantial estimation bias. Robust procedures attempt to reduce the disproportionate influence of exceptional observations without eliminating all sensitivity to important variation. Statistical validation must also distinguish ordinary random error from structural change in the population being studied. These challenges are particularly visible in medical, environmental and economic datasets, where the consequences of unreliable prediction can be considerable. Mathematical statistics therefore remained a foundational discipline rather than a preliminary stage to be superseded by machine learning. The expansion of Czech data science demonstrated the continuing necessity of this distinction.

Probabilistic modelling offered a complementary way to represent scientific uncertainty. Rather than returning only a single predicted outcome, probabilistic methods assign distributions to possible observations, parameters or hidden states. Bayesian formulations combine prior assumptions with information supplied by new data, while frequentist methods examine estimators and decision procedures through their sampling behaviour. Neither framework removes uncertainty merely by performing more calculations. Researchers must still specify models, assess identifiability and test whether assumptions remain credible. Work connected with the Institute of Information Theory and Automation has long addressed probabilistic decision-making and adaptive systems. Those themes furnished a natural background for the development of uncertainty-aware artificial intelligence. In scientific applications, distributions can communicate a range of plausible outcomes that point predictions would conceal. Their usefulness nevertheless depends on calibration, validation and an explicit account of the uncertainty being represented. The Czech research landscape linked these mathematical concerns with engineering decisions and data-driven systems.

Graphical models exemplify the interaction between probability theory, combinatorics and computer science. A graphical representation describes conditional relationships among

variables through a network structure. Directed and undirected models encode different kinds of dependence and support different approaches to inference. The mathematical problem is not simply to draw a useful diagram but to determine which independence statements follow from the specified representation. Czech work on conditional independence, notably research associated with Milan Studený, contributed to the rigorous analysis of these relations. Such contributions help distinguish formally justified inference from intuitive interpretations of complex datasets. As networks became common in computational biology, social analysis and intelligent systems, the relevance of these mathematical foundations increased. Algorithms for inference and learning within graphical models also raised practical problems of complexity and scalability. The result was a research area in which abstract structure and empirical application remained closely linked.

Artificial intelligence also developed through the mathematical study of constrained discrete decisions. Many problems in scheduling, allocation, configuration and automated reasoning can be represented as constraint satisfaction tasks. The central question is whether assignments exist that satisfy a collection of specified relations. Algebraic research on constraint satisfaction, including work by Libor Barto and international collaborators, clarified the importance of structural properties of the underlying relations. These results belong to theoretical computer science, but they provide an essential foundation for intelligent computational systems. They show that algorithmic tractability may depend on deep mathematical classifications rather than the immediate design of faster hardware. In practical applications, the same distinction separates difficult instances from entire problem families with efficient solution procedures. The contribution of Czech researchers therefore extends beyond writing software for individual problems. It includes identifying the abstract conditions under which classes of computational tasks become manageable. This is a particularly clear example of mathematical innovation that can later influence applications far removed from its original theoretical setting.

Formal verification and automated reasoning provide another connection between mathematical logic and artificial intelligence; verification asks whether a proposed system satisfies precisely stated properties under the rules of a formal model. Automated reasoning seeks procedures that construct, check or guide deductions without depending entirely on informal human judgement. These problems are particularly demanding when software interacts with dynamic physical environments or with other autonomous components. Czech research groups have investigated satisfiability, formal methods and algorithmic inference within academic computer science. Work at the Czech Institute of Informatics, Robotics and Cybernetics of the Czech Technical University in Prague illustrates the growing proximity of automated deduction and machine learning. Its research descriptions distinguish automated theorem proving and formalisation from statistical training, while examining ways to make the approaches mutually supportive. Learned heuristics may help select promising proof steps, but the validity of a formal proof still rests on logical rules. The distinction matters because computational plausibility cannot replace demonstrable correctness. This line of research reinforces the role of mathematical foundations in assessing intelligent systems.

Symbolic regression offers an instructive alternative to models whose internal operations are difficult to interpret. The method searches for mathematical expressions that reproduce

observed relationships, often through combinations of evolutionary search and numerical optimisation. Its output may take the form of an equation that scientists can compare with known physical laws or theoretical expectations. Work by researchers in the machine-learning group at the Czech Institute of Informatics, Robotics and Cybernetics has explored data-driven construction of dynamical models using symbolic regression. Such methods do not guarantee the discovery of the true mechanism governing a system. Their practical value depends on the quality of observations, permissible expression families and tests against independent evidence. Nevertheless, an interpretable candidate equation can support scientific criticism in ways that an opaque predictor may not. The approach also reconnects data-driven learning with the classical mathematical ambition to describe change through concise formal relations. Its development in Czech engineering research demonstrates how machine learning can contribute to model discovery rather than prediction alone. Here innovation lies in the exchange between numerical search, mathematical representation and experimental verification.

Reinforcement learning and optimal control represent a further area of methodological convergence; optimal control usually begins with a mathematical description of system dynamics and seeks actions that optimise a criterion over time. Reinforcement learning often learns useful decisions through experience when an adequate transition model is unavailable or incomplete. Their common mathematical language includes dynamic programming, optimisation, probability and the representation of sequential decisions. Research associated with Robert Babuška and colleagues in Prague has examined learning methods for control and nonlinear system identification. Such work is especially relevant to robotics, where action changes the future observations on which subsequent decisions depend. Learning performance must be assessed not only through average rewards but also through stability, constraints and behaviour under unfamiliar conditions. A controller that performs successfully in a simulation may fail when physical disturbances differ from its training environment. Combining domain knowledge with data-driven methods is therefore a scientific necessity rather than merely an engineering preference. Czech research at this interface offers an example of applied mathematics shaping the responsible design of intelligent technology.

The expanding importance of high-performance computing altered the practical scale of Czech computational research. Large scientific simulations require immense numbers of operations, but the workload often depends equally on communication between processors and efficient data movement. Parallel programming therefore introduces additional mathematical questions concerning decomposition, synchronisation and error propagation. IT4Innovations National Supercomputing Center, situated at VSB–Technical University of Ostrava, became a major institutional response to these challenges. Its development created a shared environment for researchers across academic disciplines and industrial applications. Access to computing resources permitted more detailed simulations and more extensive comparisons between alternative numerical models. However, the existence of powerful machines did not remove the need for sound algorithms or appropriately chosen discretisations. A poorly conditioned calculation can consume considerable computing time without producing trustworthy results. The emergence of national-scale computing consequently increased the value of mathematical

understanding rather than diminishing it. This transformation became especially visible as artificial-intelligence workloads joined established simulation applications.

The chronology of the Ostrava infrastructure illustrates the rapid expansion of computational capacity during the 2010s; iT4Innovations began operating major national supercomputing resources in 2013 with the Anselm system. The Salomon supercomputer followed in 2015, extending the capacity available for demanding scientific calculations. Barbora, installed in 2019, supplied additional specialised computing resources. These installations supported research communities undertaking numerical simulation, optimisation and data analysis. They also encouraged the development of scalable software that could exploit computing architectures substantially different from ordinary workstations. The scientific achievement should not be confused with the acquisition of machines alone. Durable benefit depended on access arrangements, software support, training and research groups capable of using the infrastructure effectively. Czech mathematical researchers consequently gained an environment in which theoretical algorithms could be examined at greater computational scale. By the end of the decade, this infrastructure had become an important bridge between national mathematical expertise and international scientific computation.

A further milestone arrived with the installation of the Karolina supercomputer in 2021. According to IT4Innovations, the system was acquired within the European High Performance Computing Joint Undertaking and achieved a theoretical peak performance of approximately 15.7 petaflops. Its architecture incorporated general-purpose computing resources alongside accelerator-equipped nodes intended for demanding workloads. The installation expanded possibilities for simulations, extensive data processing and artificial-intelligence calculations. Crucially, accelerated hardware could perform certain operations much more efficiently without making every computational problem equally easy. Mathematical methods had to be adapted to memory limits, communication overheads and the structure of parallel algorithms. Training a large predictive model could therefore raise different bottlenecks from solving a sparse discretised system of differential equations. These distinctions affected how research groups designed their numerical experiments and evaluated computational efficiency. Karolina symbolised a change in scale, but its scientific significance depended on the methods implemented upon it. The event also demonstrated the increasingly European organisation of Czech research infrastructure.

High-performance computing helped transform mathematical models into increasingly detailed representations of complex processes; atmospheric phenomena, fluid flow and material behaviour can involve interacting mechanisms across several spatial and temporal scales. Numerical models must approximate these processes while preserving properties relevant to the underlying science. Greater resolution can reveal important effects, but it can also increase computational cost beyond practical limits. Adaptive methods respond by allocating numerical effort unevenly, concentrating resources where error estimates suggest they are most needed. Parallel algorithms distribute calculations across processors, although achieving efficient communication remains difficult. The development of Czech scientific computing provided new opportunities to combine such numerical techniques with data-driven estimation. Machine-learning components could sometimes accelerate approximations or assist in identifying

uncertain parameters. Yet a learned shortcut requires independent tests before its results can be accepted as substitutes for established calculations. The scientific opportunity lay in combining computational speed with demonstrable numerical reliability.

Uncertainty quantification became increasingly important as computational models were used for consequential decisions. Every prediction depends on assumptions about equations, parameters, measurements and numerical approximation. Some uncertainties arise from limited information and may diminish when additional evidence becomes available. Others reflect variability intrinsic to the system being studied and cannot simply be eliminated by greater computational effort. Statistical sampling, sensitivity analysis and probabilistic simulation provide ways to investigate the consequences of these uncertainties. Their computational cost can become substantial when each evaluation requires a complex numerical solution. Surrogate models and statistical emulators attempt to approximate expensive simulations more economically. Machine learning may contribute to constructing such approximations, but their error must be distinguished from uncertainty in the original physical model. Czech expertise in statistics, numerical methods and optimisation positioned research groups to address these connected problems. An intellectually responsible account of computational innovation must therefore consider both increased predictive power and the limits of confidence in the predictions.

Scientific applications in biology illustrate why data-driven computation required mathematical interdisciplinarity; biological observations are often heterogeneous, incomplete and affected by experimental conditions. Models of gene expression, molecular interactions or ecological networks may require statistical learning alongside mechanistic interpretation. Graph-based representations can describe interactions, while differential equations express hypothesised changes through time. High-dimensional data analysis introduces regularisation and optimisation problems that rarely appear in simpler classical models. Czech mathematical and computational institutions participated in research environments where such methods could be connected to life-science questions. Their contribution was not necessarily the discovery of a new biological mechanism. It could consist instead of designing an estimator, an algorithm or a computational workflow that made a difficult analysis more reliable. Scientific interpretation remained the responsibility of interdisciplinary collaboration with researchers familiar with the underlying biological system. This division of expertise demonstrates how mathematical innovation can support discoveries without being identical to those discoveries.

Engineering applications similarly connected computational mathematics with physical experimentation. Robotics, materials research and industrial processes generate measurements that can inform dynamic models and operational decisions. Control systems must translate predictions into actions while respecting safety and performance requirements. Mathematical optimisation supports the selection of settings, schedules or trajectories under practical constraints. Data-driven algorithms may provide flexibility when conventional models are uncertain, but this flexibility can introduce sensitivity to unfamiliar operating conditions. Research at Czech technical universities explored combinations of machine learning, numerical modelling and control. In such settings, mathematical validation includes the behaviour of the complete system rather than the accuracy of an isolated predictive component. Engineers also

require models that can be updated as physical equipment changes. The movement between experiments, algorithms and formal analysis becomes iterative rather than strictly sequential. The resulting research culture illustrates the practical importance of maintaining dialogue between theoretical mathematics and engineering expertise.

Economic and organisational systems provided a different field for computational mathematical methods; financial variables, supply networks and transport processes are influenced by changing incentives and institutional rules. The data describing these systems may not obey stationary probability distributions over long periods. Statistical learning can recognise patterns, yet the presence of a pattern does not establish a stable causal relationship. Optimisation can identify preferable actions only after objectives, constraints and trade-offs have been formulated. Czech research traditions in econometrics, stochastic decision-making and operations research provided tools for approaching such problems. More powerful computation enlarged the number of scenarios that could be explored but did not resolve disputes about model specification. Decision-support systems therefore needed interpretable assumptions and communication of uncertainty. This requirement became especially apparent when computational analysis was used outside controlled laboratory environments. The practical value of mathematical innovation rested on its capacity to make decisions more accountable, not merely faster.

The growth of artificial intelligence also changed the meaning of computational reproducibility. A classical mathematical proof may be checked independently from the machine that first assisted its discovery. Large computational experiments, by contrast, can depend on software versions, hardware precision, training data and random initialisation. Differences in any of these elements may alter the numerical results or the behaviour of a learned model. Reproducible research therefore requires more than publishing a summary of the final findings. Investigators need to describe data preparation, evaluation procedures, numerical settings and important sources of variability. This expectation is particularly demanding when privacy restrictions or proprietary datasets prevent unrestricted data sharing. Czech institutions participating in international computational science faced the same methodological challenge as their counterparts elsewhere. Formal documentation and transparent reporting became important components of scientific quality. Such practices connect the ethos of mathematical proof with the changing operational realities of data-driven research.

The historical movement towards artificial intelligence also created questions about scientific explanation; a machine-learning model can predict outcomes accurately without revealing why those outcomes occur. Conversely, a theoretically meaningful equation can offer explanatory insight while giving imperfect predictions under uncertain conditions. It is therefore misleading to arrange mathematical methods on a single scale from obsolete theory to advanced artificial intelligence. Different approaches serve different scientific purposes and have different standards of evidence. Czech investigations in symbolic regression, statistical modelling and formal reasoning exemplify the possibility of combining these approaches constructively. Model selection becomes a question of intellectual suitability as well as numerical performance. Researchers must ask which assumptions are plausible, what information is available and how conclusions will be validated. A mature computational culture treats interpretability, efficiency

and predictive success as related but distinct objectives. This broader understanding provides a more defensible measure of innovation than the size of a neural network alone.

The mathematical foundations of artificial intelligence also remain connected with questions about computational limits. Some problems become tractable through clever algorithms, while others retain difficult worst-case complexity even on advanced hardware. Approximate methods can provide useful answers without guaranteeing a globally optimal solution. Randomised algorithms may improve practical performance but require careful statements about probabilities and failure modes. Formal reasoning can establish correctness for a specified model without proving that the model captures every relevant aspect of reality. Deep-learning methods can identify complicated empirical patterns while remaining vulnerable to distributional shifts. These qualifications are not arguments against computational innovation. They define the mathematical conditions under which claims about capability can be assessed responsibly. Czech theoretical computer science, logic and optimisation research contributed intellectual tools for distinguishing solvable problems from poorly understood ones. The increasing prominence of artificial intelligence consequently reinforced the importance of foundational mathematics.

International cooperation played a significant role in integrating Czech computational mathematics into broader research agendas; european programmes encouraged collaboration among universities, Academy institutes, research infrastructures and industrial partners. Large computational projects often require different specialisations, including mathematical analysis, software engineering, statistics and domain-specific science. Czech researchers could contribute distinctive expertise without being responsible for every component of a multinational system. Participation in European computing initiatives also enabled access to resources beyond the capacities of individual national centres. The infrastructure at IT4Innovations, including the EuroHPC-supported Karolina system, reflected this form of distributed collaboration. It would nevertheless be misleading to attribute all results produced on shared facilities to a single country or institution. Scientific authorship, institutional participation and the geographical location of computation need to be distinguished. The Czech case shows how national mathematical capability can be strengthened through reciprocal international connections. This networked model became an increasingly characteristic feature of mathematical research by 2025.

Education and research training were essential to sustaining these developments. A modern computational mathematician requires competence in analytical reasoning, probability, numerical approximation and algorithmic complexity. Depending on the application, additional knowledge of software architecture, optimisation or scientific data management may be necessary. Czech universities responded to the growing convergence of mathematics and informatics through research-oriented teaching and collaboration with specialised institutions. Doctoral researchers could encounter problems whose solution depended simultaneously on proof, numerical experimentation and disciplinary interpretation. Access to high-performance computing created possibilities for advanced practical training that earlier generations had rarely enjoyed. Nevertheless, familiarity with a computing platform did not replace the need to understand convergence or statistical inference. The increasing availability of automated tools

made critical mathematical judgement more important rather than less. International scientific mobility also exposed younger researchers to different research practices and evaluation standards. The long-term strength of Czech computational mathematics therefore depended on people capable of moving confidently between theory and implementation.

By 2025, computational mathematics in Czechia had become a diverse collection of connected research practices rather than a single unified discipline; theoretical work continued in approximation, optimisation, probability, logic and the analysis of algorithms. Applied investigations addressed scientific simulation, intelligent control, complex networks and large-scale data analysis. National infrastructure made more ambitious computation possible, while European cooperation extended the resources available to Czech teams. The central achievement of this period was the enlargement of the questions that mathematical researchers could formulate and investigate with credible methods. Yet greater computational power did not make mathematical proofs, transparent assumptions or independent validation unnecessary. The same algorithms could be scientifically informative in one context and misleading in another. Understanding those differences remained central to research quality and responsible application. Czech contributions are therefore best recognised through identifiable methods, rigorous findings, scientific collaborations and the capabilities sustained by institutions. In this perspective, computational innovation constitutes a continuing transformation of mathematical inquiry rather than its replacement by automated calculation.

Part Three

Leading Institutions and Scientific Schools of Czech Mathematics

Prague Mathematical School: Charles University and the Development of Fundamental and Discrete Mathematics

The Prague mathematical school represents a network of scholars rather than a single doctrine transmitted unchanged across generations. Its contemporary identity is inseparable from Charles University, whose mathematical traditions preceded the establishment of the independent Czech Republic by several centuries. The Faculty of Mathematics and Physics provided an institutional setting in which established analytical and geometric work could coexist with rapidly developing discrete mathematics. After 1993, these traditions faced an open European academic environment and more demanding forms of international assessment. Research groups increasingly defined themselves through specialised problems, collaborative publications and doctoral supervision rather than merely through departmental continuity. Yet the persistence of seminars, mathematical language and shared research questions supplied intellectual stability during administrative change. Prague's contribution consequently arose from the interaction between an inherited scholarly culture and opportunities for disciplinary renewal. Understanding that interaction is essential before individual discoveries or research centres can be assessed. The following analysis treats the faculty as a changing research ecosystem rather than a collection of independent biographies.

The institutional history of the faculty begins well before the period examined in this book. In 1952, the mathematical and physical disciplines were separated from the Faculty of Science in a reorganisation of Charles University. The resulting Faculty of Mathematics and Physics developed specialised departments capable of supporting different mathematical traditions within a common educational framework. In 1956, Eduard Čech founded the Mathematical Institute of Charles University, creating a durable focal point for research and advanced instruction. The founding generation had worked in a Central European tradition where topology, geometry and mathematical analysis were closely connected. Subsequent decades brought intellectual achievements alongside restrictions on academic autonomy and international movement under state socialism. The democratic transformation after 1989 reopened scientific communication before the political creation of the Czech Republic in 1993. The historical significance of independence therefore lies in altering institutional circumstances rather than originating Prague's mathematical expertise. Those foundations remained relevant even when the problems pursued after 1993 differed considerably from those of the founding generation. Institutional continuity therefore should not be confused with intellectual immobility.

The changes of the early 1990s did not dismantle the faculty's mathematical disciplines, but they redefined their external relationships. Academic departments acquired broader scope to renew curricula, establish international partnerships and participate in competitive grant programmes.

Mathematical research retained a particular advantage because a theorem or proof can circulate internationally without requiring an expensive experimental installation. Nevertheless, even theoretical groups depended on reliable libraries, computing access, travel funding and opportunities for younger researchers. The adaptation of publication practices to internationally visible journals became an important measure of the changing professional environment. Czech mathematicians were increasingly evaluated alongside colleagues from institutions with more substantial resources. For the Prague school, this circumstance made specialised intellectual strength and international collaboration especially valuable. The resulting transformation was gradual, uneven across subjects and closely connected with institutional traditions. The central challenge was to make inherited expertise productive within a more open yet more competitive system.

One characteristic of Charles University's mathematical landscape is its internal diversity. The Mathematical Institute has sustained work in analysis, geometry and mathematical modelling, while other departments have developed algebra, logic, statistics and discrete methods. Such differentiation provides the intellectual space required for research that ranges from highly abstract structures to equations describing fluids and materials. It also prevents the Prague school from being identified exclusively with the internationally visible successes of graph theory. The institutional boundaries between mathematics and computer science have often been productive rather than restrictive. Students encountering a problem in one field could find appropriate methods in neighbouring seminars and research groups. These connections were particularly significant as computational complexity and combinatorial structures became increasingly central to mathematical research. Their development illustrates a broader transformation of university mathematics from isolated specialisms into intersecting intellectual programmes. This diversity is an essential reason for treating Charles University as a mathematical school in the pluralistic modern sense. Its departments share an institution without being constrained by a common theoretical programme.

Mathematical analysis remained a central foundation of research at Charles University throughout 1993–2025. Questions concerning function spaces, convergence, regularity and the behaviour of operators retained their intrinsic theoretical importance. They also provided the language needed to formulate differential equations arising in mechanics, mathematical physics and other sciences. Prague analysts worked within an international tradition in which precise definitions and sharp estimates determine whether a physical model can be placed on a rigorous mathematical basis. The relationships between Sobolev spaces, integrability conditions and boundary behaviour are examples of problems whose significance extends beyond a single equation. Research in these areas developed through specialist seminars and collaboration with the Mathematical Institute of the Czech Academy of Sciences. This interaction should be understood as a division of complementary expertise rather than as a simple transfer of research from one institution to another. Analytical advances acquired their greatest influence when new abstract results could clarify concrete questions of existence or stability. In this field, the refinement of a hypothesis can itself be a substantial scientific advance.

The study of functional spaces illustrates the continuity between Prague's classical mathematical strengths and newer analytical problems. Functions involved in physical models

may be too irregular to admit the derivatives required by elementary calculus. Modern analysis therefore works with weak derivatives, distributional interpretations and carefully chosen spaces in which limits remain controllable. These conceptual tools allow researchers to ask whether a model possesses a solution before attempting to calculate one. At Charles University, analytical and modelling traditions brought such questions into regular contact with fluid mechanics and nonlinear partial differential equations. The mathematical challenge was not simply to obtain estimates but to establish precisely which assumptions made them valid. An apparently technical embedding theorem might determine whether nonlinear terms survive a limiting procedure. This is why the historical development of functional analysis cannot be separated from the evolution of rigorous mathematical modelling. Such investigations make the distinction between formal manipulation and mathematical justification especially clear. They also explain why advanced instruction in analysis remains closely linked with current research.

The Mathematical Institute's work in mathematical modelling provides a particularly clear institutional example. Its division devoted to modelling investigates problems where continuum mechanics and differential equations demand rigorous analysis. Josef Málek is associated with research on non-Newtonian fluids and mathematical descriptions of complex material responses. Such substances cannot always be represented by a simple proportional relationship between stress and rate of deformation. Their behaviour requires constitutive assumptions that must be reconciled with energy estimates and conditions for existence of solutions. Studies conducted with collaborators helped connect abstract analytical methods with questions arising in applied mechanics. This is a form of innovation in which the principal contribution may be a mathematically justified model rather than a new physical instrument. It demonstrates the reciprocal relationship between the university's foundational strengths and problems originating outside pure mathematics. Research of this kind makes the university a participant in mathematical innovation even when its findings cannot be expressed as a patent.

Milan Pokorný and other researchers working on nonlinear partial differential equations exemplify another aspect of the Prague analytical environment. Compressible and incompressible flow models raise different mathematical questions because density, velocity and pressure interact through nonlinear constraints. The existence of weak solutions is distinct from uniqueness, smoothness or their behaviour over long time intervals. These distinctions matter when a theoretical model is invoked to interpret a numerical simulation. University research therefore contributes not only to computation but to the standards by which a computational result can be interpreted. International collaborations allow local groups to work on problems whose mathematical literature is distributed across numerous countries. Doctoral students learn that progress often consists of proving a carefully delimited proposition under explicit assumptions. Such scholarly discipline has remained a defining characteristic of rigorous analytical education at the faculty. The analytical tradition thus supplies a crucial bridge between theorems and reliable simulations. Its usefulness depends on preserving the precision of each mathematical assumption.

Geometry at Charles University also developed through continuity and conceptual expansion. Eduard Čech's historical legacy created an enduring reference point, but the research agenda of the post-1993 period involved questions far beyond classical topology. Differential geometry

supplied mathematical tools for describing curvature, connections and the transformation of geometric data. Representation theory and differential operators further connected geometric structures with mathematical physics. Research associated with Vladimir Souček and colleagues helped maintain international links in these areas. Geometric approaches often reveal relationships between apparently different equations by identifying common structural properties. This capacity to translate a problem into an invariant language is particularly valuable in an interdisciplinary mathematical setting. The Prague school consequently preserved geometric research not as a museum of earlier achievements but as an active source of contemporary techniques. International recognition of these methods rests on their mathematical substance rather than on institutional prestige alone.

The relationship between differential geometry and mathematical physics was strengthened by regular scholarly exchange. The Mathematical Institute of Charles University has organised conferences and seminars devoted to geometry and physics, including an established winter-school tradition. Such meetings allow researchers from different countries to compare methods that may have originated in distinct academic communities. Graduate students can encounter current work on geometric differential operators alongside specialists who helped develop the relevant theories. The value of this activity cannot be measured only through papers published during a single conference. Long-term connections influence doctoral projects, visiting appointments and the formation of shared research questions. In the Czech context, relatively concentrated institutional networks made repeated interaction between mathematicians and physicists especially feasible. Seminar continuity therefore operated as a form of research infrastructure in its own right. A conference series can preserve continuity even when the researchers participating in it change considerably. In this sense, scholarly meetings are investments in a community's future intellectual capacity.

Algebra developed at the faculty both as a fundamental discipline and as a language for theoretical computer science. Abstract operations, homomorphisms and structural properties permit mathematical problems to be treated independently of a particular representation. This viewpoint becomes especially powerful when an algorithmic task can be classified by the algebraic properties of its constraints. Libor Barto's research illustrates the productivity of this connection during the later period covered by the book. His work on the constraint satisfaction problem contributed to the algebraic analysis of computational complexity. A finite collection of permissible relations may appear to define a concrete combinatorial task, yet its tractability can depend upon deep structural properties. This relationship between universal algebra and algorithms became an internationally significant direction of Czech mathematics. It also offers an example of how a university research school can redefine the boundary between traditionally separate subjects. It is particularly important to distinguish this theoretical advance from the mere application of existing software to constraint problems.

Mathematical logic formed another bridge between foundational inquiry and computation at Charles University. Logical research addresses the expressive power of formal languages, the structure of mathematical proofs and the limits of algorithmic reasoning. Prague researchers worked in an environment influenced by the contributions of Petr Hájek and Pavel Pudlák, although their institutional affiliations and research paths were not identical. Jan Krajíček's work

on bounded arithmetic and proof complexity established further connections between logical methods and computational lower bounds. Questions about the length of proofs can illuminate why some apparently simple computational problems remain difficult. Such research depends on exact distinctions between what a formal system proves and what may be true in its intended interpretation. Its significance cannot be reduced to the practical use of software for verifying statements. Rather, the Prague mathematical environment helped bring foundational concepts into conversation with contemporary complexity theory. This kind of work helps explain why foundations of mathematics retained a place within modern computational research. Formal reasoning remained both a subject of inquiry and a method for investigating computation.

The faculty's greatest international visibility in discrete mathematics has been closely associated with Jaroslav Nešetřil. He had already established a research career before 1993 and became a full professor at Charles University that year. His interests include graph theory, Ramsey theory, graph homomorphisms and structural properties of sparse classes. These subjects concern finite configurations but often require powerful general principles to understand their behaviour. Nešetřil's academic contribution involved both original research and the organisation of communities capable of sustaining ambitious programmes. His collaborations extended across national boundaries and resulted in books and papers widely used by graph theorists. The development of his school illustrates how an established university department could become a focal point of international mathematical interaction. It also demonstrates why institutional history and the history of mathematical ideas should be examined together. His achievements also offered doctoral students a model of how sustained theoretical work can reshape an international field.

Ramsey theory gives a particularly revealing example of the intellectual orientation of Prague combinatorics. It asks which patterns must appear when sufficiently large structures are partitioned or coloured in prescribed ways. Classical formulations involve graphs and sets, but modern research increasingly studies structures equipped with additional relations and restrictions. Nešetřil and collaborators explored structural Ramsey properties and connections with categorical and model-theoretic methods. Such results are important because they replace isolated counting arguments with principles capable of explaining whole classes of configurations. They also illustrate the unexpectedly close relationship between finite combinatorics and questions about symmetry or definability. The Prague seminars fostered collaboration among researchers approaching these problems from different mathematical perspectives. The resulting research culture valued conceptual unification as much as the solution of a particular extremal problem. The results frequently reveal patterns of order in situations that initially appear dominated by arbitrary choice. That intellectual perspective remained a recurring theme in Prague's discrete mathematics.

Graph homomorphisms became another distinctive component of the Prague research programme. A homomorphism preserves adjacency in a way that allows one graph to be mapped into another while retaining a specified relation. This apparently elementary concept connects graph colouring, combinatorial order and algorithmic constraint satisfaction. The monograph Graphs and Homomorphisms, written by Pavol Hell and Jaroslav Nešetřil in 2004, provided a systematic account of this perspective. It helped make structural methods accessible to

mathematicians and computer scientists working on related problems. The resulting framework made it possible to ask not only whether a graph admits a particular mapping but also how whole families of such mappings behave. Such research reveals the power of a carefully chosen abstraction to unify numerous seemingly separate questions. The Prague school's contribution was therefore conceptual and methodological as well as technical. The method illustrates how mathematical generalisation can generate both conceptual clarity and algorithmic consequences.

Research on sparse graphs expanded the reach of this structural approach during the 2000s and 2010s. Sparsity does not merely mean that a particular graph has relatively few edges. Modern theory examines graph classes through their behaviour under substructures, shallow minors and associated notions of expansion. Nešetřil and Patrice Ossona de Mendez developed a broad framework connecting sparse structures with structural graph theory and algorithms. Their 2012 monograph Sparsity: Graphs, Structures, and Algorithms became a major reference for the subject. The theory provides tools for identifying when classes that appear combinatorially complicated possess exploitable regularities. Connections with logic and algorithmic meta-theorems made the results significant beyond traditional extremal graph theory. In this way, Prague research helped formulate a language through which several areas of discrete mathematics could communicate. The eventual international impact of this programme extended beyond the chronological moment of any one paper. Its influence reflects decades of cumulative collaboration and refinement.

Discrete geometry provided a complementary illustration of the faculty's international mathematical influence. Jiří Matoušek contributed influential research connecting combinatorial reasoning with geometric configurations and computational complexity. Questions about arrangements of points, ranges and convexity have immediate geometric formulations but often demand subtle probabilistic or topological arguments. His books and research articles helped establish techniques that circulated widely among researchers in computational geometry and combinatorics. The importance of his work lies partly in the way it joined precise theoretical estimates with algorithmic questions. It also demonstrates that the Prague mathematical school did not rely upon a single research group or a single class of problems. Related work by Pavel Valtr and other scholars further enriched the environment of geometric combinatorics. Their interactions contributed to a distinctive tradition in which geometric intuition was subjected to rigorous discrete analysis. These developments make geometric combinatorics indispensable to an account of the university's mathematical achievements.

The growth of this research community required organisational structures beyond an ordinary departmental timetable. The Centre for Discrete Mathematics, Theoretical Computer Science and Applications, known as DIMATIA, was organised in 1996 and began its regular activities in January 1997. Its foundation involved Charles University, the Czech Academy of Sciences and a Prague institution specialising in chemical technology. The centre linked mathematical researchers with colleagues working in algorithms, logic and applications. Early activities included seminars, visiting appointments, collaborative projects and international workshops. Accounts of its first year record talks by distinguished foreign mathematicians and the arrival of

researchers for extended stays. DIMATIA was therefore not merely a new administrative title attached to existing individual projects. It provided a durable setting in which the Prague school could make international collaboration part of its everyday practice. The centre's history provides unusually concrete evidence of the internationalisation described in the first part of this book. Its activities joined scientific exchange with the formation of a stable local research community.

The personnel of DIMATIA reflected an unusually productive combination of generations and specialisms. Jaroslav Nešetřil acted as director, with Jan Kratochvíl among the leading organisers of the centre's early work. The associated Prague mathematicians included Jiří Matoušek, Martin Loebl, Martin Klazar, Pavel Valtr and researchers linked with theoretical computer science. Their common research environment supported work on graph representations, combinatorial constructions and algorithmic complexity. Visiting scholars contributed methods developed in neighbouring European and North American institutions. These exchanges changed the intellectual opportunities available to young Czech researchers entering the profession. International collaboration could now be experienced through weekly seminars and sustained joint work, rather than only occasional travel abroad. The centre consequently became an institutional expression of the post-1993 reconnection of Czech mathematics with wider scientific networks. Their collective work is best understood as a research network rather than a succession of unrelated personal successes.

A further organisational step came with the Institute for Theoretical Computer Science, established at Charles University from July 2000 as a national research centre. Its stated purpose was to strengthen work in theoretical computing and closely related mathematics while providing opportunities for young researchers. The institute built upon research networks already cultivated by DIMATIA and the Department of Applied Mathematics. The relationship between these structures illustrates how mathematical communities may require different forms of organisation at different stages of development. A research centre can coordinate projects and support early-career scholars, while a university department maintains the longer continuity of courses and appointments. Both functions were essential to the development of theoretical computer science in Prague. The institutional changes also reveal that algorithmic research had become a sufficiently mature mathematical field to justify dedicated infrastructure. Its infrastructure was primarily intellectual and organisational rather than experimental in the conventional physical-science sense. This institutional arrangement also supported the gradual recognition of computing theory as a mathematical research field in its own right. The transition was organisational as well as conceptual.

The Department of Applied Mathematics occupied a crucial position within this broader network. Its research interests span optimisation, operations research, geometry, topology, graph theory, algorithms and abstract mathematical structures. This range may initially appear too broad to form a coherent departmental identity. Its coherence becomes clearer when viewed through the common use of discrete reasoning and the formulation of precisely constrained mathematical problems. In the post-1993 period, teaching and doctoral supervision were increasingly connected with externally funded international projects. Students could enter research at the boundaries between pure mathematics and algorithm design. This made the department

important for the scientific reproduction of the Prague discrete-mathematics school. The combination of advanced research with foundational university courses helped sustain intellectual continuity across generations. The department thereby linked undergraduate foundations with questions debated at the international research frontier.

The study of computational complexity further strengthened the connection between mathematics and informatics at the faculty. Researchers investigated algorithms not merely as instructions for computers but as mathematical objects whose efficiency and limitations can be proved. Work by Jiří Sgall and colleagues addressed questions relevant to approximation, scheduling and online algorithms. These problems are motivated by the fact that decisions must sometimes be made without complete information about future inputs. Their analysis requires carefully defined performance guarantees and comparisons with an idealised optimum. Other Prague research groups applied combinatorial and logical methods to parameterised complexity and constraint problems. Such topics demonstrate that theoretical computer science is not a departure from mathematical rigour but another arena for its application. They also explain why mathematics departments and computing institutes increasingly shared research personnel and seminars. These contributions exemplify the distinctive mathematical value of proving performance guarantees rather than reporting isolated computational successes. They remain relevant wherever scarce resources or incomplete information shape algorithmic decisions.

The theory of algorithms created practical opportunities without requiring every research outcome to produce a commercial product. Improvements in graph algorithms may affect network optimisation, database problems or the design of computational systems. Yet the immediate scientific achievement is often a theorem establishing correctness, complexity or a structural decomposition. This distinction is essential when evaluating innovation in mathematical research institutions. International grant programmes increasingly required descriptions of potential impact, but the route from proof to application remained uncertain and frequently indirect. The Prague school responded by combining problem-oriented research with long-term investigation of fundamental structures. Such a balance protected abstract inquiry while allowing ideas to cross into computer science and engineering. It is one of the characteristic institutional strengths of a university environment encompassing several neighbouring disciplines. As a result, evaluation should consider conceptual advances and the intellectual pathways they create.

Probability and mathematical statistics also maintained an established institutional presence at Charles University. The department responsible for these subjects traces its origins to the faculty's establishment in 1952, with earlier traditions shaped by Jaroslav Janko, Ladislav Truksa and later Jaroslav Hájek. Its modern research includes nonparametric methods, robust statistics, asymptotic theory, change-point analysis, extreme-value methods and stochastic modelling. These directions demonstrate that the university's mathematical strengths were not confined to deterministic or discrete structures. The statistical analysis of uncertainty provides necessary foundations for econometrics, finance, biostatistics and scientific data analysis. During the post-1993 decades, such applications became increasingly important as computational data resources expanded. At the same time, the underlying research continued to demand rigorous mathematical proofs. The department therefore represents another

instance of continuity between inherited fundamental knowledge and changing practical questions. The coexistence of probability, analysis and discrete mathematics added considerable breadth to the faculty's scientific environment. Such breadth matters for interdisciplinary problems that do not fit a single established mathematical field.

Doctoral education was one of the principal mechanisms through which Prague's research schools renewed themselves after independence. A doctoral student in mathematics learns to identify a narrowly formulated question whose answer is not already contained in the literature. This process depends on seminars, specialised reading, supervision and repeated exposure to criticism from other researchers. International exchanges helped students encounter mathematical communities with different techniques and conventions. Centres such as DIMATIA added further opportunities through visiting researchers, workshops and collaborative discussions. Mathematical training consequently became less confined to the intellectual resources of a single departmental environment. At the same time, the presence of strong local mentors remained essential because short visits cannot substitute for sustained guidance. The university's long-term scientific standing depended upon the interaction of these local and international forms of education. This arrangement also allowed research groups to preserve their traditions while adopting new methods from abroad.

Scientific publications and scholarly meetings constituted another dimension of institutional influence. Mathematics can develop through small collaborative groups, but the resulting work must enter a wider community capable of scrutinising and extending it. Prague mathematicians published articles and monographs with international co-authors while organising specialist seminars and conferences. In graph theory and combinatorics, these activities connected researchers across Europe and beyond. In geometry and analysis, recurring meetings sustained conversations about mathematical methods that developed over many years. Books written by researchers associated with the faculty also shaped graduate education outside Czechia. Their value lies not solely in summarising completed results but in establishing common terminology and promising research questions. International visibility was therefore generated through a cumulative scholarly process rather than through isolated headline discoveries. The scientific community continued to depend on these less visible forms of collective labour. They make international achievement possible without always appearing in lists of major discoveries.

The development of the Prague school nevertheless depended on material and organisational conditions that were not equally favourable at every moment. Competitive grants supported research visits and specialised projects but could create discontinuity when funding cycles ended. Mathematics required less capital-intensive equipment than experimental physics, yet salaries, doctoral stipends and access to scholarly literature remained important. Talented researchers were able to pursue international opportunities that were intellectually beneficial but could complicate domestic recruitment and retention. Major research groups sometimes enjoyed stronger visibility than smaller departments or emerging specialisms. The distinction between pure and applied research also created tensions when universities were asked to demonstrate immediate economic impact. These constraints help explain why institutional success must be assessed through long-term capacity rather than the number of prominent publications alone. A historical account that includes these limitations provides a more

convincing picture of Czech mathematical development. Understanding these conditions prevents the narrative from becoming an unqualified celebration of internationalisation.

By 2025, Charles University had preserved a recognisable mathematical identity while participating fully in an international disciplinary landscape. Its analysts and geometers continued research traditions that reach back to earlier Czech mathematical generations. Its algebraists and logicians investigated structures whose importance had grown through their interaction with computational complexity. Its combinatorial community developed internationally influential approaches to graph homomorphisms, Ramsey theory and sparse structures. Research in mathematical modelling connected abstract methods with problems concerning fluids and other complex systems. Organisational initiatives such as DIMATIA demonstrated how collaborative research could be anchored in a university while remaining open to partners abroad. The institution's greatest achievement was not the dominance of one mathematical subject over all others, but the preservation of a setting in which different forms of rigorous inquiry could develop together. The Prague mathematical school thus offers an instructive example of scientific continuity transformed into international research capacity between 1993 and 2025. This combination of intellectual depth and openness is the central conclusion of the institutional case study. It provides the foundation for comparing Prague with other Czech mathematical centres in the following chapters.

Institute of Mathematics of the Czech Academy of Sciences: Fundamental Research and International Mathematical Excellence

The Institute of Mathematics of the Czech Academy of Sciences occupies a distinctive position within the research landscape of contemporary Czechia. Its history reveals how specialised mathematical knowledge can survive a major change of state while adapting to a different academic order. The institution did not arise from a blank slate when the independent Czech Republic was established in January 1993. It inherited research programmes, professional relationships and intellectual traditions assembled over successive generations. The mathematical questions pursued in its laboratories and seminar rooms were not determined solely by political boundaries. Their development depended upon accumulated techniques, difficult unresolved problems and an international conversation that extended beyond Central Europe. Yet the new national framework altered the financial and administrative conditions under which that conversation could continue. It also required mathematicians to reassess their institutional priorities and relationships with universities. The institute's post-1993 trajectory therefore offers a particularly revealing case of continuity combined with intellectual and organisational renewal.

The antecedents of the present institute reach back to the institutional formation of mathematical research in post-war Czechoslovakia. Archival records identify the Institute of Mathematics of the Czechoslovak Academy of Sciences as an organisation operating from 1953

until the dissolution of the federal Academy in 1992. Earlier structures included mathematical units associated with the Czech Academy of Sciences and Arts and a central mathematical institute. These organisational changes reflected a growing conviction that advanced research required more than individual university chairs. Mathematical institutes could concentrate specialised scholars, maintain long-term seminars and preserve continuity across changes in teaching arrangements. They also allowed research programmes to develop beyond the immediate demands of degree curricula. The historical division between Prague and Brno formed part of this institutional inheritance. The contemporary Czech institute thus emerged from an existing network rather than from an entirely new disciplinary initiative. This longer view is indispensable for distinguishing the legal watershed of 1993 from the intellectual origins of Czech mathematics. The original institutional setting therefore remains relevant to the interpretation of later achievements.

Several foundational figures demonstrate why the institute's scientific identity could not be reduced to its administrative history. Eduard Čech contributed lasting constructions in topology, including the cohomological concepts associated with his name. Jaroslav Kurzweil helped reshape integration theory and investigated stability problems in differential equations. Ivo Babuška became internationally influential through approximation theory and the analysis of numerical methods, particularly finite elements. Jindřich Nečas established major results concerning partial differential equations and the mathematical theory of continua. Miroslav Fiedler's work connected matrix theory with the structural study of graphs. Vlastimil Pták advanced functional analysis and the theory of iterative processes. These mathematicians did not represent a single narrowly defined research method, and their careers unfolded in different institutional and international settings. Together, however, their results created an enduring repertoire of problems and techniques upon which later generations could build.

The transition of 1993 changed the legal home of these traditions without annulling their intellectual achievements. Records preserved by the Masaryk Institute and Archives of the Czech Academy of Sciences distinguish the former Czechoslovak institute from its Czech successor. The new Institute of Mathematics retained a place within the successor Academy and continued its work in Prague and Brno. This continuity mattered because mathematical research frequently evolves through seminars and mentorship networks that cannot be recreated rapidly. It also preserved institutional memory concerning unfinished work, scholarly correspondence and specialised libraries. The political liberalisation that began in 1989 had already widened opportunities for professional exchange. Independence subsequently gave that opening a new legal and financial structure. The decade thus contained more than one turning point, and each must be assigned its proper historical role. Political reform enabled renewed communication, whereas the institutional settlement of 1993 provided a durable Czech framework for research. Neither historical moment can be understood accurately if it is detached from the other.

For a research institution devoted mainly to fundamental mathematics, the effects of systemic transformation were often indirect but nevertheless substantial. Mathematics does not ordinarily depend upon the experimental apparatus required by particle physics or molecular biology. Its essential resources include uninterrupted research time, access to scholarly literature and contact with other specialists. Stable appointments and promising opportunities

for younger researchers are therefore especially important. Under new funding arrangements, continuity increasingly had to be reconciled with competition for external grants. Researchers needed to formulate programmes for evaluation while retaining the freedom to follow problems whose solutions might remain distant. International publications gradually acquired greater institutional significance as indicators of research quality. These developments did not necessarily transform the content of individual theorems, but they altered the circumstances in which such theorems could be pursued. The resulting balance between autonomy and accountability became a defining issue in the institute's post-socialist development.

The institute's research model rested on specialised teams rather than a comprehensive attempt to represent every mathematical discipline equally. This selectivity followed from the country's finite research capacity and from the highly differentiated nature of modern mathematics. Research groups developed expertise around interrelated problems that could sustain work over many years. Such groups also needed to remain permeable to unexpected conceptual developments and international collaboration. Mathematical analysis continued to provide an especially important foundation. Differential equations and functional analysis connected abstract theory with questions from mechanics, physics and numerical computation. Other directions included geometry, topology, algebra and the foundations of mathematical reasoning. The coexistence of these fields encouraged methodological exchanges without dissolving their disciplinary identities. The institute consequently functioned as a network of intellectual specialisations connected by common standards of proof and mathematical argument. Its research breadth thus represents a deliberate distribution of expertise rather than accidental fragmentation.

The European Mathematical Society's institutional profile makes clear that research priorities have been shaped substantially by initiatives originating among mathematicians themselves. This bottom-up organisation is particularly appropriate in fields where scientific opportunity cannot always be predicted by external planning. A novel argument in topology may generate questions that reach analysis or theoretical computer science years later. A technical advance in numerical linear algebra may unexpectedly influence the feasibility of a computational model. Institutional leadership therefore faces the task of supporting promising directions without prescribing their mathematical conclusions. Advisory structures and external evaluation provide additional perspectives on research quality. These mechanisms can encourage coherence between long-term inquiry and public responsibility. However, their value depends upon recognising that depth in mathematics is not adequately measured by immediate technological application. The institute's modern identity reflects this continuing negotiation between mathematical independence and institutional strategy.

The Department of Abstract Analysis illustrates the breadth concealed within a seemingly narrow disciplinary title. Its research investigates structures through tools drawn from logic, set theory, topology and functional analysis. Descriptive set theory permits complex spaces and mappings to be classified according to carefully articulated regularity conditions. Banach space theory examines the structure of complete normed spaces and the behaviour of operators acting upon them. Related investigations engage with categories and non-commutative algebras, including C-star algebras. Such subjects may appear distant from practical calculation, but they clarify the

conceptual conditions under which mathematical representations are valid. Their methods often travel between otherwise separate areas of modern mathematics. An abstract classification theorem can reveal a common architecture beneath results that were initially formulated in unrelated languages. The department therefore exemplifies innovation through conceptual unification rather than through the production of a new physical device. Such contributions can be foundational even when their significance is initially apparent only to specialists.

The evolution-equations research community represents another major line of institutional continuity. Its central concern is the mathematical description of systems that change over time. Partial differential equations formalise processes such as fluid movement, thermal exchange and material deformation. Researchers examine whether meaningful solutions exist and whether the underlying model is mathematically coherent. They also investigate uniqueness, regularity and the dependence of solutions on initial conditions. These questions acquire special importance when the equations describe nonlinear interactions that cannot be resolved through elementary formulas. Weak-solution methods permit systems to be studied when classical differentiability is unavailable. Energy inequalities and compactness arguments then provide alternative forms of control over the behaviour of solutions. The resulting theory establishes what can be known reliably about a model before numerical simulation or technological application begins.

Eduard Feireisl's research offers a prominent example of this tradition entering new international territory. His work concerns fluid dynamics, nonlinear partial differential equations and the mathematical foundations of continuum thermodynamics. The European Research Council supported his project Mathematical Thermodynamics of Fluids between 2013 and 2018. That project brought together questions about compressible viscous fluids, heat conduction and the stability of mathematical descriptions. Its outcomes included advances in the theory of dissipative measure-valued solutions. Such solutions permit mathematical analysis when sequences of approximations develop oscillations or concentrations beyond conventional compactness assumptions. The approach also helps clarify the relationship between weak analytical solutions and convergent numerical schemes. Feireisl's programme demonstrates how a national research institution can contribute concepts adopted across an international research field. Its significance lies in the structure of mathematical explanation as much as in its relationship to fluid mechanics. The international recognition of this programme also reinforced the visibility of Czech nonlinear analysis.

A related line of work connects evolutionary equations with realistic material models. Mathematical descriptions of liquids and solids must represent constitutive laws as well as conservation principles. These laws may be nonlinear, rate-dependent or sensitive to temperature and internal structure. Analysis becomes particularly demanding when the models involve interacting variables with different degrees of regularity. Theoretical progress requires methods that preserve physical constraints while accommodating mathematically weak solutions. Research associated with the institute has contributed to this general programme through fluid-mechanical and thermodynamic problems. Collaboration with university mathematicians and specialists in continuum mechanics broadens the range of relevant

questions. Such exchanges also demonstrate why the boundaries between pure and applied mathematics are often more porous than institutional classifications suggest. A theorem motivated by physical modelling may ultimately enrich the general theory of nonlinear equations.

Constructive methods of mathematical analysis provide a complementary route from mathematical theory to reliable computation. The corresponding department investigates numerical approaches to partial differential equations and the structure of large algebraic systems. Its work encompasses finite-element and related discretisation methods, a posteriori error analysis and numerical linear algebra. An approximation is not useful merely because a computer can produce it. Researchers must establish whether numerical results converge to a mathematically meaningful solution as discretisation is refined. They must also estimate the error introduced by finite computational resources and imperfect data. A posteriori estimators support adaptive calculations by identifying where additional resolution is needed. Parallel domain-decomposition and multilevel techniques distribute large computations across interconnected processors. These methods connect rigorous analysis with the increasingly substantial computational infrastructures of contemporary science. Reliability and efficiency must therefore be pursued together rather than treated as competing objectives.

Numerical linear algebra is particularly important because large scientific models typically lead to systems of algebraic equations. Such systems may be sparse, ill-conditioned or too extensive for direct calculation using ordinary computing resources. Iterative methods exploit algebraic structure to generate increasingly accurate approximations. Preconditioning attempts to improve the behaviour of these iterations without changing the underlying scientific problem. Matrix theory provides a language for explaining spectral properties that influence convergence. Domain-decomposition algorithms divide a computational region while preserving communication across its interfaces. Their mathematical analysis is inseparable from the architecture of the machines on which they run. The institute's involvement in these topics helps connect classical Czech strengths in linear algebra with present-day scientific computing. It also illustrates why theoretical reliability remains essential even when impressive computational power is available.

The Jindřich Nečas Centre for Mathematical Modeling has provided an important platform linking the institute with university research. Its intellectual agenda connects nonlinear analysis, partial differential equations and mathematical modelling. Such collaborations enable different mathematical communities to address problems requiring several complementary forms of expertise. University groups contribute graduate education and a continual influx of young researchers. Academy-based scholars can provide specialised long-term research environments and continuity of complex programmes. Joint seminars bring together theoretical considerations and problems arising from mechanics or computation. Collaboration also facilitates the transfer of techniques between abstract regularity theory and numerical approximation. The centre's existence demonstrates that institutional boundaries need not correspond to intellectual divisions. In modern Czech mathematics, durable networks can be as important as formal research departments. The centre thus contributes to research capacity through sustained cooperation rather than administrative expansion alone.

Algebra, geometry and mathematical physics constitute a further research environment within the institute. The corresponding department brings together questions in representation theory, geometric structures and mathematical models of space-time. Algebraic topology and homological methods offer tools for identifying relations that remain invariant under appropriate transformations. Differential geometry provides a framework for analysing curvature, connections and geometric fields. Problems associated with gravitation call for mathematically precise treatment of the equations used to describe space-time. These subjects depend upon subtle interactions among algebra, analysis and geometry. The department's configuration reflects a recognition that disciplinary frontiers often generate new mathematical questions. Rather than treating mathematical physics as an application appended to pure mathematics, its research places physical concepts within a shared theoretical investigation. This approach broadens the institute's contribution to international mathematical discussions.

Geometry and topology also display the persistence of older Czech traditions within contemporary problem-solving. The influence of Eduard Čech is particularly visible in the historical development of algebraic and general topology. Yet continuity does not mean that current researchers simply repeat established theorems. Contemporary approaches investigate new structures, new invariants and refined forms of equivalence. Category-theoretic methods can make relationships between constructions explicit across distinct mathematical settings. Geometric classification may connect unexpectedly with analysis or theoretical physics. The value of a research tradition is therefore measured partly by its capacity to generate new questions. Institutional support permits these investigations to mature over periods longer than ordinary project cycles. In this respect, abstract mathematics remains a major component of the institute's scientific identity. Historical inheritance becomes productive when it supports investigation rather than prescribing its conclusions.

Mathematical logic and theoretical computer science represent a distinct but closely connected strand of research. Their subjects include foundational questions about formal languages, proofs and the mathematical limits of computation. Logical methods can reveal whether certain types of statement are derivable within specified systems. Complexity theory concerns the resources required to solve mathematically defined computational problems. Combinatorial structures provide concrete objects on which formal reasoning can be tested. These investigations intersect with set theory and with classifications of abstract spaces. They also help establish reliable conceptual foundations for digital systems and algorithmic reasoning. The presence of such work within a mathematical institute illustrates a broad understanding of mathematical sciences. It also prevents the institutional story of Czech mathematics from being reduced entirely to analysis and differential equations.

A defining feature of the institute's contemporary structure is the interaction between established programmes and newer configurations of researchers. Departmental arrangements have changed over time as scientific interests and collaborative opportunities developed. The formation of a group devoted to algebra, geometry and mathematical physics illustrates such organisational flexibility. Meanwhile, research in analysis has continued to diversify internally, from abstract structure to evolutionary equations and constructive numerical methods. This division is not a hierarchy between pure and applied investigation. Each field poses its own

standards of originality and often contributes techniques to neighbouring groups. Departmental boundaries serve administrative and collaborative purposes rather than fixed definitions of intellectual identity. Evaluation reports offer snapshots of this evolving research ecology at particular moments. A historical interpretation must therefore distinguish temporary organisational arrangements from longer mathematical lineages. The institutional map is consequently an evolving record of scientific priorities and scholarly initiative.

International collaboration is essential to the scientific standing of an institute whose fields are intrinsically transnational. Major mathematical questions generally circulate through journals, specialist conferences, visiting appointments and sustained correspondence. The institute's annual reports identify international projects, exchange activities and scholarly meetings among its principal functions. Cooperation also enters doctoral education through shared supervision and access to foreign research groups. External colleagues contribute critical assessment that can sharpen arguments and identify overlooked connections. International funding competitions place specific institutional achievements within a wider comparative framework. The European Research Council grant awarded to Feireisl is one particularly visible example, but internationalisation is not exhausted by major awards. It is equally present in ordinary co-authorship, seminars and the long-term movement of ideas. The institute's influence is therefore best understood through the quality of its mathematical participation rather than through a simple count of foreign partnerships.

Scientific publication remains another major dimension of this international role. A mathematical article must make explicit the assumptions, definitions and logical argument supporting its conclusions. Peer review can test these arguments and situate them within a broader literature. The institute contributes to the dissemination of mathematics through articles, books, conference proceedings and scholarly editorial activity. Libraries and access to research publications sustain the intellectual conditions needed for such work. The digitalisation of scientific communication changed the speed at which mathematicians could consult and share findings. Nevertheless, faster circulation did not diminish the importance of painstaking verification. The scholarly authority of a mathematical result depends on reproducible reasoning rather than on the visibility of its announcement. This distinction gives mathematical publication a character that differs in meaningful respects from experimental reporting. These practices are fundamental to maintaining trust within international mathematical communities.

The institute's relationship with graduate education is especially important for the renewal of research generations. The institution cooperates with universities in doctoral programmes and provides training opportunities for younger mathematicians. Doctoral researchers must learn not only advanced techniques but also the judgement needed to recognise substantial mathematical questions. Seminars create environments in which tentative arguments can be examined before they reach publication. Experienced scholars pass on knowledge that is difficult to capture fully in textbooks or formal lectures. Research mobility permits students to encounter alternative mathematical styles and communities. Strong training arrangements can also reduce the isolation of highly specialised disciplines. In return, emerging researchers contribute new interests and methodological combinations to established groups. The

continuity of Czech mathematical schools consequently depends upon educational relationships as much as upon archival or administrative preservation.

The institute operates within a national funding environment that combines institutional support with competitive research projects. Public research organisations require stable resources for staff, administration and basic scientific services. Competitive grants can finance focused teams, new collaborations and the investigation of emerging problems. These mechanisms serve different purposes and should not be treated as interchangeable. Research on difficult theoretical questions may develop over periods that exceed the duration of an ordinary grant. Conversely, project funding can create opportunities for younger mathematicians to establish independent scientific identities. External evaluation contributes evidence about the quality of research and the institutional use of public resources. The challenge is to avoid reducing intellectual ambition to what can be predicted within a short reporting period. Sustainable mathematical excellence requires both accountability and confidence in the uncertain temporal rhythm of discovery. The appropriate balance depends partly on the differing time horizons of mathematical problems.

Evaluations conducted by the Czech Academy of Sciences provide one means of assessing the institute's international position. The institute's 2021 annual reporting describes a structured assessment process involving foreign evaluators and selected scientific outputs. Those results differentiated world-leading research from internationally excellent and other categories of performance. Such assessments can help leadership identify genuine strengths and areas requiring attention. They must nevertheless be interpreted with awareness of disciplinary diversity and the necessarily selective evidence evaluated. A breakthrough in one mathematical field may take years to influence neighbouring subjects. Some highly technical contributions acquire significance through later applications or conceptual developments. International peer assessment is thus more informative when considered alongside sustained scholarly achievement. It provides evidence of standing, but it cannot replace a historical account of the mathematical work itself.

The institute's scientific identity also reflects the geographical concentration and distribution of Czech mathematical expertise. Prague provides a dense network of Academy units, universities and research seminars. Brno carries its own longstanding mathematical traditions and professional communities. Cooperation between these centres gives the institute access to complementary intellectual resources. Czech mathematical research extends further through universities in Plzeň, Olomouc, Ostrava and other regions. A national institute should not be treated as identical with the entire national discipline. Its significance arises partly from the way it connects specialised Academy research with this wider university network. Shared projects and professional societies can help prevent fragmentation within a relatively compact research system. The resulting landscape is better described as interconnected than as governed by a single mathematical centre. Collaborative geography consequently matters even in a discipline whose central objects are abstract.

The early 2020s highlighted the importance of flexibility in maintaining research continuity. The coronavirus pandemic disrupted ordinary travel, conference arrangements and some forms of face-to-face mathematical exchange. Digital seminars offered alternative channels for

intellectual communication, although they could not fully reproduce every aspect of in-person collaboration. Research programmes also encountered broader pressures associated with changing costs and competition for scientific personnel. At the same time, expanding computational resources created new opportunities for numerical and data-intensive work. Research in artificial intelligence increased interest in rigorous mathematical questions about optimisation, approximation and uncertainty. The institute's combination of abstract and constructive disciplines positioned it to engage with aspects of these transformations. However, the significance of such engagement depends on identifiable mathematical results rather than the adoption of fashionable terminology. The contemporary challenge is therefore to preserve deep mathematical inquiry while responding selectively to new scientific questions.

The interaction between fundamental research and practical application is perhaps the most important interpretative theme of the institute's history. A theorem in functional analysis may be motivated by a purely structural question yet later become relevant to differential equations. A result about weak solutions may clarify which physical simulations can be regarded as mathematically reliable. Progress in numerical linear algebra can make previously intractable scientific calculations feasible. Conversely, physical and technological problems may reveal deficiencies in existing mathematical concepts. These exchanges are not simple one-way transfers from theory to industry. They form a cycle in which mathematical abstraction and the demands of explanation stimulate one another. The institute's departments illustrate distinct stages within this cycle while retaining their own disciplinary responsibilities. Their combined contribution shows why innovation in mathematics includes new ideas, proofs, algorithms and rigorous modelling frameworks. The resulting conception of research impact is broader than immediate commercialisation.

By 2025, the Institute of Mathematics had maintained its standing as an important institution of Czech fundamental research while participating in an increasingly international scientific environment. Its continuity after 1993 rested upon inherited schools, adaptable research teams and lasting relationships with universities. The growth of modern analysis, evolutionary equations and constructive numerical methods broadened the scientific questions it could address. Research in algebra, geometry, logic and mathematical physics preserved the essential autonomy of theoretical investigation. International collaborations and research funding enabled selected programmes to contribute to wider European mathematical developments. At the same time, evaluation, recruitment and the maintenance of stable research conditions remained continuing institutional responsibilities. The institute's historical achievement cannot be summarised adequately by a single discovery or the construction of a particular facility. It lies in sustaining a community capable of producing mathematical knowledge across political transitions and successive methodological transformations. This experience illustrates how an established institution can preserve an intellectual inheritance while repeatedly enlarging the possibilities of its future research.

Prague Schools of Mathematical Logic, Information Theory, and Applied Mathematics

The development of mathematical logic and applied mathematics in Prague after 1993 cannot be reduced to the history of a single university department. Two institutes of the Czech Academy of Sciences provide particularly instructive evidence of the city's wider intellectual structure. The Institute of Computer Science cultivated theoretical questions concerning computation, formal reasoning, and the mathematics of information. The Institute of Information Theory and Automation pursued complementary problems involving uncertainty, adaptation, measurement, and the control of complex systems. Both organisations inherited expertise that predated Czech independence and therefore entered the new era with established methodological foundations. Political and institutional change nevertheless altered their access to international partnerships, competitive funding, and emerging computational technologies. Their subsequent trajectories reveal how abstract mathematical research and applications developed through sustained contact rather than through institutional merger. Prague consequently became a setting in which several distinct research cultures could address connected questions about knowledge and decision making. The achievements of these institutes should be understood as contributions to international research fields, not as inventions attributable to national identity alone.

The intellectual foundations of the two institutes reach back to the Czechoslovak period, when mathematical methods supported both fundamental investigations and technical research. The Institute of Computer Science developed from an Academy computing environment into a research organisation with a substantial theoretical mission. The Institute of Information Theory and Automation maintained traditions rooted in cybernetics, information processing, automatic control, and probabilistic inference. These histories mattered after independence because scientific communities retain knowledge through people, seminars, research questions, and specialised publication practices. A change in legal jurisdiction could not instantaneously reproduce the expertise accumulated in such communities over decades. At the same time, the opening of research networks after 1989 and the institutional watershed of 1993 encouraged methodological reconsideration. Researchers could compare local traditions more directly with international developments in computational complexity, Bayesian methods, and formal semantics. Continuity therefore did not imply intellectual immobility or the preservation of every earlier research priority. It supplied the resources from which selective transformations became scientifically possible. Its history is therefore one of adaptation through existing expertise rather than replacement of established scholarship.

The creation of the Czech Academy of Sciences in 1993 placed established research units within a sovereign national system of scientific governance. For mathematical institutions, the significance of this transition was chiefly organisational rather than experimental or technological. Mathematical work requires sustained intellectual time, access to literature, and communities capable of testing arguments critically. Its infrastructure also includes seminars, software, specialist libraries, and international networks whose costs are less conspicuous than those of large laboratories. Competitive grants gradually became an increasingly important

complement to institutional funding and influenced how programmes were articulated. Teams had to explain the originality of theoretical problems while demonstrating the credibility of their proposed research methods. This requirement encouraged clearer project identities, although mathematical discoveries cannot be scheduled as predictable engineering deliverables. Research governance also had to accommodate collaborations with universities because many investigators taught or supervised doctoral students. Thus the post-1993 institution was neither a detached academy enclave nor simply an extension of a technical university.

Mathematical logic occupied a distinctive position in Prague because it linked foundational questions with the design of computational reasoning systems. Petr Hájek played a formative role in showing that reasoning about vague statements can be treated with mathematical rigour. His monograph Metamathematics of Fuzzy Logic, published in 1998, offered a systematic account of formal many-valued calculi. The significance of that work lies in its treatment of graded truth through explicit syntax, semantics, and deductive relationships. In this setting, an inference rule is not accepted merely because it resembles an intuitive description of uncertainty. Its validity must be investigated within a carefully specified logical system and compared with the structures that interpret its expressions. This approach provided a theoretical language for discussing approximate reasoning without equating it with informal heuristic computation. Hájek's research also connected the Prague community with wider international discussions of algebraic logic and non-classical deduction. The institutional legacy is visible in the continuing interest of Academy researchers in graded, substructural, and modal logical systems. This achievement made Prague a recognisable contributor to the formal study of vagueness in international logic.

Subsequent Prague research expanded the study of non-classical logics beyond the original presentation of mathematical fuzzy logic. The central questions included expressiveness, completeness, algebraic semantics, and the computational difficulty of deduction. Petr Cintula and collaborators developed systematic approaches to graded and fuzzy logical frameworks within an international research environment. Zuzana Haniková and other investigators studied relationships between formal calculi, algebraic structures, and computational problems. Such work asked which forms of inference remain valid when familiar classical assumptions are weakened or replaced. Substructural logics offered particularly useful settings for examining the consequences of restricting conventional structural rules. Dynamic and modal approaches made it possible to represent knowledge, action, and changing informational states with greater precision. These programmes joined mathematical philosophy to concrete algorithmic questions without making either dimension subordinate to the other. They also demonstrated why computational applications require foundational clarity before software implementation becomes intellectually reliable.

Proof theory provides another bridge between logical foundations and theoretical computer science in the Prague research landscape. A proof can be treated not only as evidence that a statement follows from assumptions but also as a structured mathematical object. Its length, transformations, admissible rules, and relation to computational complexity become questions for systematic investigation. The Czechoslovak and Czech tradition associated with Petr Hájek and Pavel Pudlák helped connect metamathematics with complexity-oriented reasoning. Later

programmes at the Institute of Computer Science continued exploring algebraic and computational methods relevant to formal deduction. Research on proof systems clarifies why two logically equivalent descriptions can behave very differently in automated reasoning. A compact mathematical specification may lead to a difficult proof search even when its interpretation is conceptually straightforward. Understanding these differences informs formal verification, satisfiability procedures, and the design of knowledge representation languages. It also protects mathematical logic from being treated merely as a catalogue of symbols employed by software engineers. Such investigations show why the complexity of formal reasoning constitutes a legitimate mathematical problem.

The Institute of Computer Science progressively developed theoretical computer science as a domain in which mathematical questions have independent intellectual significance. Its research profile includes combinatorics, computational complexity, and mathematical logic as closely connected fields. Extremal graph theory asks how structural constraints determine the possible configurations of a finite or infinite discrete system. Computational complexity investigates which resources are intrinsically required to perform a specified computational task. Mathematical logic studies the inference principles by which properties of such systems can be expressed and established. These directions share an interest in how mathematical structure limits or enables computation, despite using different technical languages. Work on Boolean functions, knowledge compilation, and computational models illustrates the importance of this shared perspective. Neither an algorithm's existence nor a formal description of its task necessarily establishes that the algorithm is practically feasible. The Prague tradition accordingly combines constructive mathematical questions with investigations of unavoidable computational limitations.

Discrete mathematical structures became increasingly prominent as digital systems created new problems about networks, constraints, and combinatorial organisation. At the Academy's Institute of Computer Science, research on extremal graphs and probabilistic combinatorics complemented work undertaken at Charles University. The study of random discrete structures supplied models for analysing typical behaviour rather than only specially constructed examples. Graph limits provided a language for comparing very large networks and investigating their asymptotic properties. Structural combinatorics connected such questions with classical methods of graph theory and with algorithmic considerations. These developments must not be confused with routine network visualisation or merely descriptive data analysis. Their contribution lies in general theorems that reveal patterns shared across extensive families of mathematical objects. Prague's research groups participated in international projects involving combinatorial structures and their computational interpretation. Their activities show how locally established expertise acquired new relevance within an increasingly networked scientific environment. These advances were pursued through specialised research groups and wider international exchanges.

The mathematical foundations of neural computation formed another substantial research direction associated with the Institute of Computer Science. Věra Kůrková contributed to investigations of approximation, learning, and complexity in models inspired by neural networks. Such studies sought to identify the mathematical conditions under which flexible computational

models represent functions accurately. Approximation theory offered a framework for distinguishing universal representational capacity from achievable accuracy under limited computational resources. Questions about function spaces and the complexity of representation therefore connected neural computing with traditional analysis. A model capable of approximating a broad class of functions may nevertheless require substantial parameterisation or extensive data. This distinction helps explain why mathematical guarantees and successful empirical training are not interchangeable forms of evidence. Research on neurocomputing gradually acquired greater visibility as machine learning expanded during the 2010s and early 2020s. The Prague contribution was especially relevant because it maintained a theoretical perspective alongside the growing popularity of applications.

The Institute of Computer Science also built an interdisciplinary research identity extending beyond classical theoretical computation. Statistical modelling was particularly important because scientific data frequently violate assumptions implicit in simple mathematical models. Robust estimation seeks procedures whose conclusions remain informative in the presence of outliers or model departures. Multivariate methods analyse relationships among variables whose dependencies cannot be understood through isolated measurements. Institute researchers worked across these methodological questions and applications in environmental, health, and social research. The value of such work rests on the explicit connection between mathematical assumptions and the nature of available evidence. Models must specify the uncertainty they represent rather than allowing sophisticated computation to conceal weak assumptions. Cooperation with substantive specialists can expose weaknesses that remain invisible within purely formal modelling exercises. This style of research expanded the public relevance of Prague's mathematical expertise while retaining mathematical accountability. Statistical reliability remained a substantive scientific question rather than an optional stage of data processing.

Complex systems research provided an additional institutional bridge between mathematical modelling, statistics, and interdisciplinary applications. Systems composed of interacting elements may display collective behaviour that cannot be inferred from the properties of individual components alone. Network representations offer a useful starting point, but their scientific interpretation requires careful choices about connectivity and dynamics. Statistical dependence, causation, and structural connection must be distinguished if resulting explanations are to remain defensible. Research groups associated with the Institute of Computer Science investigated complex networks and brain dynamics using mathematical and computational methods. Their work illustrated how theoretical graph concepts could be brought into contact with measured biological processes. It also highlighted the limitations of transferring network metrics between disciplines without examining their scientific meaning. International cooperation was indispensable because such projects often combined mathematical expertise with specialised experimental or clinical knowledge. The resulting interdisciplinarity involved reciprocal methodological learning rather than the simple delivery of ready-made algorithms.

The Institute of Information Theory and Automation developed a somewhat different but complementary mathematical research profile. Its programmes combined probability theory,

statistics, optimisation, control theory, econometrics, and signal processing. The institute's central problems often began with a system that could be observed only imperfectly and influenced through uncertain decisions. Researchers therefore needed mathematical descriptions of hidden states, evolving parameters, measurement error, and competing objectives. Probability distributions supplied one language for describing uncertainty, while optimisation identified possible courses of action. The theory of dynamic systems offered a framework for analysing how decisions change future states and observations. These perspectives connected abstract mathematical reasoning with engineering, economics, and the management of complex processes. The institute's research organisation supported multiple departments with different methodologies rather than a single unified modelling technique. Its contribution is consequently best understood as a sustained network of mathematical specialisms. The resulting institutional profile remained flexible enough to accommodate changing technologies and theoretical priorities.

Miroslav Kárný's research exemplifies the Prague tradition of Bayesian decision making under uncertainty. A Bayesian approach updates uncertain beliefs as evidence becomes available and thereby supports sequential decisions. Kárný investigated adaptive systems, parameter identification, and the mathematical design of probabilistic decision policies. His work on fully probabilistic design sought to formulate decisions through probability distributions rather than through isolated deterministic commands. This perspective forces researchers to specify desired behaviour and uncertainty in a coherent analytical framework. It also makes assumptions about prior information, learning, and preference representation central objects of investigation. Applications may concern industrial control or other settings, but their conceptual foundation remains a theory of rational adaptation. The approach is especially useful for showing that decisions depend on models as well as on observed numerical values. It represents a distinctive line of research in which statistics and control theory are treated as mutually informing disciplines.

The evolution of Bayesian modelling after 1993 also demonstrates how specialised theoretical methods can acquire wider relevance over time. Earlier traditions of system identification provided tools for adapting a model when new observations contradicted previous estimates. Later research considered how several decision makers could combine information or operate under partially different objectives. These challenges required more than numerical optimisation because assumptions about information sharing had to be made explicit. Fully probabilistic control design, developed by Kárný and collaborators, offered a mathematical language for evaluating such strategies. The methodology did not eliminate the need for domain knowledge or remove uncertainty from the resulting decisions. Instead, it specified how uncertainty enters the selection and revision of a policy. Its connection with machine learning became more visible as adaptive decision processes gained prominence internationally. Prague's contribution thus lay in developing general principles that could survive changes in application technology. The programme therefore linked learning, uncertainty, and action within a common mathematical vocabulary.

Milan Studený developed another influential research programme at the intersection of information theory, statistics, and artificial intelligence. His monograph Probabilistic

Conditional Independence Structures, published in 2005, examined the mathematical representation of independence relationships among variables. Conditional independence is fundamental because a complex distribution can sometimes be described through a much smaller set of relevant dependencies. Graphical models provide one familiar representation, but the underlying mathematical questions are not exhausted by graph drawing. Studený studied non-graphical representations and structural imsets as formal tools for reasoning about independence. This work clarified what different independence statements imply and how they may be organised consistently. Such results matter for probabilistic modelling because an incorrect independence assumption can produce misleading inference even with abundant data. The research exemplifies a Prague contribution that is theoretical in method yet directly relevant to machine learning. It also illustrates the productive overlap between Academy-based statistics and international information science.

Statistical pattern recognition became one of the most visible applied-mathematical strengths of the Institute of Information Theory and Automation. Researchers investigated how image information can be represented, classified, restored, and interpreted under imperfect observations. Mixture models and Markov random fields provided probabilistic descriptions of visual texture and the relationships between neighbouring image elements. Such models seek to distinguish relevant signal structure from sensor noise, illumination changes, and other disturbances. Image restoration, segmentation, and feature selection consequently require a combination of optimisation and probabilistic inference. The institute's work on recognition methods demonstrated that visual computing is not merely a problem of faster hardware. It depends upon mathematical assumptions concerning the generation of observations and the structure of their uncertainty. Some methods offer flexible representations, while others provide clearer interpretability or more reliable behaviour under specific constraints. This tension encouraged sustained interaction between theoretical statistics and computational experimentation. Its influence extended to the theoretical justification of data-driven reasoning methods.

Computer vision at the institute also developed through close connections between mathematical modelling and physical image formation. An observed image depends not only on an object but also on geometry, lighting, optics, and the characteristics of the recording device. Methods that disregard these influences may perform well in controlled conditions yet fail in realistic environments. Prague researchers therefore investigated image invariants, deconvolution, and model-based recognition to address identifiable sources of variation. Statistical modelling helped quantify uncertainty where a unique reconstruction could not be guaranteed. Numerical algorithms translated these mathematical formulations into feasible procedures for large image datasets. The interaction of physical knowledge with probabilistic analysis made the resulting systems more scientifically interpretable. It also provided opportunities for cooperation with engineering and industrial research communities. In institutional terms, this field illustrates how applied mathematics can produce both methodological publications and useful computational tools.

Control theory supplied an additional axis along which Prague mathematics entered technological research without losing its theoretical identity. A control problem asks how

interventions can influence a changing system while respecting limitations and uncertainty. Stability, observability, controllability, and robustness provide different mathematical perspectives on whether a proposed control method can succeed. The Institute of Information Theory and Automation maintained dedicated expertise in control theory and adaptive systems throughout the period examined. Model uncertainty encouraged approaches that revised estimates or considered alternative operating conditions. Optimisation methods supported the selection of actions while probability theory clarified uncertainty about future outcomes. Research in this area could interact with industrial processes, signal systems, and complex infrastructure. Nevertheless, a mathematical guarantee established under a model remains conditional on the model's relevance to reality. This qualification is essential to an academically responsible account of progress in automatic control. The combination of geometry and probability helped translate modelling assumptions into operational algorithms.

Optimisation research connected the institute's theoretical programmes with mathematical decision making in complex operational settings. Problems could involve nonlinear constraints, uncertain objectives, or interacting processes that prevent straightforward analytical solutions. Variational methods and convex analysis provided tools for determining whether mathematically meaningful solutions exist. Numerical optimisation then sought procedures capable of approximating those solutions under finite computational resources. The institute's research in applied mathematics included optimisation theory and related problems in the calculus of variations. Such work overlapped with the national tradition of mathematical analysis without simply duplicating it. Where fundamental analysis establishes structural properties, application-oriented optimisation often examines how those properties guide feasible calculation. This relationship is particularly important when numerical algorithms must operate reliably despite noisy or incomplete input data. Prague's research environment allowed these abstract and practical concerns to remain in close contact.

Econometrics offered a further route through which statistical methodology could contribute to questions of public and economic relevance. Economic observations are shaped by policy changes, expectations, institutional constraints, and disturbances that cannot be controlled experimentally. Mathematical models therefore require explicit assumptions about causality, temporal relationships, and uncertainty. At the Institute of Information Theory and Automation, econometric research formed part of a wider mathematical profile. Statistical estimation and optimisation supported forecasting and the examination of alternative policy or management scenarios. These techniques should not be represented as instruments for predicting economic outcomes without qualification. Their value depends on transparent model comparison, sensitivity analysis, and awareness of structural change. Collaboration between mathematicians and economists helped connect formal methods with interpretation of actual institutions. The resulting research demonstrates how mathematically disciplined analysis can support judgement without replacing it. The distinction between explanatory interpretation and predictive performance remained especially important in these applications.

Scientific journals, seminars, and conferences constituted crucial intellectual infrastructure for the Prague schools discussed here. The journal Kybernetika provided an established channel for research in systems, control, information, and related mathematical fields. Specialist seminars

allowed developing results to undergo criticism before or alongside formal publication. In mathematical logic, the enduring seminar tradition associated with Petr Hájek fostered continuity across changes in personnel and institutional organisation. International meetings exposed Czech researchers to alternative approaches and helped identify common theoretical problems. Such exchanges are essential where progress depends on arguments that may be difficult to evaluate outside a specialised community. Editorial and refereeing responsibilities also connect local research programmes with international standards of evidence. The institutes' intellectual significance therefore cannot be measured only through individual monographs or exceptional projects. Durable professional communities are themselves part of the infrastructure through which mathematical knowledge is produced.

European integration changed the scale and organisation of cooperation available to Prague's mathematical institutions after 2004. Participation in multinational projects brought researchers into contact with groups that possessed complementary theoretical and applied expertise. Collaborative funding also required new administrative capacities and more systematic presentation of research objectives. The Institute of Computer Science later participated in international initiatives concerning theoretical computation, logic, and network mathematics. Its documented project history includes collaborative programmes on formal methods and on analytical foundations of neurocomputing. The Institute of Information Theory and Automation likewise operated within an expanding environment of international mathematical and technological collaboration. Such programmes helped circulate ideas and provided younger researchers with access to larger scientific communities. Yet international integration should not be equated with the automatic achievement of scientific excellence. The importance of each project ultimately depends on the knowledge generated and the durability of its research relationships. Scientific exchange thus strengthened research culture as well as the visibility of individual organisations.

The relationship between the Academy institutes and Prague universities played an important role in the reproduction of research expertise. Academy institutions generally did not function as independent degree-awarding universities, but their investigators could supervise and train students through formal partnerships. Doctoral researchers benefited from entering laboratories and seminars already engaged in international scientific discussion. In mathematical logic, such arrangements exposed students to proof methods, semantics, and open foundational problems. In statistics and control, they provided experience of modelling under conditions where data and assumptions were imperfect. Joint activities also encouraged researchers to translate specialised investigations into teachable conceptual frameworks. This movement between education and research strengthened the continuity of individual scientific schools. It allowed Academy projects to contribute to human-capital formation beyond their immediate publication outputs. The resulting institutional relationship was cooperative rather than a simple division between research and teaching.

Despite their proximity and overlapping interests, the two institutes should not be treated as interchangeable organisations. The Institute of Computer Science emphasised mathematical foundations of computation, formal logic, and selected areas of statistical and complex-systems research. The Institute of Information Theory and Automation placed greater emphasis

on probabilistic inference, dynamic decisions, control, signal interpretation, and optimisation. These descriptions identify broad orientations rather than rigid disciplinary boundaries. Both institutes contributed to mathematical aspects of artificial intelligence and to the analysis of complex data. Their distinct research histories nevertheless influenced which problems they regarded as central and which methods they cultivated most deeply. Such differentiation can strengthen a national research system by providing complementary intellectual resources. It may also require deliberate collaboration when a problem crosses the boundaries of established specialisms. A history of Prague mathematics must therefore preserve both institutional specificity and the connections between research communities. This institutional balance helps explain the breadth of mathematical research maintained within Prague.

The growing prominence of artificial intelligence during the 2010s and early 2020s changed how many established mathematical techniques were publicly perceived. Optimisation, probability, logic, and function approximation appeared increasingly as components of modern intelligent systems. For Prague's Academy researchers, these were not entirely new subjects introduced by the contemporary popularity of machine learning. Their earlier work had already investigated formal inference, uncertain decisions, adaptive control, and neural approximation. What changed was the volume of data, the available computational infrastructure, and the diversity of potential applications. This shift created opportunities for transferring mathematical results into new computational settings. It also increased the risk of mistaking impressive demonstrations for general theoretical guarantees. Research institutions therefore had a continuing responsibility to distinguish empirical performance from proven properties. The mathematical foundations of artificial intelligence remained valuable precisely because they provided instruments for making this distinction.

Several enduring limitations accompanied the achievements of Prague's mathematical research institutions between 1993 and 2025. The competition for internationally mobile researchers could place pressure on the continuity of smaller specialist groups. Reliance on temporary project funding sometimes complicated the support of long-term foundational questions. Interdisciplinary work required sustained communication across communities that used different technical vocabularies and standards of evidence. The rapidly changing computational environment increased demand for software, data resources, and specialist expertise that had not previously been central to all mathematical fields. At the same time, institutions had to balance nationally relevant research needs with competition in internationally visible areas. Successful cooperation depended on administrative support as well as scientific goodwill. These challenges do not negate the achievements recorded during the period, but they qualify any narrative of uninterrupted progress. Understanding institutional constraints is necessary for interpreting how mathematical excellence can be sustained beyond individual projects. Their progress required sustained specialist knowledge rather than reliance on technological trends alone.

By 2025, the Prague schools of mathematical logic, information theory, and applied mathematics had developed into an interconnected yet differentiated research environment. The Institute of Computer Science preserved and expanded theoretical inquiry into formal reasoning, computation, statistical modelling, and complex systems. The Institute of

Information Theory and Automation sustained complementary programmes in uncertainty, probabilistic decision making, control, optimisation, and image analysis. The work of Petr Hájek demonstrated the intellectual power of rigorous non-classical logic, while Milan Studený clarified structures underlying probabilistic independence. Miroslav Kárný's probabilistic approaches to adaptive decisions exemplified the integration of inference and control. Other researchers contributed to mathematical analysis of neural approximation, discrete structures, and computational representations of scientific evidence. Together, these directions illustrate how Prague transformed inherited intellectual strengths through international collaboration and interdisciplinary adaptation. Their significance lies not in making mathematics subordinate to artificial intelligence but in establishing mathematical standards for understanding intelligent systems. The history of these institutes consequently offers a distinctive account of continuity, specialisation, and innovation in Czech mathematical science.

Czech Technical University in Prague: Applied Mathematics, Numerical Methods, and Mathematical Physics

The development of mathematical research at the Czech Technical University in Prague (CTU) between 1993 and 2025 illustrates a particular relationship between abstract reasoning and technological practice. Unlike a university organised primarily around the traditional divisions of pure mathematics, CTU cultivated mathematical knowledge within engineering, nuclear science, architecture, computing and physical research. Its mathematical community consequently operated across several faculties rather than within a single self-contained institute. This institutional distribution gave researchers continuing contact with problems arising in laboratories, construction projects and industrial systems. It also presented the challenge of sustaining rigorous mathematical inquiry under the pressure of immediate technical usefulness. The independent Czech Republic inherited both the university's longstanding engineering tradition and specialised programmes created during the Czechoslovak period. Following independence, these resources entered an environment of competitive grants, international evaluation and closer integration with European science. The resulting trajectory should therefore be understood as a changing network of disciplines, people and methods rather than a simple history of one mathematics department. This perspective gives institutional history a mathematical meaning that cannot be reduced to administrative chronology.

CTU's position in Czech mathematical science cannot be explained without its older institutional inheritance. The development of technical higher education in Prague created a durable connection between mathematical instruction and the solution of engineering problems. During the twentieth century, increasingly specialised engineering disciplines required advanced differential equations, numerical techniques and mathematical physics. The Faculty of Nuclear Sciences and Physical Engineering (FNSPE) emerged within this broader national investment in

scientific and technical expertise. Nuclear research placed unusual demands on analytical modelling, probability, spectral theory and computational accuracy. Departments concerned with mechanics and civil engineering likewise depended on mathematics for structural analysis and fluid dynamics. By 1993, many such capabilities had already been consolidated through decades of teaching and research. The new political circumstances made their international renewal possible without requiring a wholesale reinvention of the underlying intellectual traditions. The continuity of ideas across political regimes is therefore a necessary part of any credible account of innovation. Long-established research habits supplied a foundation upon which newer numerical and theoretical approaches could develop.

The democratic transition after 1989 preceded the formal establishment of the Czech Republic in January 1993. This distinction matters because the university's scientific opening was already under way when a new national administrative framework came into existence. Greater opportunities for travel and access to international journals altered the conditions under which researchers could test their ideas. University teams became increasingly exposed to foreign standards of peer review, collaboration and doctoral supervision. Mathematical groups faced an additional responsibility to demonstrate the relevance of their theoretical work within internationally competitive scientific conversations. Applied projects were progressively expected to document not merely computational outputs but also the validity of their models and numerical approximations. Funding arrangements therefore began to reward clearer research programmes and durable partnerships. The institutional transformation of CTU was gradual, uneven across faculties and inseparable from the preservation of expertise formed before independence. The decisive question is how altered institutions changed the kinds of mathematical problems researchers could pursue.

The Faculty of Nuclear Sciences and Physical Engineering became a distinctive centre for the interaction of mathematical theory and physical applications. Its Department of Mathematics contributed both fundamental training and research in subjects directly relevant to sophisticated physical models. Such a setting gave students exposure to analysis, algebra, probability, numerical methods and mathematical statistics alongside specialised engineering subjects. The combination helped to prepare researchers who could move between formal derivation and computational investigation. Some research groups concentrated on mathematical physics, whereas others examined stochastic systems or practical modelling problems. Their coexistence made it possible to approach the same technical problem from analytically different perspectives. It also reduced the risk that applied mathematics would be treated as software use detached from mathematical justification. The faculty's subsequent research profile reflects the long-term value of maintaining advanced mathematical capabilities within a technically oriented institution. Its influence is best understood through the research competence developed within successive generations of scholars. As a result, the faculty provided a setting in which demanding theoretical methods remained relevant to experimental science.

A particularly consequential initiative occurred in 1993 with the establishment of the Doppler Institute for Mathematical Physics and Applied Mathematics. The institute was conceived as a means of encouraging advanced research while supporting postgraduate education in

mathematical physics. Its foundation coincided with the earliest phase of the independent Czech scientific system. The coincidence was significant because it demonstrated that institutional renewal could also emerge from research communities themselves. Rather than replacing university teaching, the institute offered a framework connecting specialised seminars, visiting scholars and collaborative investigations. Its participants included scholars associated with CTU and other Czech scientific institutions. External grants supported activities that extended beyond ordinary faculty resources. The institute thus became a useful example of how a relatively small mathematical network could attain an international orientation through specialisation and sustained intellectual exchange. The institute's founding offers a clear point of departure for assessing this transformation.

The Doppler Institute's activities developed around mathematical questions associated with modern physical theories. Its seminars provided a setting in which operator theory, geometry and quantum mechanics could be examined together. Research meetings also made it possible to expose doctoral students to subjects normally distributed across several university departments. The institute developed connections with the Czech Academy of Sciences and Charles University, together with collaborators from abroad. Such relationships helped Prague-based researchers participate in larger mathematical conversations without relinquishing the advantages of a locally coherent community. An institute of this kind operates as an intellectual network as much as an administrative unit. Its contribution cannot therefore be evaluated adequately by counting only buildings or permanent posts. The persistence of conferences, seminars and publications offers a more revealing indication of the research culture it cultivated. These relationships supplied continuity even when individual research projects or funding arrangements changed. Its evolution therefore illuminates a wider pattern of mathematical internationalisation through small and focused networks.

Pavel Exner became one of the most visible figures associated with Prague's mathematical physics community. His research addressed spectral problems arising from quantum structures whose geometry constrains particle motion. These questions require detailed operator methods rather than the direct transfer of classical physical intuition. Work on quantum graphs and waveguides connected rigorous mathematics with models of nanoscale and mesoscopic systems. A graph can encode the connectivity of a thin physical network, but its mathematical operator must still be defined with carefully justified boundary conditions. Exner's publications and collaborations illustrated how an apparently simple geometric representation can support profound spectral questions. International engagement was integral to this research tradition rather than an accessory added after a result had been obtained. Through this work, CTU's mathematical-physics environment gained visibility in a field whose methods crossed the boundaries of analysis, geometry and theoretical physics. This intellectual setting helps explain the enduring connection between Exner's results and Prague-based research.

Spectral theory provides a particularly instructive example of the intellectual depth possible within technical universities. The spectrum of an operator can reveal which modes a physical system supports and how these modes change under perturbations. In quantum mechanics, seemingly modest alterations of a domain or its boundary may create or remove localised states. Rigorous conclusions demand attention to the operator domain, self-adjointness and the

distinction between discrete and continuous spectra. Research associated with the Prague mathematical-physics community investigated such questions in models of waveguides and quantum networks. The mathematical results matter independently of whether every model immediately becomes an experimental device. At the same time, they supply a disciplined language for assessing what simplified physical descriptions can and cannot represent. This combination of conceptual rigour and potential application became a recognisable feature of the institute's research culture. In this respect, theoretical progress also served as a critical examination of the limits of physical analogy. It would be misleading to equate mathematical depth with the speed at which a physical application becomes commercially useful.

The relation between thin physical networks and idealised quantum graphs raised a further mathematical problem. A physically narrow tube still occupies a multidimensional region, whereas the limiting graph possesses edges and vertices with very different analytical properties. Demonstrating that one spectral problem approaches another requires more than visual resemblance between their geometries. Appropriate limiting operators and modes of convergence must be identified and justified. Studies involving Exner and international collaborators contributed to the understanding of these limiting processes. They also drew attention to the influence of boundary conditions and vertex geometry on effective quantum behaviour. Such work shows how mathematical physics can refine an entire modelling framework rather than merely solve an isolated equation. The Prague contribution belongs to a wider international enterprise, and the distinction between national participation and collective achievement remains essential. The value of such limiting analysis extends to the modelling assumptions adopted in subsequent research.

The institute's conference activity gave its research profile an additional international dimension. Meetings in the Mathematical Results in Quantum Theory series brought specialists together around problems in operator theory, spectral analysis and quantum systems. Other seminars and workshops considered non-self-adjoint Hamiltonians, algebraic methods and related mathematical structures. Academic meetings were not simply ceremonial markers of an institution's international status. They facilitated criticism of proofs, exposure to alternative techniques and the formation of research collaborations. For doctoral students, sustained participation in this environment could transform an unfamiliar theoretical question into a concrete research programme. Regular intellectual contact also helped smaller groups maintain visibility despite possessing fewer resources than major Western European centres. Conference culture therefore formed part of the infrastructure of mathematical excellence, even though its achievements were primarily intellectual rather than technological. The meetings also helped connect local doctoral education with established international research communities. An enduring meeting series can shape a research field by determining which questions become collectively visible.

The Faculty of Civil Engineering cultivated a different but complementary model of mathematical research. Its Department of Mathematics combined responsibility for teaching with work in mathematical and numerical analysis. Researchers studied partial differential equations, dynamical systems, functional analysis and problems with direct engineering interpretations. Structural design, geodesy and environmental processes create mathematical questions that

are not resolved by a standard textbook algorithm. For example, real boundary conditions can be irregular, input data uncertain and constitutive assumptions only approximately known. Numerical methods must therefore be analysed for their stability and convergence under realistic constraints. The department's cooperation with engineering colleagues provided opportunities to test theoretical methods against such difficulties. This pattern illustrates the importance of having mathematicians involved from the formulation of an engineering problem rather than only at the final calculation stage. The resulting exchange strengthened research questions as well as their numerical solutions.

At the Faculty of Mechanical Engineering, mathematical investigation became closely connected with the simulation of moving fluids and interacting structures. Ordinary and partial differential equations supply the governing mathematical language for many such processes. Yet industrial design rarely presents the ideal boundaries, steady conditions or simple material responses assumed in elementary derivations. Computational fluid dynamics must account for discretisation errors, turbulence modelling and constraints on available computing resources. The faculty's work included the development and testing of numerical schemes for compressible and incompressible flows. Research collaborations linked mathematical modelling with turbomachinery, environmental flow and biomedical problems. The mathematical interest lies in assessing which approximations preserve the behaviour most relevant to the original system. This work exemplifies how technical mathematics can contribute to both theoretical understanding and engineering reliability. Theoretical scrutiny was consequently central to the practical value of engineering simulation. Industrial relevance did not diminish the need to analyse the mathematical structure of the governing equations.

Numerical analysis remained fundamental because complex mathematical models generally cannot be solved in closed form. A numerical result is persuasive only when the relation between the continuous problem and the discrete approximation is understood. Mesh refinement, conservation properties and error estimation consequently belong to the mathematical content of simulation. Finite-volume methods are especially relevant when fluxes and conservation laws govern physical evolution. Finite-element approaches provide a flexible framework for problems involving complex geometries and differential operators. The choice between numerical methods depends on the governing equations and the properties of the desired solution. Prague's technical research environment encouraged this methodological reasoning because simulations were often intended to inform decisions outside mathematics. Scientific credibility depended on showing why a computational answer was trustworthy, not merely producing a visually convincing result. Such analysis made computational confidence a matter of mathematical evidence rather than visual impression.

The treatment of uncertainty became increasingly important as mathematical simulations moved closer to practical decisions. Engineering data may contain measurement errors, and material parameters can vary between experimental specimens. Even when a deterministic mathematical model is well posed, its predictions may be sensitive to uncertain input quantities. Probability theory, sensitivity analysis and stochastic modelling offer different ways of describing this uncertainty. At CTU, mathematical training in probability and statistics complemented the traditional emphasis on differential equations and numerical analysis. Research in stochastic

methods also opened opportunities for connections with biological and social models. The value of such methods is not that uncertainty can always be eliminated through calculation. It is that researchers can distinguish robust predictions from conclusions whose reliability depends heavily on unverified assumptions. This distinction became especially important when models informed decisions with material consequences. Quantifying uncertainty also encouraged more explicit communication between modellers and those using their predictions.

The Mathematical Modelling Group within the FNSPE Department of Mathematics provides a particularly concrete example of this research orientation. Its publication record includes mathematical studies of evolving interfaces, curvature-driven motion and computational schemes for differential operators. Such subjects combine analytical descriptions of continuous phenomena with methods intended for numerical implementation. Models of interface evolution occur in materials science, fluid mechanics and other fields where the geometry of a boundary changes over time. Their computation may become difficult when curvature, topology or nonlinear feedback influences the motion. Researchers must then consider how a numerical scheme represents geometric features while maintaining computational efficiency. The group's projects illustrate the sustained institutional value of combining applied analysis with computational experimentation. They also demonstrate why mathematical modelling should be understood as research into the behaviour and limitations of models rather than the routine use of established software. The publications demonstrate how specialised modelling problems can generate questions of wider mathematical interest.

An important characteristic of CTU was the coexistence of several centres undertaking related mathematical work. Departments serving civil engineering and mechanical engineering often addressed partial differential equations and computation from different practical starting points. The FNSPE connected some of these mathematical methods with physical theories and advanced analytical questions. Computing specialists supplied algorithms, implementation techniques and knowledge of computational architectures. This institutional pluralism could encourage collaboration but also risk the duplication of effort. Productive cooperation required shared definitions of problems, accessible data and agreement over standards of numerical verification. When these conditions were met, work across departments could link fundamental derivation with sophisticated application. The distributed character of mathematical research was consequently a structural feature of the university, not a defect to be explained away. The research environment was strongest where disciplinary differences encouraged complementary forms of expertise. This exchange was particularly productive when theoretical and experimental partners agreed on what counted as validation.

The renewal of mathematical education was inseparable from the transformation of research after 1993. Advanced instruction had to preserve rigorous foundations while responding to changing computational tools and international scientific practice. Mathematics students needed more than competence in symbolic calculation to participate in modern research. They also required training in numerical methods, probability, algorithmic thinking and the critical reading of research papers. Engineering students, meanwhile, benefited from understanding the assumptions under which familiar modelling techniques are valid. Doctoral supervision offered a particularly important bridge between regular curricula and specialised research communities.

Research institutes and collaborative seminars introduced students to questions whose answers were not already known. In this way, the university's mathematical contribution depended as much on reproducing intellectual competence across generations as on producing individual publications. The renewal of research therefore depended on teaching practices as well as on external collaboration.

International mobility gradually changed the professional environment of the university's mathematical researchers. Collaborations with other European universities and institutes made it easier to compare analytical methods and computational approaches. Visiting researchers introduced alternative research questions, while Czech doctoral students gained experience in different academic cultures. Such mobility was especially important in mathematical physics, where specialised communities were widely dispersed. Yet internationalisation did not mean abandoning local research traditions or accepting external research agendas without criticism. The most valuable exchanges allowed Czech groups to contribute problems, proofs and methods of their own. Foreign collaboration could therefore strengthen the autonomy of a local school precisely by making its work more widely contestable. The history of CTU illustrates how research quality can depend on both a durable home institution and open transnational networks. International contact offered a mechanism for testing locally developed ideas against broader mathematical standards. Intellectual independence and international partnership thus appeared as mutually reinforcing rather than opposing aims.

Competitive project funding played a substantial role in the transformation of CTU's mathematical sciences. Grant support enabled groups to organise meetings, sustain collaborations and undertake research that exceeded routine teaching resources. Such mechanisms also made investigators responsible for specifying research questions and communicating anticipated scientific contributions. For theoretical mathematics, the outcome of an investigation cannot always be predicted as precisely as the completion of a technical deliverable. Research proposals therefore had to balance intellectual risk with the requirements of accountability. Applied mathematics faced a related tension because industrial partners often expected concrete computational results on relatively short timescales. The Doppler Institute's dependence on grant resources exemplifies how a specialised research organisation could function within this environment. Long-term scientific strength nevertheless required the accumulation of expertise that no individual project could fully finance or reproduce. This balance between continuity and competition remained a defining challenge for research institutions.

Czech accession to the European Union in 2004 widened the institutional context of these developments. European research cooperation offered additional opportunities for international projects, shared research standards and researcher mobility. The relevance for mathematics extended beyond the acquisition of grants because collaboration exposed national teams to larger and more diverse scientific communities. Mathematical modelling became increasingly important in European discussions of advanced manufacturing, environmental protection and scientific computation. Theoretical mathematical physics likewise benefited from international meetings and programmes focused on specialised analytical problems. Still, accession did not instantly transform every CTU department into an internationally leading research centre.

Different groups responded according to their inherited strengths, administrative capacity and established partnerships. The 2004 watershed is therefore best understood as an expansion of opportunities whose scientific consequences unfolded over subsequent years. The intellectual consequences of integration must therefore be traced through actual research activity. European opportunities were consequential chiefly when they could be translated into durable mathematical expertise.

Increasing computational power altered what could be attempted in mathematical modelling during the 2000s and 2010s. Larger simulations made it possible to investigate finer spatial details or more realistic interactions between physical components. They also generated substantial requirements for parallel algorithms, data handling and reproducibility. A poorly conditioned numerical system can remain inaccurate even when executed on a powerful machine. Computational scale therefore heightened rather than diminished the importance of mathematical analysis. Research teams needed to connect discretisation strategies with efficient linear algebra and stable iterative procedures. In engineering contexts, calculation time could itself constrain which models were useful for practical decision-making. The growing computational dimension of CTU research illustrates the mutual dependence of mathematical insight and technological capability. Mathematical reliability remained inseparable from the practical possibilities supplied by computation.

Optimisation and control theory supplied another connection between mathematical abstraction and engineered processes. Many technical tasks ask not only what a system will do but how it can be guided towards a preferred outcome. Such problems often combine differential equations, parameter constraints and competing objectives. The mathematical challenge is to establish whether a proposed solution exists and how sensitive it is to uncertain information. Numerical optimisation adds a further problem of finding computationally tractable approximations. Research and teaching in applied mathematical methods supplied the conceptual tools needed to examine these questions. Their practical applications ranged from process design to the interpretation of experimental data and resource allocation. The relevance of mathematical control at CTU therefore arose from engineering demands but depended on theoretical ideas developed well beyond any single industrial sector. The theoretical analysis of feasible decisions was consequently part of mathematical innovation itself. In practice, useful decisions depended on both appropriate optimisation criteria and reliable mathematical representations.

The university's mathematical identity also became visible through its research seminars and specialist publications. An academic school is more than the presence of a distinguished professor or an impressive list of course titles. It requires recurring conversations in which younger mathematicians can test conjectures, inspect proofs and acquire methodological judgement. The Doppler Institute's seminars offered one environment for this intellectual reproduction. Other departmental research groups provided comparable opportunities around differential equations, numerical methods and applied probability. Publication in international journals made claims open to verification by scholars beyond the national community. The combination of local continuity and international scrutiny was especially important for a relatively small national research system. It helped prevent mathematical work from becoming

either narrowly inward-looking or disconnected from the institutional foundations that sustained it. The central methodological lesson is that mathematically similar notation need not imply equivalent models.

By the 2010s, CTU mathematical research was increasingly connected with broader interdisciplinary and computational agendas. Mathematical physics continued to investigate quantum structures and operator-theoretic questions. Modelling groups examined nonlinear evolution, fluid processes and numerical approximation. Statistical and stochastic methods became relevant to more diverse forms of scientific data. The growing availability of advanced computation changed expectations concerning the resolution and complexity of engineering simulations. These trends should not be mistaken for a replacement of established mathematical disciplines by a single new field. Instead, established tools acquired additional contexts of application, while new mathematical problems arose from those contexts. The university's development thus illustrates a cumulative form of innovation in which abstract knowledge and technical practice repeatedly transformed one another. The institutional durability of seminars often mattered more than any one administrative reorganisation. It is through such recurring exchanges that individual results acquire the character of a scientific tradition.

The early 2020s reinforced the importance of combining rigorous models with computationally efficient implementations. Research published by CTU mathematical modelling groups addressed, among other subjects, numerical schemes for fractional operators and curvature-driven evolution. Such investigations indicate continuing interest in how nonlocal or geometrically complex processes can be computed reliably. Work connecting numerical models to measured data also illustrated the interaction between mathematical analysis and experimental systems. At the same time, research in mathematical physics retained its emphasis on careful spectral and geometric reasoning. Developments in machine learning offered additional computational tools without removing the need to establish the assumptions and limits of a model. The central intellectual challenge remained the translation of a physical or technical question into a mathematically defensible form. In this sense, the period to 2025 continued rather than broke with the university's earlier research trajectory. Its emerging directions were shaped by interaction rather than the abandonment of established fields.

Any assessment of CTU's contributions needs to distinguish several forms of scientific achievement. Producing a new spectral theorem differs from implementing an efficient numerical scheme, even when both may inform physical research. Establishing a modelling group and maintaining its doctoral culture differs again from obtaining a single high-profile publication. Applied projects may yield validated methods, specialised software or improved engineering understanding rather than a universally applicable theorem. These outcomes cannot be compared responsibly by one simple measure of productivity. The university's distributed institutional structure makes such distinctions particularly important. Moreover, many notable publications emerged through international collaboration and should not be described as exclusively national achievements. A defensible history therefore evaluates research contributions through their actual intellectual content and the institutional conditions that made them possible. These projects show the continued demand for mathematical

justification in data-intensive research. Claims of national excellence are strongest when they identify the exact results and collaborations involved.

The same historical record also reveals persistent limitations in the development of technical mathematical science. Project-based funding can be poorly aligned with theoretical investigations requiring many years of sustained concentration. Engineering obligations can restrict the time available for pursuing problems without immediate commercial relevance. Highly specialised areas depend on small communities that may be vulnerable to changes in personnel or doctoral recruitment. International competition for researchers adds pressure to sustain attractive professional conditions. Advanced computational methods demand continued investment in software competence and access to appropriate infrastructure. Nonetheless, CTU developed an institutional environment in which fundamental analysis, mathematical physics and engineering modelling could support one another. Between 1993 and 2025, its distinctive contribution was the preservation and renewal of mathematics as both an autonomous intellectual activity and an indispensable instrument of technological inquiry. The history of the university is thus most convincing when different achievements are assessed on their own terms.

Brno Mathematical School: Masaryk University, Algebraic Traditions, Geometry, and Mathematical Analysis

The mathematical school of Brno entered the Czech Republic's independent era in 1993 with an intellectual history extending far beyond the political boundaries of that year. Its principal university centre was Masaryk University, where research in algebra, geometry, analysis and differential equations had accumulated over successive academic generations. The division of Czechoslovakia changed the national institutions within which these traditions operated but did not dissolve the mathematical questions, professional relationships or teaching practices that sustained them. For Brno, continuity was particularly visible in the transmission of abstract mathematical methods through seminars, doctoral supervision and specialist publications. The mathematical community had already formed productive links with colleagues in Prague, Bratislava and research centres elsewhere in Europe. After 1993, these links were gradually situated within a more open and competitive international environment. The resulting history is therefore better understood as a transformation of established scientific capacity than as the construction of a new school from nothing. This distinction provides the necessary starting point for an assessment of mathematical innovation in Brno between 1993 and 2025.

Among the figures essential to the earlier history of Brno mathematics, Otakar Borůvka occupies a distinctive position. His work on graph theory, algebra and differential equations helped establish intellectual connections between subjects that later developed into separate research specialisms. His early solution to an electrical-network optimisation problem is widely recognised as an antecedent of minimum spanning tree algorithms. His later differential-

equation research influenced mathematicians concerned with oscillation, transformation methods and qualitative theory. Borůvka also supported institutional mathematics in Brno, including the mathematical journal Archivum Mathematicum, founded in 1965. These achievements preceded the period under investigation and must not be represented as discoveries made after independence. They nevertheless provide an instructive explanation for the breadth of interests preserved within the university's mathematical environment. The post-1993 school inherited not only several specialised subjects but also a model of mathematical reasoning in which deep abstraction and concrete problems could illuminate one another. Such intellectual inheritance helps explain why the modern Brno school cannot be reduced to a single department or mathematical technique.

The Faculty of Science at Masaryk University provided a principal institutional setting for this continuity. Mathematical teaching and research were organised through units whose responsibilities evolved with curricular reforms and changes in university governance. The academic community had to balance introductory instruction for large numbers of students with specialised research requiring concentrated time and long-term collaboration. That balance was particularly important in abstract mathematics, where scientific results do not necessarily depend upon laboratory facilities but do depend upon sustained intellectual communities. During the 1990s, increasingly open international mobility expanded opportunities for researchers to attend conferences and work with foreign colleagues. Competitive grant mechanisms also affected how research topics were proposed, financed and evaluated. The university setting enabled young scholars to connect advanced coursework with current mathematical literature and participation in seminars. These changes did not eliminate the older scholarly culture of patient theorem building, but they altered the conditions under which that culture was reproduced.

One of Brno's most recognisable contributions to late twentieth-century and contemporary mathematics arose from category theory and universal algebra. Jiří Adámek and Jiří Rosický developed a body of work concerned with the organisation of mathematical structures through morphisms, limits and accessibility conditions. Their 1994 monograph Locally Presentable and Accessible Categories provided a systematic account of concepts linking category theory with universal algebra, model theory, logic and computer science. Locally presentable categories offer ways of describing broad classes of objects through suitably controlled constructions from smaller constituents. Accessible categories extend this perspective and make it possible to compare structures that emerge in apparently different mathematical settings. The intellectual importance of this research lies in its capacity to uncover common organising principles rather than merely to introduce another specialised vocabulary. Because the monograph appeared shortly after 1993, it also illustrates continuity across the historical watershed. A substantial programme of abstract investigation was already established and was able to contribute to international mathematical discussion in the newly independent Czech context. The work remains an appropriate example of theoretical innovation whose effects reach beyond a single mathematical discipline.

The relationship between category theory and algebra is central to understanding this contribution. Universal algebra studies operations and identities across classes of

mathematical structures, seeking results independent of their particular representations. Category theory broadens that perspective by treating structure-preserving maps as fundamental elements of mathematical explanation. In Brno, the interaction of these approaches supported research into injectivity, orthogonality, directed colimits and the behaviour of categories under different constructions. Adámek and Rosický's 1993 article on injectivity in locally presentable categories is one documented example of this research direction. Its results concern structural conditions on classes of objects and explicitly involve assumptions from set theory. Such work shows that algebraic generalisation can intersect with foundational questions rather than remaining separate from them. The research also forms a natural bridge to theoretical computer science, where algebraic and categorical frameworks help describe computational processes.

The scientific influence of the Brno algebraic school cannot be measured adequately by counting publications alone. Abstract results gain importance when they change how other mathematicians formulate questions or transfer arguments between subject areas. Research into locally presentable categories offered a language for understanding families of structures defined by operations, equations and suitable closure properties. It also clarified how logically formulated conditions could be reflected in categorical constructions. Later mathematical work built upon these connections while extending them to new contexts. The development of such theories requires international dialogue because their consequences emerge through contributions from many different research communities. Brno researchers participated in that dialogue through scholarly books, journal articles and sustained collaboration. Their contribution exemplifies a distinctive form of mathematical innovation, in which conceptual unification becomes as significant as a new algorithm or numerical result. Within the university, this tradition also furnished sophisticated material for advanced postgraduate study.

Geometry constitutes another major component of the Brno mathematical identity. Research associated with Ivan Kolář, Jan Slovák and their collaborators addressed geometric structures through differential geometry and the systematic study of natural operations. Their work connected classical constructions on manifolds with modern questions involving bundles, differential operators and invariance. The monograph Natural Operations in Differential Geometry, published in 1993 by Kolář, Peter W. Michor and Slovák, became a substantial reference for this approach. Its central concern was not a catalogue of isolated geometric examples but a general framework for operations that behave consistently under changes of coordinates and geometric maps. The timing of its publication again underlines the difference between a political turning point and the beginning of a scientific programme. Several years of mathematical development were embodied in a major international publication at the outset of the new Czech period. The research established common ground between local geometric calculations and global structural principles. This framework would continue to influence the university's geometric investigations well beyond its initial publication.

Natural differential operators arise when geometrical constructions depend on a manifold in a way that respects appropriate mappings. Examples include familiar operations on tensor fields and connections, but the underlying classification problems can become highly abstract. The Brno school developed conceptual techniques for investigating which constructions are

genuinely geometric rather than artefacts of a particular coordinate description. This distinction is fundamental to differential geometry because meaningful results should survive legitimate transformations of their representations. Work on natural bundles and operators therefore combined rigorous algebraic reasoning with geometric intuition. It also offered connections to mathematical physics, where invariant formulations are indispensable for describing physical laws. The intellectual trajectory illustrates how a university research group can develop a coherent specialism while interacting with broader international developments. The resulting approach remained relevant to discussions of differential invariants and geometric structures throughout the period covered by this book. Its significance lies particularly in the methodological discipline imposed by naturality and invariance.

Parabolic geometries became a further distinctive research direction within the extended Brno geometric community. These structures generalise several familiar forms of geometry by describing them through Cartan connections and associated homogeneous models. They provide a setting for studying conformal and projective structures, among other geometric phenomena. Jan Slovák participated with Andreas Čap and other collaborators in research connecting representation-theoretic methods with invariant differential operators. The 2009 monograph Parabolic Geometries I: Background and General Theory by Čap and Slovák systematised the foundations of this field. Such research is demanding because geometry, Lie theory and sophisticated algebraic techniques must be employed simultaneously. Its international relevance shows that specialised university groups in Brno could contribute to the theoretical architecture of modern geometry. The resulting scientific programme also demonstrates that interdisciplinary work may occur entirely within fundamental mathematics, at the interface of fields once taught as separate subjects.

Geometric research in Brno also maintained contact with topology and the classification of bundles. Questions about vector bundles, characteristic classes and manifolds require algebraic techniques that describe global structure from local information. Published work by mathematicians associated with the region, including Martin Čadek and Jiří Vanžura, illustrates this intersection. The classification of vector bundles is not simply an exercise in formal taxonomy, because it identifies structural possibilities and obstructions in geometry. Such investigations may rely upon homotopy-theoretic and cohomological techniques that extend beyond differential geometry in its narrowest sense. They therefore reinforce the picture of Brno as a centre of several mutually connected abstract traditions. These lines of enquiry also create a framework in which graduate students encounter mathematics as a network of methods rather than a rigid division of subjects. The resulting intellectual environment can sustain new questions long after individual projects have concluded. This long temporal horizon is characteristic of successful fundamental mathematical schools.

Mathematical analysis and differential equations formed an equally significant part of the Brno landscape. The heritage of Borůvka’s investigations of differential equations created an enduring link between qualitative analysis and more general questions about dynamical behaviour. Research after 1993 included ordinary differential equations, functional differential equations and the asymptotic behaviour of their solutions. Analysts investigated conditions under which solutions exist, remain stable, oscillate or exhibit prescribed long-term properties. Such

questions can be formulated abstractly, yet they also arise when mathematical models describe mechanical or natural processes. The mathematical interest frequently lies in establishing rigorous results without relying upon closed-form solutions. Brno's research culture thus preserved a connection between theoretical proofs and the behaviour of evolving systems. The development of this programme was supported by seminars, doctoral training and continuing participation in international meetings.

Work on difference equations and dynamic equations offered another means of extending classical analytic questions. Discrete-time systems are essential when processes are observed periodically or modelled through successive computational steps. The methods used to analyse them often resemble those developed for differential equations, but discrete structures can display distinctive forms of instability and oscillation. Research in these directions encouraged dialogue between analysis, applied mathematics and computational techniques. The mathematical challenge was to identify qualitative properties that remain valid across different classes of equations. Comparison principles, asymptotic analysis and suitably formulated boundary conditions can all contribute to such investigations. Their importance becomes clearer when a mathematical model must distinguish between transient behaviour and long-term regularity. In Brno, these themes provided a natural continuation of the established differential-equations tradition while accommodating new forms of mathematical modelling. They also helped integrate abstract analysis with subjects of interest to engineering and scientific computation.

The presence of several distinct analytical approaches did not mean that all were organised within a single research team. Some projects emphasised ordinary differential equations, while others concentrated on numerical techniques or qualitative properties of nonlinear problems. The value of the wider Brno environment lay in the proximity of colleagues able to discuss different aspects of common mathematical questions. Regular seminars allowed theoretical arguments to be tested against alternative formulations and examples. Doctoral students could acquire both specialised expertise and an appreciation of the methodological boundaries of their chosen problems. International conference participation provided a second arena in which local assumptions and results could be scrutinised. The resulting research network functioned through intellectual exchange rather than through a single centralised agenda. This diversity helps explain the persistence of the Brno mathematical school despite changing institutional and funding arrangements.

Applied mathematical analysis developed through growing interaction with numerical modelling and scientific computation. Differential equations used in continuum mechanics and technological systems usually cannot be solved analytically in realistic geometries. Consequently, researchers need approximation methods whose accuracy and stability can be justified mathematically. Theoretical questions about existence, uniqueness and regularity are therefore related to the reliability of algorithms used in computation. University research and cooperation with engineering institutions made it possible to discuss these two perspectives together. Brno's mathematical development should nevertheless not be mistaken for the emergence of a single comprehensive computational institute within Masaryk University. Rather, the city's broader research ecology distributed related expertise across different universities and

scientific organisations. The Faculty of Science contributed to that ecology through mathematical foundations, training and participation in interdisciplinary research. Such an arrangement illustrates the value of institutional complementarity within a regional scientific centre.

The relationship between mathematics and computer science deepened across the period from 1993 to 2025. Formal descriptions of computational processes depend on mathematical ideas concerning structures, functions, relations and logical inference. Algebraic and categorical research offers ways of organising these ideas at a high level of abstraction. Geometry and analysis, by contrast, contribute mathematical techniques for optimisation, simulation and the understanding of continuous systems. These connections do not erase differences between theoretical mathematics and the engineering of software or computing devices. They reveal instead that computation frequently requires concepts developed independently of immediate practical demands. Brno mathematicians participated in this exchange through collaborative projects, lectures and published research. The resulting connections enriched both mathematical education and the regional discussion of scientific innovation.

The evolution of undergraduate and postgraduate mathematical education was integral to institutional development. A university research school survives only if younger mathematicians can learn advanced methods and eventually contribute new work. At Masaryk University, the relationship between specialist courses and research groups helped transmit expertise in algebra, geometry and analysis. Graduate education required students to move beyond reproducing established proofs towards identifying unresolved problems and evaluating potential methods. International mobility increased the range of mathematical approaches to which students and early-career researchers were exposed. The availability of English-language literature and electronic research resources made participation in international discussion progressively more immediate. These changes were especially useful in abstract subjects, where access to recent papers and specialists can matter more than access to physical equipment. The school's development therefore depended upon scholarly training as much as upon the formal expansion of its institutional structures. Education was not simply a consequence of research strength but one of its principal mechanisms of renewal.

Scientific publishing provides an unusually clear record of continuity in Brno mathematics. Archivum Mathematicum was founded in 1965 and continued to be published by the mathematical community of Masaryk University throughout the post-1993 decades. Its volumes contain research articles in a range of mathematical fields, recording both local editorial work and contributions from a wider international community. The preservation of the journal during institutional and technological changes demonstrates a durable commitment to mathematical communication. Its online archive provides direct evidence of publication activity across the period examined here. As scholarly publishing became increasingly electronic, the journal made research more readily accessible to readers beyond the university. Peer review remained central to its scholarly function, distinguishing mathematical argument supported by proof from unsupported speculation. The journal thus operated not merely as an institutional symbol but as a continuing mechanism of international scientific exchange.

The history of Archivum Mathematicum also illustrates changing standards of mathematical visibility. A journal rooted in a regional university may nevertheless circulate internationally when its articles address problems recognised by global research communities. Indexing, electronic dissemination and bibliographical coverage influence whether mathematicians can discover and cite published work. The journal's institutional information identifies its coverage by mathematical reviewing services and major bibliographic databases. Such recognition does not, by itself, establish the importance of every article or the strength of every research group. It does, however, help demonstrate that the publication participates in recognised procedures for communicating mathematical research. The existence of a digital archive further enables historical comparisons between early post-independence papers and more recent contributions. This continuity makes the journal an especially useful primary resource for studying mathematical science in Brno between 1993 and 2025. Its relevance extends beyond numerical publication indicators to the preservation of the region's intellectual record.

International conferences constituted another prominent element of the city's mathematical culture. The EQUADIFF series, devoted to differential equations and their applications, originated in 1962 and became a longstanding forum linking mathematical communities across Europe. Brno hosted EQUADIFF 9 from 25 to 29 August 1997, when 268 participants from 32 countries attended according to the conference preface. The meeting was organised by Masaryk University in cooperation with academic and professional mathematical institutions. Its programme and published contributions demonstrated the continued international relevance of differential-equation research during the first post-Czechoslovak decade. A conference of this character provides a form of scientific infrastructure distinct from a laboratory or computing centre. It connects established researchers, doctoral students and specialists working on different but related theoretical problems. The 1997 meeting is therefore a concrete measure of Brno's capacity to convene an international research community. It also indicates that the reorganisation of national institutions did not interrupt inherited scholarly traditions. The documented international participation makes the meeting particularly useful for historical assessment.

Brno again hosted an EQUADIFF meeting in 2009, confirming the continuing character of that association. This event occurred after the Czech Republic's accession to the European Union and during a period of expanding European scientific cooperation. The conference tradition was thus able to accommodate a substantially changed institutional and international environment. The importance of repeated meetings should not be assessed solely through participant totals or lists of speakers. They create opportunities for long-term intellectual relationships that often precede joint publications and subsequent research visits. Differential equations provide particularly fertile ground for such connections because they arise across pure analysis, numerical mathematics and scientific modelling. Through this international channel, research questions developed in Brno could be compared with those pursued by specialists elsewhere. Conference organisation consequently formed part of the city's intellectual role within European mathematical science.

The mathematical conference culture was not restricted to differential equations. Geometry, algebra and category theory also depended upon workshops, specialist seminars and

international visits. For highly abstract research, a focused meeting can permit sustained examination of a few difficult questions rather than a broad survey of completed results. The resulting exchange may clarify definitions, expose counterexamples or reveal connections between apparently unrelated theorems. Such effects are hard to quantify but remain central to the actual practice of advanced mathematics. Researchers associated with Masaryk University took part in international scholarly networks through their publications and collaborations. The circulation of ideas between Brno and other centres helped prevent regional mathematical traditions from becoming intellectually isolated. At the same time, locally organised meetings gave the city an active role in shaping discussion rather than merely receiving developments from abroad. This reciprocity is a defining feature of an internationally integrated research school.

The geographical position of Brno reinforced this international activity. Its location within Central Europe made interaction with Austrian, Slovak, Polish and German mathematical institutions practically feasible. Historical links with Bratislava persisted after the dissolution of Czechoslovakia, particularly within the shared traditions of mathematical conferences and publications. These relationships were embedded in disciplinary communities whose boundaries did not coincide neatly with state borders. European integration after 2004 expanded institutional opportunities, but it did not create mathematical cooperation from nothing. A more accurate account recognises that formal European programmes strengthened networks that often had older intellectual foundations. For Masaryk University, participation in those networks offered both opportunities and competitive pressures. International research visibility increasingly depended on original contributions capable of attracting attention beyond familiar regional circles.

A notable feature of the Brno school is the interaction between research in abstract structures and research into geometric transformations. Both domains ask how mathematical objects behave under maps that preserve specified forms of structure. The categorical approach expresses this question through morphisms and general constructions, whereas differential geometry formulates it through manifolds, bundles and invariant operations. The methods are not identical and should not be combined into a misleading claim of disciplinary unity. Nevertheless, their conceptual proximity encourages an intellectual environment attentive to symmetry, structure and naturality. This orientation helps explain why Brno became associated with internationally visible contributions in more than one branch of abstract mathematics. It also offers a distinctive educational advantage, allowing students to encounter different meanings of structural reasoning. Such connections represent one of the city’s most interesting contributions to the development of modern mathematical thought. They illustrate innovation through the refinement and transfer of conceptual methods.

The city’s analysis and geometry traditions also maintained connections with mathematical physics. Geometric constructions provide the language for describing symmetry, curvature and invariant physical theories. Differential equations express the evolution of physical systems, while functional and variational methods support rigorous analysis of their solutions. These fields intersect most productively when researchers preserve the precision of each mathematical framework. Geometric results may illuminate the formulation of physical models without immediately producing an experimental prediction. Similarly, analytical theorems may

clarify the mathematical limits of a model independently of numerical simulation. Brno's fundamental research therefore contributed to the general knowledge on which interdisciplinary mathematical physics can draw. This contribution should be distinguished from the technical activities of specialised experimental physics institutions. Its importance lies in the analytical and geometric structures that make sophisticated physical reasoning possible. Such distinctions are necessary whenever scientific contributions cross disciplinary or institutional boundaries.

The period 2011–2020 brought increasingly data-intensive methods to international science, but fundamental mathematical research in Brno retained its independent intellectual aims. Category theory, geometry and qualitative analysis were not rendered obsolete by the growing prominence of computation. On the contrary, formalisation and computational representation can generate new questions about mathematical structures and proof. Researchers had to decide where computational exploration could assist theoretical work and where rigorous deduction remained indispensable. This distinction is especially important in mathematics because persuasive numerical examples cannot establish a universal theorem. The continuing development of abstract research within Masaryk University demonstrates that innovation should not be identified exclusively with digital applications. Computational tools can extend the range of mathematical experimentation while leaving the standards of proof intact. The coexistence of these approaches became part of the changing research environment of the early twenty-first century.

During 2021–2025, the established research traditions entered another phase of international scientific communication. The same fundamental problems of structure, transformation, existence and stability remained intellectually productive. At the same time, contemporary scientific questions encouraged further contact among mathematics, computer science and applications. The continuity of research programmes was visible in ongoing university publishing and the organisation of mathematical meetings. The fifteenth EQUADIFF conference took place in Brno in July 2022, continuing the sequence begun in the Czechoslovak period. Its proceedings appeared in 2023 in issues of Archivum Mathematicum devoted to ordinary differential equations, partial differential equations, and numerical analysis and applications. This publication arrangement linked an international conference directly to an established local mathematical journal. It demonstrated that the historical scientific infrastructure of Brno could still support new research exchange within the period under consideration. The meeting provides an instructive concluding example of continuity operating through modern institutional arrangements.

An assessment of Brno's mathematical achievements also requires caution about attribution. A major book or theorem may be written by a scholar affiliated with Masaryk University while drawing upon decades of international cooperation. The existence of a university affiliation does not mean that every component of a collaborative result originated within one institution. Similarly, the historical prominence of a mathematical school should not be used to assign contemporary research achievements to predecessors who worked in a different era. The distinction between inherited traditions and results established during 1993–2025 is therefore essential. Adámek and Rosický's categorical research and Kolář and Slovák's geometrical

programme can be located through identifiable publications and research records. The conference and journal histories provide equally concrete evidence for continuity of scholarly institutions. Together, such sources make possible a more rigorous account than an unsourced catalogue of reputations.

The development of mathematics at Masaryk University also reveals the practical limits of institutional comparison. Different disciplines require different forms of support, and publication patterns in abstract geometry cannot be assessed through the same criteria as large engineering projects. The productivity of fundamental mathematics often depends upon small groups and sustained periods of concentrated work. International collaboration may be intensive even when the resulting project has few formal participants or modest equipment costs. Research assessment must therefore recognise the originality and influence of mathematical arguments rather than relying exclusively upon organisational scale. The continued activity of specialised groups in Brno indicates the value of an academic environment that accommodates such differences. Preserving that environment also requires stable teaching and doctoral supervision, which connect research excellence with institutional sustainability. These considerations are relevant to the wider development of Czech mathematical science after independence. They show why diversity across mathematical schools can be a national strength rather than a sign of fragmentation.

By 2025, the Brno mathematical school associated with Masaryk University remained identifiable through its interlocking strengths in algebra, category theory, geometry and analysis. Its history combined the pre-independence legacy of figures such as Borůvka with original research published throughout the post-1993 period. Internationally recognised books and journal articles demonstrated that Brno mathematicians could contribute conceptual advances to fundamental mathematics. The journal Archivum Mathematicum provided a durable channel for communicating research and preserving the scholarly record. The EQUADIFF tradition showed that the city also possessed the intellectual and organisational capacity to convene international mathematical communities. These features were complemented by doctoral education, cross-border research networks and connections with computer science and applied analysis. No single achievement exhausts the significance of this development, because the school’s strength emerged from continuity among institutions, ideas and researchers. The experience of Brno therefore illustrates how an established university mathematical tradition could adapt to the independent Czech Republic and remain active within international science between 1993 and 2025.

Applied Mathematics and Computational Research in Brno: Brno University of Technology and Interdisciplinary Scientific Centres

The Brno University of Technology (BUT) occupies a distinctive position in Czech mathematics because its research questions arise at the intersection of theoretical reasoning and

technological design. Its mathematical identity cannot be understood solely through a department devoted to abstract research, since mathematical expertise is distributed among mechanical engineering, electrical engineering, information technology, civil engineering and materials science. After the establishment of the Czech Republic in 1993, these faculties adapted established analytical and numerical traditions to a research environment increasingly dependent on competitive grants and international collaboration. The practical constraints of machines, structures and signals encouraged researchers to connect formal descriptions with measurements, simulation and design decisions. Yet applied mathematics in a technical university is not simply the routine use of calculations supplied by other disciplines. Choosing a model, establishing whether its solution exists and determining the reliability of numerical results require mathematical judgement in their own right. The development of this judgement provides a useful perspective on the intellectual history of engineering research in Brno. By 2025, computation had become a major means by which mathematical ideas passed between laboratories, industrial partners and international research networks. This institutional profile illustrates how a technical university can sustain mathematical scholarship without reproducing the structure of a classical mathematics faculty.

The institutional environment of BUT carries a longer history than the period examined in this volume. Its antecedents in technical education created a culture in which mechanics, geometry and quantitative reasoning were essential to professional formation. The reorganisation of Czech higher education after 1989 and independence in 1993 altered the possibilities for research partnerships and academic mobility. Faculty members could engage more extensively with European collaborators and scientific literature while responding to changing requirements for project evaluation. Such changes did not immediately produce a unified mathematical research centre with a single programme. Instead, mathematical activities evolved through specialised institutes located close to engineering problems and their experimental evidence. This arrangement fostered intellectual diversity but also made interdisciplinary coordination more demanding. The value of mathematical competence was increasingly assessed through both scientific publication and the capacity to solve difficult applied problems. The subsequent growth of research centres should therefore be viewed as an extension of accumulated university expertise rather than an abrupt creation of new mathematical capacity. The institutional trajectory remains inseparable from the wider transformation of Czech research governance.

The Institute of Mathematics within the Faculty of Mechanical Engineering provides the clearest institutional example of the university's mathematical foundation. Its teaching includes linear algebra, analysis, differential equations, probability, statistics and numerical methods, while advanced programmes connect these subjects with engineering practice. The institute identifies applied mathematical analysis, finite element methods, numerical solutions of differential equations and optimisation among its research areas. Applied algebra, discrete mathematics, differential geometry, modelling of forming processes and digital image processing broaden this profile. This combination makes the institute a meeting place for subjects that are often distributed between separate departments elsewhere. In particular, the mathematical engineering programme provides a pathway through which students acquire a theoretical

background alongside programming and contemporary computational tools. Such training is relevant to the maintenance of research groups because computational projects need specialists capable of evaluating algorithms rather than merely operating software. The institute's research agenda demonstrates the continuity between core mathematical education and advanced engineering investigation. It also cautions against describing mathematical research in Brno exclusively through the better-known university traditions of Masaryk University.

Finite element analysis is particularly revealing because it turns continuous mathematical models into finite-dimensional systems while retaining a connection with the original differential equations. Engineers employ the method to approximate stresses, temperatures, vibrations and other fields whose spatial variation cannot normally be described by elementary formulae. The quality of a numerical solution depends on the mathematical formulation of the underlying boundary-value problem. A mesh that looks geometrically detailed may still produce a misleading approximation if stability and convergence have not been established. Researchers must consequently distinguish errors introduced by discretisation from uncertainty in material parameters and boundary conditions. Adaptive refinement offers one response by concentrating computational effort in regions where a calculated error indicator suggests that resolution is insufficient. Such strategies are relevant to mechanical design because computational resources are never unlimited and engineering decisions often require repeated simulation. The Faculty of Mechanical Engineering's declared focus on finite element methods situates these questions within a sustained programme of applied analysis. Its significance lies in linking mathematical assurances about approximation to technically interpretable predictions. The underlying numerical questions remain active even as available computational hardware changes.

Numerical solutions of differential equations form a second methodological axis of applied mathematics at BUT. A physical system may be governed by conservation laws, constitutive relationships or evolution equations whose behaviour cannot be captured adequately by closed-form expressions. Computational methods allow researchers to investigate these systems under experimentally relevant geometries and operating conditions. Nevertheless, the mathematical formulation determines which phenomena a simulation can plausibly represent. For time-dependent models, the choice of time step influences accuracy and may affect the numerical stability of the entire calculation. Nonlinear terms can introduce multiple solution branches or sensitivity to initial conditions, complicating both convergence analysis and interpretation. Researchers therefore need to connect algorithm design with qualitative knowledge about the underlying equations. This relationship between analysis and computation became increasingly important as engineering models grew more complex between 1993 and 2025. It remains a defining distinction between numerical mathematics and the uncritical production of computer-generated results.

Optimisation places mathematical research at the point where prediction is converted into design. An engineer may be able to simulate the performance of a component but still need to determine which configuration satisfies competing objectives. Weight, cost, energy consumption, structural reliability and manufacturing feasibility cannot always be improved simultaneously. Mathematical optimisation makes these trade-offs explicit through objective

functions, feasible sets and constraints. In practical problems, the relationship between design variables and system response may be nonlinear, nonconvex or expensive to evaluate. This makes gradient information, sensitivity analysis and approximate surrogate models important methodological resources. The Institute of Mathematics explicitly includes optimisation of engineering processes in its research profile. That orientation connects mathematical analysis with industrial decisions while preserving the distinction between an optimal solution of a formal model and a desirable decision in the real world. The difference becomes critical whenever model assumptions omit safety, maintenance or social considerations. Mathematical clarity also assists communication among researchers whose professional priorities differ.

The mathematical modelling of forming processes illustrates the particularly close relationship between engineering science and nonlinear analysis. Manufacturing operations change the geometry and internal state of materials through complex combinations of force, temperature, contact and deformation. A useful description may require continuum mechanics, constitutive equations and numerical procedures for moving or changing boundaries. The relevant material response frequently depends on deformation history rather than only on the current state. Consequently, identifying parameters from experiments and testing sensitivity can be as important as solving the governing equations. Numerical schemes must also handle localised deformation without introducing artificial behaviour that dominates the prediction. Because this problem extends from mathematical theory to production technology, it naturally encourages collaboration among mathematicians, materials researchers and mechanical engineers. Its inclusion among the institute's research topics shows how an ostensibly specialised industrial problem can demand sophisticated mathematical methods. The resulting knowledge may be transferable to other nonlinear processes, even when the original application remains specific.

Applied algebra and discrete mathematics provide a contrasting yet complementary direction within the Faculty of Mechanical Engineering. Where continuum models describe fields varying through space and time, discrete methods examine finite structures, relations and combinatorial choices. Graph-based descriptions can represent connections among components, information pathways or sequencing constraints in complex systems. Algebraic methods can clarify symmetries, transformation rules and the organisation of algorithmic objects. These techniques are especially valuable when the engineering question concerns configuration rather than the evolution of a physical field. They also illustrate why the mathematical competence required by contemporary engineering cannot be reduced to differential equations. The institute's recognition of discrete subjects strengthens its ability to support interdisciplinary projects involving informatics and optimisation. Mathematical abstractions can make computational procedures easier to verify and generalise across distinct applications. Their contribution is often less visible than that of a large simulation, but they can determine whether the simulation workflow is logically coherent. The distinction between continuous and discrete approaches is often productive rather than restrictive.

Differential geometry and computer graphics connect exact mathematical representations with the description of manufactured forms. Geometric models provide coordinates, curves, surfaces and transformations that allow a complex object to be designed and analysed. The mathematical treatment of curvature and parametrisation becomes important when local shape

affects mechanical behaviour or production accuracy. A geometric representation must also be compatible with the discretisation used by numerical solvers. The translation from design geometry to computational mesh is therefore not a purely technical conversion. Errors in that transition can influence calculated fields before the numerical solver begins its work. Research that combines differential geometry with applied computation offers a means of controlling this interface. At BUT, the presence of these subjects within mathematical training and research supports interactions between design, manufacturing and analysis. Their continued importance reflects the fact that digital engineering still depends on precise mathematical descriptions of physical shape.

Digital image processing represents another area in which mathematical methods engage directly with measured information. An image is neither a transparent copy of reality nor a set of observations free from measurement error. The reconstruction of meaningful features requires choices about filtering, enhancement, segmentation and uncertainty. Mathematical optimisation, probability and numerical linear algebra provide foundations for many such choices. Geometric knowledge becomes important when images are related to physical objects or when features must be tracked across changing viewpoints. The institute's inclusion of digital image processing within its research profile demonstrates the breadth of its applied agenda. Connections with industrial inspection and materials analysis are methodologically plausible, although specific outcomes require documentation rather than assumption. The broader scientific significance lies in the movement from raw numerical measurements to interpretable structures. Such work anticipated the increasing convergence of classical image analysis and data-driven computational methods during the 2010s and 2020s. These questions increasingly connect traditional mathematics with contemporary data science.

Mathematical engineering programmes contribute to research by transmitting both established theory and changing computational practice. Students must understand linear algebra and calculus before they can evaluate sophisticated optimisation or simulation algorithms. They also need experience in selecting numerical representations that are appropriate to an engineering problem. Programming competence becomes scientifically meaningful when it enables reproducible calculations and transparent model testing. Doctoral education adds another dimension by requiring researchers to formulate questions for which an existing software package is insufficient. This creates a route from the practical demands of an engineering faculty to original mathematical inquiry. The presence of advanced mathematical instruction within technical specialisations helps connect fundamental subjects with emerging technological problems. It also supports collaboration because mathematicians and engineers must develop a shared language for assumptions, uncertainties and verification. The institutional development of mathematics at BUT should therefore be assessed partly through the education and circulation of specialised research personnel.

The Faculty of Electrical Engineering and Communication demonstrates a different mode of mathematical integration. Its research in signal processing, communication systems and sensing relies on the analysis of uncertain and often noisy information. Fourier methods, linear algebra, estimation theory and stochastic models supply much of the underlying mathematical vocabulary. In digital communication, abstract properties of signals must be connected with

constraints imposed by electronic devices and transmission environments. The development of algorithms consequently depends on both mathematical performance criteria and physical characteristics of the systems in which they operate. This combination makes collaboration among mathematicians, physicists and engineers particularly valuable. It also explains why research infrastructure devoted to communication technologies can become a site of substantial mathematical innovation. The university's Centre of Sensor, Information and Communication Systems offers an instructive institutional example. Its activities place mathematical information processing within research programmes that remain explicitly oriented towards communication and sensing technologies. The relevant mathematics is often invisible to users of the final communication technology.

The Centre of Sensor, Information and Communication Systems, known as SIX, was established in August 2010 through cooperation among departments of radioelectronics, telecommunications, microelectronics and physics. Its institutional history traces the collaboration back to a research plan initiated in 1998 and continued through a successor programme for 2005–2011. The centre consequently emerged from an established network rather than from an entirely new collection of unrelated laboratories. An implementation phase followed in 2010–2012, testing operation was planned for 2013 and full operation for 2014. The stated research areas include signal processing, sensor systems, radio communication, mobile communications, cybersecurity and high-frequency technologies. The relevance to mathematics is especially clear in the design of inference and processing algorithms for incomplete or corrupted signals. Mathematical modelling can connect statistical assumptions to the practical limits of detection, reconstruction and communication reliability. The SIX trajectory illustrates how a technical research school may become institutionally visible when longstanding departmental expertise receives shared facilities and coordinated management. It also demonstrates the value of distinguishing foundational mathematical contributions from the engineering products in which they may eventually be embedded.

Statistical signal processing is a useful example of the scientific challenges addressed by information-oriented centres. A sensor delivers observations influenced by noise, calibration errors and environmental variation rather than direct access to a perfectly known quantity. Estimators must therefore use explicit assumptions about the signal and its uncertainty. Bayesian and frequentist frameworks provide different tools for describing how information changes when measurements become available. A filtering algorithm may work well under idealised noise yet fail when real measurements exhibit unmodelled dependence. The mathematical assessment of robustness is consequently as important as the nominal accuracy of an algorithm. In networked systems, timing, communication restrictions and missing data introduce additional sources of complexity. These issues connect traditional statistical inference with numerical optimisation and information theory. Research in the SIX environment helps illustrate why applied mathematics increasingly became inseparable from modern communication technology. The same concerns arise across numerous applications beyond communication engineering.

The Centre for Research and Utilisation of Renewable Energy Sources, or CVVOZE, broadens this computational landscape towards energy systems. Located within the Faculty of Electrical

Engineering and Communication, it addresses scientific and technological problems connected with renewable energy and their utilisation. Such problems frequently require the modelling of interacting components rather than the isolated analysis of a single device. Optimisation becomes relevant when renewable generation must be balanced with variable demand and system constraints. Uncertainty enters through meteorological conditions, operational disturbances and the quality of observed data. Control methods must reconcile immediate responses with longer-term operational objectives. The underlying mathematics ranges from differential equations and network models to numerical optimisation and probabilistic prediction. The centre therefore offers an institutional setting in which mathematical concepts are translated into problems of energy technology. Its value to an account of Czech mathematical development lies in this interface, not in treating the centre as though it were an independent mathematics institute.

The Faculty of Civil Engineering creates another environment in which mathematical models influence practical decisions. Structural reliability depends on understanding how a building or infrastructure system responds to loads, material variation and gradual deterioration. Computational mechanics supplies approximate solutions to mathematical models of stress, deformation and dynamic response. The interpretation of such results also requires attention to the limits of input data and to uncertainties that cannot be eliminated by finer discretisation. Statistical descriptions of material performance contribute to risk-based assessment, while optimisation can assist the choice of design alternatives. Complex infrastructure projects therefore depend on analytical, numerical and probabilistic methods working together. The relationship is particularly visible in research on advanced construction materials and structural diagnostics. As monitoring technologies improved after 1993, the connection between experimental observations and computational representations became stronger. Applied mathematics helped establish the conceptual bridge through which those different forms of evidence could be brought into a common engineering argument. These connections illustrate the interdisciplinary character of quantitative engineering science.

The Advanced Materials, Structures and Technologies centre, abbreviated AdMaS, is a prominent example of research infrastructure within the Faculty of Civil Engineering. Its development project ran from 2011 to 2014, and university documentation identifies July 2014 as the beginning of operations. The centre investigates materials, structures and technologies relevant to buildings, transportation and urban infrastructure. Its research agenda includes methods of diagnosing structures and evaluating the consequences of physical influences on their performance. A subsequent project description explicitly refers to mathematical, physical and computational models for assessing reliability, service life and durability. This language reveals how numerical representation is woven into the research mission rather than added after experimental investigation. The centre's infrastructure creates opportunities to compare models with measurements from materials and structural testing. Nevertheless, a model is scientifically useful only when the relationship between assumptions and observations remains open to criticism. AdMaS illustrates a path through which European-supported research infrastructure expanded the practical reach of applied mathematics in Czech engineering.

Reliability modelling requires a careful distinction between deterministic simulation and the probability of failure. A numerical model can indicate the response of a structure under specified inputs without establishing how likely those inputs are to occur. Probabilistic assessment combines uncertain loads, material properties and model imperfections within an explicit quantitative framework. The treatment of rare events is particularly difficult because direct observation may supply little evidence about exceptional conditions. Approximation methods can reduce computational demands, but they introduce errors that must be evaluated independently of statistical uncertainty. This interaction among stochastic analysis, numerical mechanics and engineering judgement is central to infrastructure durability studies. It encourages researchers to examine not only nominal performance but also sensitivity to uncertain assumptions. The AdMaS programme offers a concrete institutional reason to discuss these ideas in the history of Brno's applied mathematical research. Such analysis illustrates how mathematical uncertainty acquires practical consequences in the design and maintenance of public infrastructure. Treating these uncertainty sources separately improves the interpretation of estimated risks.

The New Technologies for Mechanical Engineering centre, commonly known as NETME, reflects the concentration of engineering research around shared facilities. Its association with the Faculty of Mechanical Engineering makes it relevant to computational work on manufacturing and mechanical systems. Sophisticated mechanical experiments often generate questions requiring inverse modelling rather than forward simulation alone. Researchers may know measured responses and seek the parameters or mechanisms that could have produced them. These inverse problems can be ill-posed, allowing distinct underlying descriptions to fit observations nearly equally well. Regularisation and sensitivity analysis therefore become necessary tools for constructing scientifically interpretable results. Large experimental datasets also encourage cooperation between mathematical modelling and information processing. The centre is best understood as part of a larger research environment in which mathematics supports engineering, rather than as evidence of a wholly separate school of theoretical mathematics. This distinction preserves the institutional accuracy of a history that includes both academic departments and interdisciplinary facilities.

The Central European Institute of Technology at Brno University of Technology, or CEITEC BUT, developed as an interdisciplinary setting for advanced materials and nano- and microtechnologies. Its shared laboratories support research that would be difficult to organise through isolated departmental instruments. Although the centre is not principally a mathematics institute, the interpretation of its experimental evidence depends on sophisticated quantitative methods. Mathematical modelling helps relate nanoscale geometry, material properties and observable physical behaviour. Inverse problems arise when researchers reconstruct inaccessible structures from indirect measurements. Optimisation also informs the design of experiments and the calibration of models against measured results. Computational methods can reduce the number of costly experiments, provided that the resulting predictions have been validated. The centre's open-access infrastructure creates the possibility of exchange between university researchers, external scientific users and industrial partners. Its role in this chapter is to demonstrate the increasing interdisciplinary reach of mathematical reasoning within the Brno

research ecosystem. The scientific value of computation depends upon the questions posed to it.

The expansion of high-performance computation altered what could reasonably be expected from engineering mathematics. Parallel algorithms enabled larger discretisations, repeated parameter studies and more complex multiphysics calculations. However, computational power does not guarantee that a model faithfully represents an observed phenomenon. The performance of a numerical scheme also depends on data movement, algorithmic scaling and communication among processors. Researchers must balance mathematical accuracy against memory constraints and the costs of computation. This can require domain decomposition, iterative linear solvers and strategies for allocating computational work across a machine. Brno's engineering faculties increasingly operated within national and European networks where access to computing resources complemented local laboratory expertise. The resulting computational culture linked mathematical ideas to research infrastructure without making those ideas reducible to hardware. By 2025, scientific computing had become an indispensable but intellectually demanding component of applied mathematical investigation.

The growth of data-oriented research created further contact between applied mathematics and informatics. An engineering experiment may produce more observations than a researcher can interpret through manual comparison or elementary statistical summaries. Machine learning offers procedures for identifying patterns, but its success depends on mathematically meaningful choices about representation and evaluation. Training data can reflect instrumental bias, operational conditions or incomplete coverage of the system under investigation. An accurate prediction within the training environment therefore does not automatically establish reliability under changing circumstances. Regularisation, cross-validation and uncertainty quantification help address different aspects of this difficulty. The analytical challenge is to distinguish correlation useful for a limited prediction from relationships supported by a credible physical model. Research combining signal processing, image analysis and computational mechanics has particular reason to make that distinction. The interaction of mathematical analysis with data science thus represents continuity with earlier modelling traditions as much as a technological change. Careful validation prevents computational novelty from being confused with scientific reliability.

Industrial research partnerships influence the direction of mathematical inquiry without determining its scientific validity. Companies may ask for faster calculations, lower material costs or more reliable quality control rather than for a new theorem. Yet a practical difficulty can expose weaknesses in existing algorithms and motivate a fundamental methodological question. A numerical method developed for one material may require reformulation before it can be applied to another physical regime. Similarly, an optimisation procedure that performs well on a small example may become unstable or prohibitively expensive at industrial scale. Researchers must therefore test whether the apparent success of a computational method survives changes in geometry, data quality and operational conditions. Applied mathematics contributes most when it treats such failures as opportunities to improve understanding rather than merely as inconveniences to be hidden. The centre-based structure of BUT creates opportunities for this reciprocal interaction between university methods and external

technological problems. The resulting innovation may consist of a more reliable procedure or a better validated model rather than a commercially identifiable product.

European integration reshaped the financial and organisational conditions under which applied research centres were established. After the Czech Republic joined the European Union in 2004, universities gained broader access to collaborative projects and investment instruments. The 2007–2013 programming period was particularly significant for developing large research infrastructures in Brno. Such programmes could finance specialised buildings and equipment that individual academic departments would have found difficult to obtain independently. They also required commitments to management, performance assessment and the sustainability of research activity after construction. The AdMaS project is an especially well-documented example of this investment model. Research-centre development nevertheless introduced a distinction between capital investment and the continuing support needed for academic staff and scientific programmes. Mathematics, which can rely less visibly on equipment than some experimental disciplines, still requires stable time for theoretical work, software development and validation. The institutional history of applied mathematics must therefore consider whether infrastructure expansion was accompanied by durable conditions for producing new knowledge. Research quality ultimately depends on people and methods as well as on physical infrastructure.

International collaboration at BUT was reinforced by the exchange of researchers, students and research practices. Engineering mathematics increasingly operated through publications, collaborative projects and software environments that crossed national borders. Joint work can combine expertise in modelling, numerical analysis and experimental verification that no single team possesses in full. The challenge is to maintain a common interpretation of a problem when disciplinary assumptions differ. An engineer may prioritise predictive performance in a particular operating range, while a mathematician may ask whether a model remains well posed beyond that range. Both perspectives can be necessary for an intellectually defensible research outcome. International partnerships also expose domestic research groups to competing methodologies and standards of reproducibility. They provide doctoral researchers with opportunities to learn how mathematical claims are tested and criticised in wider scientific communities. The significance of such cooperation is cumulative, developing through repeated collaboration rather than through individual high-profile events alone.

Scientific publications provide one measure of mathematical research, but institutional evidence requires cautious interpretation. University webpages describe research priorities, while project reports specify objectives, resources and periods of implementation. Neither type of source independently establishes that every anticipated mathematical result was achieved. A rigorous history therefore distinguishes between an announced research direction, an operational facility and a demonstrated scientific contribution. Peer-reviewed articles and monographs are necessary when attributing particular methods or theorems to named researchers. Infrastructure reports remain valuable for reconstructing the conditions in which mathematical work was undertaken. Similarly, descriptions of educational programmes clarify how institutions reproduced expertise but do not automatically measure research excellence. This evidential distinction is particularly important in interdisciplinary institutions where

mathematical contributions may be embedded in engineering outputs. The present account consequently interprets organisational development as an enabling condition for innovation, not as a substitute for evidence about specific results. The distinction also protects historical analysis against overstated claims of achievement.

By the end of 2025, the mathematical research environment associated with BUT was distinguished by its distributed and application-oriented character. The Institute of Mathematics sustained formal mathematical teaching and research in numerical analysis, differential equations, optimisation and geometry. Electrical engineering centres connected statistical and computational methods with communication, sensing and energy systems. Civil and mechanical engineering research made extensive use of simulation, reliability analysis and models of material behaviour. Interdisciplinary centres such as CEITEC BUT enlarged the contexts in which quantitative methods could support complex investigations. These activities were connected by a shared dependence on mathematical reasoning even though they belonged to different institutional and professional traditions. The principal historical transformation was therefore not the replacement of classical mathematical methods by digital technology. It was the widening of the settings in which those methods were developed, tested and applied under increasingly demanding standards of scientific collaboration. Brno University of Technology consequently forms an indispensable part of the story of Czech mathematics as a practical and intellectually productive research discipline.

Regional Mathematical Schools: Olomouc, Ostrava, Plzeň, České Budějovice, and Other Czech Research Centres

The geography of Czech mathematical research after 1993 cannot be reduced to the distinction between Prague and Brno. Smaller university cities retained intellectual traditions that had developed within the former Czechoslovak educational system. Their departments entered independence with different resources, teaching obligations and disciplinary identities. Nevertheless, many possessed established communities capable of producing original mathematical research. European integration subsequently offered opportunities to connect these communities more closely with international networks. Their achievements emerged through specialisation rather than the replication of a single national research model. Olomouc developed distinctive strengths in algebra and mathematical analysis. Ostrava cultivated connections between geometry, spectral theory and mathematical physics. Plzeň and České Budějovice demonstrate particularly clearly how mathematical investigation could interact with technological and biological problems.

The expression regional mathematical school requires a careful analytical definition. It should not imply that each university possessed a unified theoretical doctrine. Instead, it refers to a sustained community of researchers, shared problems, institutional continuity and recognisable channels of scientific exchange. A department may sustain several such communities with only

limited intellectual overlap. Research excellence likewise cannot be inferred from university size or the number of programmes offered. It depends upon identifiable publications, collaborations and the capacity to train new researchers. The history of regional mathematics is consequently a history of differentiation within an interconnected scientific system. Attention must be paid to the distinction between an institution existing in 2025 and a result actually achieved during the preceding thirty-two years. This distinction is particularly significant for research centres established after European structural funding became available. A historical account should therefore connect present-day departmental profiles with documented changes over time.

Palacký University Olomouc offers a compelling example of continuity extending far beyond Czech independence. Mathematical education in the city is linked institutionally to university traditions that began centuries earlier. The university was re-established after the Second World War and subsequently developed specialised mathematical departments. During the socialist period, teaching and research acquired more differentiated organisational forms. By 1993, these departments therefore inherited both academic experience and identifiable disciplinary specialisations. The Department of Algebra and Geometry continued the study of algebraic structures and geometric problems. A second important centre was the Department of Mathematical Analysis and Applications of Mathematics. Both operated within the Faculty of Science and maintained connections with doctoral education. Their subsequent development illustrates the significance of institutional stability for mathematical fields requiring extended theoretical work.

An important institutional change in Olomouc occurred in 1996. The former Department of Mathematical Analysis adopted the name Department of Mathematical Analysis and Applications of Mathematics. The new designation conveyed a broader connection between fundamental analytical theory and application-oriented questions. Research activities encompassed differential equations and dynamical systems. Numerical mathematics, optimisation and mathematical statistics formed further elements of the departmental profile. Theoretical problems associated with decision-making also acquired a place within this environment. These directions were not necessarily components of a single research programme. Rather, they provided complementary mathematical languages for problems involving evolution, uncertainty and constrained choice. Such diversification helped the department participate in interdisciplinary activities without abandoning its analytical foundations. The change of name thus serves as a useful historical marker, although it should not be mistaken for proof of an instantaneous transformation.

Research on differential equations in Olomouc contributed to the wider Czech engagement with nonlinear dynamics. These equations provide a mathematical description of systems whose states evolve through time. Questions about the existence of solutions must often be separated from questions about their uniqueness and long-term behaviour. Periodic solutions and boundary-value problems create additional difficulties that are not resolved merely by numerical computation. The Olomouc tradition included work associated with Jan Andres and collaborators in nonlinear and multivalued dynamical systems. Such research connected classical differential equations with fixed-point methods and more general mathematical frameworks. Its importance is methodological, because difficult physical or biological models

frequently require existence theorems before reliable calculations are possible. International publication and collaboration allowed regional work to enter discussions shared by analysts elsewhere in Europe. The resulting profile demonstrates how a university outside the two largest metropolitan centres could contribute to the general theory of dynamic systems.

Algebra and geometry created another enduring research identity in Olomouc. The Department of Algebra and Geometry has maintained work on algebraic structures alongside geometry and mathematics education. Universal algebra studies common properties of operations and relations without restricting attention to one familiar numerical system. This perspective creates connections with logical calculi, order theory and formal descriptions of information. Researchers including Radim Bělohlávek became associated with important work on fuzzy relations and formal concept analysis. Other mathematicians pursued the structures of lattices, logical algebras and related systems. These topics demonstrate that foundational algebra can influence methods used in computer science. Their mathematical contribution must nevertheless be assessed through the precise propositions and publications involved rather than through a generic claim of technological impact. The school's significance rests partly in sustaining specialised algebraic discourse across changes in national academic administration. Its doctoral programmes provided a means of reproducing that expertise.

The relationship between algebra and computer science was particularly visible in approaches to graded information. Classical formal concept analysis represents relationships between objects and attributes using mathematical order structures. Extensions to graded or fuzzy settings permit more nuanced treatment of attributes that do not behave as binary properties. The corresponding mathematics involves lattices, residuated structures and Galois-type connections. Radim Bělohlávek's work supplied a recognisable Czech contribution to this research conversation. Such work belongs simultaneously to algebraic theory and the mathematical foundations of knowledge representation. Its relevance should not be confused with an assertion that every resulting technique became an industrial application. Rather, it illustrates how abstract mathematical structures create rigorous languages in which possible applications can subsequently be formulated. The interdisciplinary character of the work helped regional mathematics develop links beyond traditional departmental boundaries. This connection made Olomouc an identifiable participant in the broader Czech landscape of logic-related mathematics. The historical record must consequently preserve the specificity of these achievements.

The publication infrastructure of Olomouc also mattered for international integration. Acta Universitatis Palackianae Olomucensis, Facultas Rerum Naturalium, Mathematica has published mathematical research since 1960. Its scope includes analysis, algebra, geometry and statistics. Such journals provide more than an archive of isolated articles. They permit research communities to maintain editorial networks, communicate specialised findings and engage with scholars abroad. After 1993, the adoption of internationally compatible publication practices became a growing concern throughout Czech higher education. Regional journals consequently operated within an environment of increasing international comparison. Their continuing relevance depended upon the quality of submitted work and professional editorial standards. A journal's existence is not itself a measure of a school's scientific influence.

Nevertheless, publication continuity represents a significant element of the institutional history of regional mathematical research.

Ostrava occupies a different position in the geography of Czech mathematics. Its academic development is closely connected with an industrial region historically shaped by mining, metallurgy and engineering. The University of Ostrava developed its independent institutional identity in the early post-socialist period. Its Department of Mathematics subsequently supported research and teaching across several mathematical fields. Mathematical physics and geometry became particularly recognisable components of its research profile. The combination reflects an approach in which abstract structures are studied partly for their ability to clarify physical models. International researchers and collaborators contributed to this intellectual orientation. The result was not a single industrial mathematics school dictated by regional economic needs. Instead, Ostrava provided an example of fundamental mathematics developing within a university situated in a strongly technical environment.

Several strands of the Ostrava research programme illustrate this orientation. Work on Lie algebras and other nonassociative structures connects algebraic classification with symmetry and mathematical physics. Investigations of structured matrices develop methods of relevance to both abstract analysis and computational questions. Spectral theory addresses operators whose properties govern the behaviour of waves and quantum systems. Quaternionic and Clifford analysis enlarge classical function theory through richer algebraic settings. Research in fractional calculus adds methods for systems involving nonlocal or memory-dependent effects. These fields share neither identical methodologies nor identical applications. What connects them institutionally is the possibility of dialogue among algebraists, analysts and mathematical physicists. Such dialogue is especially valuable in smaller research communities where an individual group's international connections can extend the methodological repertoire of a department. The example therefore deserves attention beyond its immediate regional setting.

Quantum graphs offer one striking example of the connection between geometry and physical interpretation. A quantum graph treats a network as a domain on which differential operators act under prescribed vertex conditions. Spectral properties can then depend on both the shape of the network and the rules governing its connections. The resulting questions intersect with scattering theory, eigenvalue estimates and quantum waveguides. Ostrava researchers have participated in investigations of Schrödinger operators and associated spectral inequalities. The names Ondřej Turek and other members of the regional mathematical-physics community appear in the university's research materials. Their work should be understood within a collaborative national and international field that also includes Prague-based mathematical physicists. It would be misleading to attribute the entire development of quantum graph theory to one regional institution. Ostrava's contribution lies in sustaining specialised expertise and conducting identifiable research within this wider network. This example shows how local strength can be enhanced by international collaboration rather than diminished by it. This distinction remains important for interpreting the institution's research history.

The circulation of mathematical knowledge through conferences remains another feature of Ostrava's profile. The Vojtěch Jarník International Mathematical Competition provides a long-running connection between mathematical education and international exchange. Although a

competition is not equivalent to a research conference, it strengthens the mathematical community and introduces students to demanding problems. The department also supports the journal Communications in Mathematics. Published research in this journal includes geometry, differential equations and related branches of mathematics. Its editorial and contributor networks reach beyond Czech borders. Together, these initiatives help create a sustained environment of academic communication. They are valuable for understanding how regional centres cultivate future researchers as well as publish existing research. Their influence should therefore be assessed institutionally and educationally alongside strictly bibliometric evidence.

The University of West Bohemia in Plzeň represents a particularly important case of applied mathematical specialisation. Established in 1991 through the merger of earlier educational institutions, it entered the 1993 period with a technical orientation. Its Faculty of Applied Sciences became a central environment for mathematics, mechanics, informatics and related disciplines. The Department of Mathematics contributed to research in mathematical analysis and graph theory. Geometric modelling and numerical mathematics were further significant areas of activity. Unlike a narrowly theoretical department, this environment could connect new mathematical concepts with engineering research. Its programmes trained specialists who needed both mathematical abstraction and practical computational methods. The intersection between mathematical theory and numerical implementation became a defining characteristic of this regional profile. It also prepared the conditions for substantial research infrastructure development during the following decades. The resulting development was gradual rather than the outcome of one event.

The methodological challenges of numerical mathematics are particularly clear in engineering applications. A mathematical model may correctly describe a physical process while still resisting exact analytical solution. Discretisation replaces continuous objects with approximating structures that computers can manipulate. This procedure introduces approximation error and may also magnify instability. Reliable numerical analysis must therefore study convergence, error bounds and sensitivity to perturbations. Plzeň research connected these general problems with differential equations, geometric structures and mechanical models. In this setting, mathematical analysis and scientific computation cannot be treated as separate enterprises. The analytical framework explains why an algorithm should work under particular assumptions. Computational experiments then test its performance within the limits of available resources. Such interactions represent a distinct type of mathematical innovation beyond a single theoretical theorem. Such differences should remain visible in any national historical synthesis.

Geometric modelling became one of Plzeň's more recognisable applied mathematical directions. Research on curves and surfaces involves exact geometrical relations as well as approximate representations of measured objects. Industrial design and manufacturing frequently require algorithms that can preserve geometric consistency under numerical uncertainty. Symbolic computations can establish algebraic relationships, while numerical algorithms handle the perturbations and scale encountered in practical tasks. Hybrid methods seek to exploit the strengths of both approaches. The university's researchers developed methods for recognising and reconstructing geometric structures. Applications include tool-

path planning and the modification of technically significant surfaces. These examples do not imply that all geometric research is motivated by production. They show instead how theoretical geometry can acquire additional questions through sustained dialogue with engineering.

A major institutional turning point came through the creation of the New Technologies for the Information Society research centre, known as NTIS. The centre was developed with European regional funding and became an established research centre of the Faculty of Applied Sciences in 2015. Its facilities helped consolidate interdisciplinary work under a shared organisational structure. Mathematical modelling was one of the recognised areas of activity rather than an incidental support service. The programme included qualitative and quantitative investigations of mathematical models. Researchers addressed continuous and discrete structures as well as numerical simulation. Such an environment could combine analysis, statistics, computation and technological investigation. The centre also connected mathematical research with broader projects in cybernetics, mechanics and information technologies. Its development illustrates how European research investment could change the opportunities available to a regional university. The scientific significance of the centre lies in the research it enables, not merely in the construction of buildings or laboratories.

The research agenda of NTIS reveals the diversity of mathematical methods needed for complex practical systems. A differential equation may express the evolution of a physical process, whereas a discrete graph can describe the structure of connections. Optimisation identifies configurations that satisfy constraints while improving a chosen objective. Statistical models provide descriptions of uncertainty and variability in observations. Geometric algorithms transform shapes into computable representations. Numerical simulation brings these components into a common computational process. Research on solvability, stability and robustness remains necessary when results must be interpreted beyond idealised conditions. These mathematical questions justify treating interdisciplinary modelling as genuine research rather than routine software use. Plzeň's institutional arrangements made it possible to approach several such questions in parallel. They also connected doctoral education more directly with the development of research methods. The historical record must consequently preserve the specificity of these achievements.

The University of South Bohemia in České Budějovice developed yet another distinctive mathematical profile. Established in 1991, the university operates within a region possessing substantial ecological and biological research traditions. Mathematics became closely connected with questions arising from the life sciences. The Department of Mathematics at the Faculty of Science supports research in numerical mathematics and geometry. Applications of mathematics in biology constitute a further principal area. This configuration differs from the predominance of mathematical physics in parts of Ostrava and from the engineering orientation of Plzeň. Its interdisciplinary character is rooted in the scientific questions investigated rather than in a simple desire to diversify programmes. Researchers formulate equations and computational models to understand processes that are difficult to isolate experimentally. Their work consequently requires close collaboration with specialists able to assess the biological assumptions of those models.

The Centre for Mathematical Biology is especially important for understanding this approach. Its stated mission combines research, teaching and cooperation with biological institutions. Mathematical models provide ways to examine patterns that may arise from interactions among organisms or within populations. They can reveal which assumptions are capable of reproducing observed changes. However, mathematical fit alone does not guarantee a satisfactory biological explanation. Parameters must be interpreted carefully, and uncertainties in field data may alter conclusions. Collaboration with biologists is therefore part of the scientific method rather than a secondary outreach activity. The centre also aims to communicate mathematical biology to students and wider scientific communities. Such activity positions České Budějovice within the international expansion of quantitative biological science. It demonstrates how regional specialisation can create original mathematical research questions from locally strong neighbouring disciplines.

Population dynamics provides a clear illustration of the questions addressed by this school. Deterministic equations can describe average changes in population size through time. Individual-based models instead represent separate organisms and their interactions, sometimes incorporating stochastic behaviour. Evolutionary game theory considers how behavioural strategies affect reproductive or competitive outcomes. Models of the Allee effect examine situations in which low population density can reduce individual fitness or population growth. These issues create challenging mathematical problems involving thresholds, stability and long-term persistence. Researchers in České Budějovice have used such frameworks to investigate ecological and evolutionary processes. The work of Luděk Berec is particularly associated with mathematical ecology, epidemiology and population dynamics. The mathematical significance comes from linking formal modelling choices to experimentally or observationally meaningful questions. Such links require continuous evaluation of the conditions under which a model's conclusions remain valid. The example therefore deserves attention beyond its immediate regional setting.

Mathematical epidemiology offers a further connection between theoretical tools and biological uncertainty. Disease transmission depends upon interactions among individuals rather than on the properties of an isolated organism. Differential equations can represent average flows between epidemiological categories. Network and individual-based models permit additional variation in contact structure or behaviour. Parameter estimates may be uncertain when surveillance data are incomplete or unevenly collected. As a result, scenarios should not be confused with unconditional predictions. The mathematical biology community in České Budějovice has addressed disease-related questions within its broader research remit. Such work acquired wider public relevance during the coronavirus pandemic, although its disciplinary development cannot be dated solely from that event. The enduring scientific issue is the relationship between mechanistic understanding and the limits of available evidence. This challenge makes mathematical biology an important constituent of the national research landscape.

The coexistence of different methodological traditions also creates opportunities for cooperation among regional centres. Numerical methods developed for engineering equations can assist researchers modelling biological systems. Techniques from spectral theory can

inform mathematical questions arising in structured dynamical networks. Algebraic representations of information may contribute to formal or data-oriented approaches. Nevertheless, interdisciplinarity should not be treated as evidence of actual collaboration without documented projects or publications. The existence of transferable methods creates possibilities, not automatic institutional partnerships. Regional mathematical communities consequently depend on conferences, doctoral mobility and individual researchers to convert potential connections into sustained work. Participation in European educational and scientific programmes has provided opportunities for such contact. National funding agencies have also supported competitive research that can cross university boundaries. The resulting network is best understood as interconnected but unevenly distributed rather than uniformly integrated. Its significance becomes clearer when the scientific work is examined in context.

Other Czech university centres added important dimensions to this geographical picture. Technical universities and smaller institutions often required mathematical expertise for mechanics, economics and computational disciplines. Their teaching functions helped maintain a population of mathematically trained researchers and graduates across the country. Some groups developed specialised publication records even when they lacked a large independent mathematics faculty. Applied mathematical research could consequently take place inside departments carrying names drawn from engineering or information technology. A comprehensive institutional history must take this organisational diversity into account. It must also recognise that the existence of a mathematically oriented course does not establish a significant independent research school. The strongest evidence comes from sustained projects, identifiable publications and internationally connected researchers. This analytical distinction prevents regional history from becoming an indiscriminate catalogue of educational provision. It also clarifies how smaller institutions may contribute valuable expertise without claiming the institutional breadth of Charles University.

The period from 1993 to 2003 placed particular demands upon regional departments. They had to preserve teaching responsibilities while adapting to changing grant and evaluation systems. International mobility opened new opportunities but could also intensify competition for qualified staff. Research communities with strong pre-existing networks were often better positioned to maintain continuity. Smaller departments faced additional challenges in supporting several specialised fields at once. The situation favoured strategic concentration on topics where experienced researchers could sustain publication and supervision. Olomouc's analytical and algebraic traditions provide one example of such continuity. Other centres gradually developed different profiles shaped by their university environments. These trajectories should not be portrayed as uniform or inevitable. They show that the national transformation of Czech mathematics took multiple local forms. Such differences should remain visible in any national historical synthesis.

European Union accession in 2004 contributed to a second institutional phase. Researchers obtained wider access to collaborative projects and opportunities for cross-border mobility. European support also altered the possibilities for building major regional research facilities. This was especially consequential in fields requiring computing resources or technological laboratories. The development of NTIS in Plzeň illustrates the infrastructural dimension of these

changes. More theoretically oriented departments could benefit from networks and mobility without requiring comparable physical investment. Such differences remind us that the resources needed for mathematical progress vary by field. An algebraic result may depend principally on intellectual time and cooperation, whereas computational modelling may require sustained access to specialist equipment. Public policy must consequently recognise multiple legitimate forms of mathematical research. A balanced national system cannot evaluate them solely through identical infrastructure indicators.

Doctoral education represents a common institutional concern across these regional trajectories. Mathematical research depends upon the ability to sustain abstract expertise across generations. A doctoral candidate must acquire both established theory and the judgement needed to formulate genuinely new questions. Smaller schools can offer close supervision and early participation in specialised research groups. Yet their programmes also require international engagement to prevent intellectual isolation. Joint seminars and visiting researchers can introduce techniques unavailable within one department. Publication in international journals offers a further test of the relevance of new results. The universities of Olomouc, Ostrava, Plzeň and České Budějovice have developed different pathways linking higher education with research. Their contribution to national mathematical science therefore includes the preparation and circulation of researchers. This function is less visible than an individual breakthrough but essential for long-term institutional resilience. Its intellectual value cannot be measured through organisational indicators alone.

Scientific journals and conferences likewise contribute to the reproduction of regional schools. The Olomouc mathematical journal provides continuity for work in several theoretical fields. Communications in Mathematics in Ostrava facilitates dissemination in geometry and related areas. International conferences and specialised meetings allow researchers to compare methods and formulate collaborative projects. These activities can reinforce local scientific identity without restricting participation to a regional audience. Their importance becomes particularly clear in fields where small groups share highly specialised interests. Nevertheless, conferences vary greatly in scientific scope and status. Historical evaluation must distinguish a well-documented research programme from occasional academic events. Similarly, journal longevity should be complemented by an examination of editorial standards and actual research contributions. Taken together, these institutions form a less conspicuous but indispensable infrastructure of mathematical communication.

Regional mathematical science also complicates simple accounts of the distinction between basic and applied research. Algebra in Olomouc can contribute to formal representations used in information science. Spectral theory in Ostrava grows from fundamental operator questions while interacting with quantum models. Plzeň's numerical mathematics requires analytical proofs even when motivated by technical applications. Biological modelling in České Budějovice develops abstract dynamical questions from empirical ecological problems. The boundary between theory and application therefore runs through many individual projects. It does not coincide neatly with the boundary between one university and another. A scientifically mature regional system permits mathematical problems to move between these contexts. Such movement can produce new theoretical insight without necessarily generating immediate

commercial products. Understanding this process is essential to the conception of mathematical innovation adopted in the present volume. This interpretation avoids overstating the effects of a single institutional reform.

The national prominence of Prague and Brno remains an important structural fact. They contain large concentrations of research institutions, doctoral programmes and internationally visible mathematical groups. Regional centres operate within this asymmetrical environment rather than outside it. Their strategic opportunities frequently depend on areas where distinctive expertise can be maintained. Geographical concentration may improve access to research networks but can also increase disparities in recruitment and funding. Regional universities therefore face a dual task of building international credibility and sustaining local research communities. The evidence considered here suggests that specialisation has often been more productive than institutional imitation. Olomouc, Ostrava, Plzeň and České Budějovice illustrate different configurations of that principle. Their significance becomes clear when individual disciplinary profiles are examined alongside the national research architecture. A plural mathematical landscape can be stronger than one measured only by its largest institutions.

By 2025, the regional geography of Czech mathematical science had acquired a recognisable pattern of complementary strengths. Olomouc contributed established communities in analysis, algebra, geometry and the mathematical treatment of information. Ostrava supported research in algebraic structures, spectral theory, geometry and mathematical physics. Plzeň combined theoretical analysis with numerical algorithms, geometric modelling and interdisciplinary technology research. České Budějovice developed a distinctive meeting point between mathematical methods and biological investigation. These institutions also sustained journals, academic meetings, doctoral training and international partnerships. Their achievements should be understood as contributions to a shared national and European environment rather than isolated provincial traditions. No single institution represents the complete range of mathematics practised outside Prague and Brno. Together, however, they demonstrate how scientific continuity after 1993 could produce differentiated forms of international participation. The regional history of Czech mathematics is therefore a history of intellectual specialisation, institutional adaptation and the continuing movement of mathematical ideas across disciplinary borders. The example therefore deserves attention beyond its immediate regional setting.

Computational Mathematics and National Research Infrastructures: IT4Innovations and the Integration of Czech Mathematics into European Science

The development of computational mathematics in the Czech Republic illustrates how scientific infrastructure can reshape an intellectual discipline without replacing its theoretical foundations. Before large national supercomputers became available, Czech mathematicians

had already established substantial traditions in numerical analysis, differential equations and optimisation. The challenge after 1993 was to connect this expertise with computing resources capable of addressing increasingly complex models. University departments and Academy institutes initially relied on institutional clusters, specialised computing laboratories and foreign collaboration. These arrangements supported important research but constrained projects involving very large simulations or numerous coupled variables. A modern scientific computation requires far more than the ability to perform arithmetic rapidly. Its validity depends on selecting an appropriate mathematical model, designing stable algorithms and evaluating approximation errors. Infrastructure becomes scientifically valuable when it enables these methodological tasks to be undertaken at otherwise inaccessible scales. This relationship between mathematical knowledge and collective computational capacity provides the central perspective of the following account.

The establishment of IT4Innovations in Ostrava in 2011 marked a decisive institutional step towards an integrated national supercomputing environment. The centre was developed under the leadership of VSB–Technical University of Ostrava with partners from several Czech research institutions. Its formation reflected a wider understanding that research competitiveness increasingly depended on shared digital infrastructure. European structural funding helped establish the physical and organisational foundations needed to support substantial computing services. The project was not simply an acquisition programme for expensive machines. It sought to create a research environment linking numerical mathematics, computing architecture and the problems of scientific users. Partners could thereby combine regional engineering expertise with broader Czech capabilities in applied mathematics. Located outside Prague and Brno, the centre also contributed to a more geographically distributed national research landscape. The involvement of multiple founding partners also helped link mathematics to the wider regional research economy. Its early development demonstrates how regional institutional cooperation could acquire a distinctly European technological dimension.

The Anselm supercomputer, introduced in 2013, gave IT4Innovations its first operational platform for substantial computational workloads. Its theoretical performance was approximately ninety-four teraflops, significant for the centre during its formative period. The installation enabled researchers to develop practical experience in parallel submission systems, allocation procedures and reproducible computing workflows. Mathematical models that had previously depended on smaller clusters could be evaluated across a larger number of processors. Such expansion was especially relevant for numerical solutions of differential equations and computational mechanics. Nevertheless, hardware performance alone did not guarantee equally rapid progress in solving mathematical problems. Algorithms required careful restructuring to limit communication overhead and inefficient distribution of work. Early operational experience consequently contributed to expertise in code profiling, performance testing and user support. Anselm should therefore be understood as both a computing resource and a practical training ground for national computational research.

The opening of the dedicated IT4Innovations building in 2014 gave the expanding project a more permanent infrastructural setting. Specialised facilities made it possible to operate high-density computing equipment alongside supporting storage and network systems. Their design mattered

because scientific calculations rely on reliable data movement, electrical supply and thermal management. Even a theoretically efficient solver can become ineffective if storage or communication performance restricts execution. The institutional consolidation also encouraged the development of teams able to maintain computing services over long periods. Researchers gained a more stable environment for collaborative experiments involving mathematics, computer science and engineering. Such facilities require continuous operating budgets rather than one-off expenditure on machines alone. This issue illustrates the distinction between creating a research infrastructure and ensuring its long-term scientific productivity. This organisational capacity would prove as consequential as the specifications of any individual computing system. By the middle of the decade, Ostrava had become an increasingly visible national location for advanced computational research.

The installation of Salomon in 2015 represented a pronounced increase in national computing capability. Its theoretical peak approached two petaflops, and it initially appeared among the forty highest-ranked systems on the TOP500 list. This development substantially widened the range of simulations accessible to researchers working at Czech institutions. High-resolution discretisations, parameter studies and computationally expensive optimisation problems became more feasible. Yet the significance of the system cannot be measured by its ranking alone. Mathematicians needed numerical methods that preserved accuracy while dividing computations across many processing units. The relationship between processor count and useful scientific throughput depended on the structure of the equations under investigation. Consequently, the arrival of Salomon reinforced the importance of scalable mathematical software and collaboration between numerical analysts and computing specialists. It also made resource allocation and transparent evaluation of proposed projects central aspects of research governance.

Parallel computing altered the practical interpretation of algorithms long studied in numerical mathematics. In serial calculations, the dominant concern may be the arithmetic complexity of an individual method. On a distributed system, communication costs, memory access and synchronisation can become equally important. Iterative solvers therefore require analysis both as mathematical procedures and as implementations on particular architectures. Preconditioning methods may accelerate convergence while simultaneously creating communication demands that limit parallel efficiency. Domain decomposition distributes a physical or geometric region into smaller computational subproblems. This strategy can exploit concurrency, although interfaces between subdomains introduce additional mathematical and software challenges. Effective design consequently joins spectral understanding, numerical linear algebra and architecture-aware implementation. The resulting questions are mathematical and architectural at the same time. Such interdisciplinary work made scalable algorithms a distinctive research priority in Czech computational mathematics.

Finite-element modelling provided an important bridge between established Czech mathematical schools and larger computing platforms. Complex engineering domains are commonly represented through meshes on which approximations to differential equations are constructed. Finer meshes can describe local structures more accurately but dramatically increase the number of unknowns. Large computing systems allow these degrees of freedom to

be handled, provided the underlying linear algebra remains manageable. Adaptive methods seek to allocate computational effort where estimated numerical errors are greatest. Their effectiveness depends on reliable estimators, mesh quality and stable treatment of boundary conditions. Researchers must also distinguish discretisation errors from inaccuracies caused by incomplete solution of linear systems. Such distinctions are crucial because spectacular visualisations do not necessarily establish numerical reliability. Through these concerns, supercomputing reinforced rather than displaced the analytical foundations of engineering mathematics.

Computational fluid dynamics became one of the domains in which mathematical analysis and high-performance computing could be integrated productively. Simulations of flow through industrial structures require models of momentum transport, viscosity and sometimes thermal effects. The governing equations frequently possess nonlinear behaviour that complicates stability and convergence. Czech traditions in mathematical fluid mechanics furnished expertise in weak solutions and the theoretical interpretation of approximations. Computational research translated parts of this understanding into numerical procedures for realistic geometries. Turbulence modelling, mesh resolution and appropriate boundary conditions remained major sources of uncertainty. For industrial applications, validation against experimental observations was therefore indispensable. Larger supercomputers made complex calculations possible but did not remove the need for physical judgement about model assumptions. In this setting, mathematical rigour becomes a practical condition for trustworthy simulation. This example demonstrates the reciprocal influence of mathematical theory, available computation and engineering evidence.

Mechanical modelling also benefited from the closer relationship between computational mathematics and specialised research infrastructure. Elastic and inelastic materials require constitutive relations that describe how forces influence deformation. The equations may involve contact, damage, plasticity or multiple interacting physical processes. Numerical treatment becomes difficult when material behaviour changes sharply or when different spatial scales interact. Parallel implementations offer additional capacity but may introduce convergence problems if the mathematical formulation is poorly conditioned. Researchers therefore combine variational methods, nonlinear solvers and error assessment with simulation software. Practical investigations range from components of industrial machinery to the mechanical performance of protective structures. These examples demonstrate that applied mathematical innovation often emerges through the refinement of methods rather than a single dramatic breakthrough. National computing facilities provide an environment in which these refinements can be tested on technically meaningful problems.

The creation of specialised research laboratories within IT4Innovations strengthened the relationship between service provision and independent scientific investigation. The Parallel Algorithms Research Lab focused on scalable numerical methods, software libraries and demanding engineering calculations. Its remit connected the performance of computer systems with mathematical questions about decomposition, approximation and optimisation. The Infrastructure Research Lab pursued computational performance, parallel application optimisation and the efficient operation of scientific workflows. These research directions

overlap but are not interchangeable. One may concern the mathematical organisation of a numerical algorithm, while the other investigates the practical execution of that algorithm on available hardware. Both are required when a computational model must be transformed into a reliable and economical scientific calculation. Research laboratories also offer continuity beyond the operational lifetime of individual supercomputers. The distinction helps explain why maintaining research staff is as important as upgrading processors. Their enduring value lies in expertise that can migrate between successive generations of hardware.

The establishment of the European High Performance Computing Joint Undertaking introduced a further dimension of international integration. The Czech Republic joined this European initiative in 2018, and IT4Innovations participated actively in its subsequent activities. The arrangement connected national investments to a coordinated European strategy for supercomputing capacity and access. For mathematical researchers, participation created opportunities to work within a larger computational ecosystem. A national facility could contribute specialised expertise while Czech users also benefited from external systems. This model differs fundamentally from viewing scientific computing as a competition to own the largest possible machine. European collaboration can distribute expertise in software, architecture, application support and research training. However, effective participation still requires strong domestic mathematical groups capable of proposing and interpreting sophisticated calculations. International infrastructure thus amplifies rather than substitutes for national scientific competence.

The launch of Barbora in 2019 demonstrated that infrastructure development could also involve complementary rather than exclusively larger systems. With a theoretical peak of approximately 849 teraflops, the machine expanded the range of available computational resources. Different scientific workloads place different demands on processor architecture, memory and communication patterns. Consequently, more than one system can provide useful flexibility for researchers whose models have contrasting computational characteristics. Allocation across machines requires an understanding of workload suitability rather than an assumption that theoretical peak speed predicts all outcomes. Smaller or specialised jobs can compete inefficiently for resources intended for very large parallel simulations. Matching models to systems therefore became an important component of responsible resource management. The operation of Barbora alongside other facilities contributed to this more differentiated approach. The resulting diversity can also improve the resilience of national computing services. It also illustrated the need to evaluate infrastructure through scientific usefulness and not simply aggregate performance.

The deployment of an NVIDIA DGX-2 system in 2019 highlighted the growing importance of computational methods associated with artificial intelligence. Accelerated hardware was suited to selected machine-learning calculations whose numerical operations could be expressed in highly parallel forms. This development expanded the range of mathematical questions addressed within a national supercomputing centre. Large-scale optimisation, representation learning and statistical approximation increasingly interacted with established simulation research. Nevertheless, machine learning did not create an alternative to mathematical reasoning. Training procedures depended on optimisation algorithms, the statistical properties

of data and decisions about model complexity. Questions of generalisation and uncertainty remained central when predictions were intended to influence real-world decisions. The presence of specialised hardware therefore stimulated methodological work as well as new application opportunities. It also demonstrated that future computational infrastructures would need to accommodate more varied scientific workloads.

The introduction of Karolina in 2021 constituted another major stage in the development of Czech scientific computing. Its theoretical peak performance reached approximately 15.7 petaflops, and the system was developed with European co-financing. Its position in international rankings drew attention to the country's expanding technological capacity. For mathematicians, the more consequential effect was access to computational experiments involving large models and advanced numerical methods. Researchers could explore multiple parameter regimes, larger ensembles and complex coupled systems with greater practical flexibility. Such opportunities are particularly valuable in uncertainty quantification, where a model must be evaluated repeatedly. Successful exploitation of Karolina nevertheless required suitable algorithms and careful assessment of computational cost. The system thus embodied a broader transition from national infrastructure development towards participation in an integrated European supercomputing environment. Its deployment followed several years of investment in expertise, partnerships and scientific planning. Its significance should be evaluated through sustained research outputs as well as initial technical performance.

Czech participation in the LUMI consortium extended access beyond machines physically located in the country. LUMI, situated in Finland, became an important European computing facility available through a multinational partnership. The arrangement illustrated a networked conception of scientific resources in which location and access no longer coincide. Researchers could pursue projects whose computational scale exceeded the capacities of a single national institution. At the same time, obtaining access did not resolve all practical barriers to high-performance scientific work. Software portability, data transfer, scheduling and technical support remained essential elements of effective collaboration. Mathematical teams consequently needed experience with heterogeneous computing environments and reproducible numerical workflows. International allocation mechanisms also encouraged more explicit descriptions of scientific merit and computational requirements. These developments placed Czech computational mathematics within an increasingly transnational research culture.

The national e-INFRA CZ arrangement joined IT4Innovations with complementary digital research infrastructures including CESNET and CERIT-SC. This organisational framework addressed a requirement sometimes overlooked in accounts of supercomputing development. Advanced numerical research depends upon networking, data management and distributed services as much as processing capability. Research groups need reliable routes for transferring data between laboratories, storage facilities and computing systems. Collaborative projects may also require secure access, persistent data repositories and shared computing environments. Bringing complementary services into a coordinated infrastructure can reduce fragmentation for users. Nevertheless, integration must accommodate different scientific practices rather than impose a single workflow on all disciplines. The development of e-INFRA

CZ therefore indicates an increasingly systemic approach to computational research support. The arrangement demonstrates how scientific infrastructures increasingly operate as connected public services. It connects numerical computation with the wider organisation of digital scholarly work.

Scientific simulation underwent a methodological change as larger datasets and more detailed models became available. Researchers increasingly needed methods for organising ensembles of calculations rather than performing only individual large simulations. Uncertainty quantification asks how numerical predictions vary when input parameters, boundary conditions or modelling assumptions change. Sensitivity analysis identifies which uncertainties have the strongest influence on quantities of interest. These tasks often multiply computational demand because the same model must be evaluated across numerous scenarios. Surrogate models and reduced-order methods offer ways of approximating expensive calculations more economically. Their quality must be assessed against the physical model and the mathematical properties of the approximation. High-performance infrastructure allows these methods to be developed, compared and validated at useful scales. The result is a closer integration of statistics, optimisation and numerical analysis within scientific computing.

Computational modelling of nanomaterials created a significant interdisciplinary connection between mathematics, physics and chemistry. The relevant scientific questions concern how atomic structure and interactions influence measurable material properties. Mathematical approaches are necessary to formulate equations, approximate energetic states and interpret multiscale behaviour. Calculations often require substantial computational resources because many particles or configurations must be considered. The Modelling for Nanotechnologies Lab provided an institutional setting for such work within IT4Innovations. Its development demonstrated that national computing infrastructure could support research far beyond conventional engineering simulations. However, numerical predictions remain dependent on the validity of underlying physical assumptions and approximation schemes. Collaboration with experimental investigators is essential when computationally suggested materials or structures require practical verification. Accurate scientific prediction requires both numerical competence and trustworthy experimental interpretation. This relationship illustrates how mathematics supplies both instruments of discovery and criteria for evaluating computational claims.

Data-intensive research became another defining feature of Czech computational science during the period under review. The Advanced Data Analysis and Simulations Lab connected high-performance computation with advanced data-processing methods. Such research often combines statistical modelling, numerical optimisation and systems for handling large data volumes. Applications may range from environmental monitoring to medical imaging and industrial diagnostics. The availability of large datasets does not automatically produce reliable scientific conclusions. Researchers must address measurement error, missing observations, selection effects and the stability of learned models. Methods derived from probability theory and mathematical statistics therefore remain central to data-driven discovery. Computational resources can accelerate modelling experiments, but sound inference requires a defensible

connection between data and the scientific question. This principle is particularly important when machine-learning tools are used alongside more traditional mechanistic simulations.

The construction of reproducible computational workflows acquired greater significance as scientific projects became more complex. A simulation may depend on software versions, compilation choices, numerical tolerances and the architecture of the machine employed. Small implementation differences can produce substantial changes in results for sensitive or poorly conditioned problems. Reproducibility consequently requires systematic documentation of modelling assumptions and technical execution conditions. Shared infrastructure encourages common practices for storing code, recording parameters and tracing computational outputs. Such practices should not be confused with proof that a model is physically accurate. Verification establishes whether equations are solved in accordance with the selected numerical method, while validation asks whether the model captures its intended phenomena. Both questions require mathematical insight and empirical evidence. In this respect, computational practice contributes to the broader ethics of reproducible scientific research. By encouraging more explicit workflows, sophisticated computing centres can strengthen standards of scientific accountability.

European research programmes offered opportunities for IT4Innovations to contribute to software and application development beyond national service provision. The LEXIS project, initiated in 2019 with IT4Innovations in a coordinating role, sought to improve access to complex scientific workflows. Its approach linked computing resources with data-oriented services and application requirements. The initiative illustrated a shift from providing processor time towards constructing more accessible computational environments. Similar projects addressed performance optimisation, scientific applications and cooperation among research institutions. Their significance for mathematics lies in making sophisticated algorithms usable within sustained research processes. Yet software platforms alone cannot resolve questions of numerical error or the scientific appropriateness of computational models. The most durable benefits therefore arise when platform design remains closely connected to the needs of mathematical specialists and domain researchers. International projects can support such connections through common standards, shared expertise and repeated collaboration.

The creation of national competence services for high-performance computing strengthened the educational dimension of infrastructure development. Researchers from disciplines with limited computational traditions often need guidance before they can exploit large parallel systems effectively. Training programmes introduce methods of parallel programming, job scheduling, numerical performance analysis and efficient data management. Mathematical instruction is equally important because inefficient algorithms cannot always be rescued by additional hardware. Workshops and specialist courses therefore connect abstract mathematical principles with practical implementation decisions. Early-career researchers can acquire skills transferable between academic fields and industrial projects. This activity also reduces the concentration of technical knowledge within a small group of highly specialised users. Broadening access does not mean eliminating the demanding intellectual work required for accurate modelling. These educational activities make computing infrastructure part of the

national research-training system. It means providing the knowledge and institutional assistance that allow more researchers to undertake that work responsibly.

The interaction between computational mathematics and Czech industry provided an additional route through which scientific methods could acquire practical influence. Engineering firms may require numerical predictions of thermal behaviour, fluid flow, structural deformation or equipment performance. Research centres can contribute expertise in developing models and adapting algorithms to such questions. Industrial problems also expose mathematical researchers to irregular geometries, uncertain inputs and restrictive computational deadlines. These demands may stimulate methodological improvements that remain relevant beyond the original application. Nevertheless, commercial usefulness and scientific novelty should be evaluated separately. A successful industrial simulation need not produce a new theorem, whereas a mathematical advance may have no immediate commercial application. Applied research becomes especially valuable when it maintains analytical clarity while responding to practical constraints. IT4Innovations offered a setting where this relationship could be developed within a public research infrastructure.

Environmental and Earth-science modelling revealed another field in which high-performance computing could support public-interest research. Numerical simulations may investigate atmospheric processes, hydrological behaviour and the stability of natural or engineered environments. Satellite observations and other measurement systems provide data that can be analysed together with mathematical models. Combining sources requires careful handling of spatial resolution, temporal variation and uncertainty. Computing resources make detailed analysis more practicable but do not determine which mechanisms should be included in a model. Interdisciplinary teams therefore need to reconcile mathematical representation with established domain knowledge. The growing use of data-driven techniques adds further questions concerning generalisation under changing environmental conditions. Robust conclusions depend on transparent assumptions and evidence rather than computational sophistication alone. The resulting collaboration can help public institutions assess risks without concealing uncertainty. These applications show why computational mathematics is essential to scientific questions whose social significance extends beyond the laboratory.

The development of quantum computing research introduced a new technological horizon for the Czech scientific-computing community during the early 2020s. Quantum algorithms involve different computational assumptions from those underlying classical numerical methods. Their potential requires investigation through linear algebra, probability, complexity theory and mathematical models of noise. The national interest in this field did not make classical high-performance computing obsolete. Instead, it created opportunities to compare problem formulations and investigate hybrid methods involving distinct computational resources. The installation of the VLQ quantum computer at Ostrava in 2025 marked an institutional milestone within this emerging direction. Claims about future advantages nevertheless require careful attention to hardware limitations and the computational cost of error correction. Mathematical evaluation is essential for distinguishing plausible research opportunities from unsupported technological expectations. The appearance of quantum facilities therefore expanded the research agenda while reinforcing the importance of theoretical scrutiny.

By 2025, IT4Innovations had evolved from a regional infrastructure initiative into a participant in several interlocking European research systems. Its development encompassed computing machines, specialist laboratories, international projects, training programmes and scientific application services. The centre did not replace university departments or Academy institutes conducting fundamental mathematical research. Instead, it created opportunities for those groups to address computationally demanding questions in cooperation with engineering and scientific specialists. This complementary role is particularly important in a medium-sized research system with finite resources. International memberships can extend access, but national institutions must still maintain the expertise required to use external facilities meaningfully. The advancement of computational mathematics therefore depends simultaneously on theory, personnel, scientific infrastructure and institutional continuity. Evaluating achievements requires attention to all four dimensions rather than treating hardware acquisitions as sufficient evidence of intellectual progress. Future capability will depend on the sustained interaction of these institutions with mathematical research groups. This perspective explains how national research capacity can be strengthened through selective specialisation and international cooperation.

The historical trajectory of Czech computational mathematics from 1993 to 2025 demonstrates a cumulative transformation rather than a replacement of established mathematical traditions. Earlier strengths in numerical analysis, differential equations and optimisation supplied the theoretical resources needed for larger computational projects. Investments in Ostrava created opportunities to test and deploy those methods within increasingly ambitious applications. European arrangements brought additional platforms, funding channels and communities of scientific users. Advances in parallel algorithms and data-intensive modelling subsequently altered what counted as a practicable research question. At the same time, increasing computational power made methodological limitations more visible rather than removing them. Stability, convergence, reproducibility, uncertainty and model validation remained indispensable criteria of scientific quality. The most durable achievement of this period was consequently the formation of an environment in which abstract mathematics could interact productively with advanced computing and interdisciplinary investigation. Czech participation in European computational science illustrates how intellectual continuity and infrastructural innovation can reinforce one another.

Conclusion

Czech Mathematics between 1993 and 2025: From Scientific Traditions to an Internationally Integrated Mathematical Research Ecosystem

Between 1993 and 2025, Czech mathematics developed within a national research environment transformed by political independence, European integration and the expansion of computational capabilities. Its history cannot be reduced to a succession of institutional reforms, because mathematical knowledge grows through intellectual traditions that frequently outlive the political systems in which they arise. The independent republic inherited rigorous schools of analysis, geometry, algebra, logic, differential equations and numerical mathematics from the Czechoslovak period. These traditions provided the conceptual vocabulary, teaching practices and professional relationships upon which later advances could be constructed. Nevertheless, continuity did not imply an absence of change, since the conditions governing research funding, academic careers and international communication were substantially altered. Czech mathematicians increasingly worked within competitive networks where the relevance of a theorem or method was assessed by a wider international community. Universities became more strongly research-oriented, whilst Academy institutes adapted their organisation and evaluation to new expectations. New computational resources subsequently extended the range of problems that mathematical research could investigate in practical settings. The resulting trajectory was one of cumulative intellectual renewal rather than the replacement of an inherited scientific culture.

The distinction between the democratic opening after 1989 and the creation of the Czech Republic in 1993 remains essential to an accurate historical interpretation. The former transformed the political circumstances of academic activity, while the latter established the sovereign legal and institutional framework for Czech research. Mathematical departments, individual research programmes and professional connections did not suddenly originate when Czechoslovakia ceased to exist. Instead, they entered a period in which their administrative affiliations, sources of support and opportunities for collaboration were progressively reconsidered. The creation of the Czech Academy of Sciences preserved significant elements of earlier scientific capacity under a new organisational structure. Its research institutes could therefore pursue long-term programmes without treating every political transition as an intellectual beginning. Such continuity was especially valuable in mathematics, where the formation of a distinctive research school may require several generations. At the same time, the possibility of collaboration with foreign scholars altered the questions, methods and standards circulating within those schools. The period beginning in 1993 should consequently be understood as the reconfiguration of an existing mathematical community within a radically changed scientific environment. This distinction remains important when evaluating the period as a whole.

Institutional continuity acquired meaning through the people and scholarly practices that it allowed to survive. Research seminars, doctoral supervision, specialist journals and informal exchanges maintained the coherence of mathematical communities even when administrative structures were changing. Prague and Brno possessed exceptionally deep traditions, but their influence was never confined to their own universities. Researchers trained in those centres contributed to departments and institutes across the country, transmitting methodological approaches and building collaborative networks. The sustained importance of mathematical analysis, differential equations and geometric investigation illustrates the durability of these inherited intellectual connections. Their value cannot be measured simply by counting institutions bearing historical names or tracing lines of administrative succession. What matters is the persistence of challenging research questions, credible standards of proof and opportunities for younger mathematicians to develop independent work. When such conditions are maintained, a scientific school becomes capable of transformation without abandoning its characteristic strengths. Czech mathematical development offers a persuasive example of this relationship between intellectual memory and methodological renewal.

The position of universities changed significantly over the three decades under consideration. Charles University continued to provide an influential environment for fundamental mathematics, discrete structures and theoretical computer science. Masaryk University sustained strong research traditions in algebra, category theory, geometry and analysis, while technical universities increasingly connected mathematics with engineering. Doctoral programmes and international collaboration made university research more visible beyond national boundaries. The distinction between institutions devoted primarily to teaching and those devoted primarily to discovery became less rigid in practice. Graduate students increasingly encountered mathematics as an active international conversation rather than a fixed body of established results. Their participation in research groups, workshops and joint projects also strengthened the circulation of new techniques between disciplines. Nevertheless, successful integration of teaching and research requires stable academic appointments and sufficient time for supervision. The ability of universities to sustain this combination remains a central condition of Czech mathematical vitality. This distinction remains important when evaluating the period as a whole.

The Academy of Sciences continued to play a complementary rather than interchangeable role within this system. Its Mathematical Institute supported specialised research in analysis, differential equations, geometry, numerical methods and neighbouring theoretical fields. Institutes concerned with computer science, information theory and automation broadened the institutional foundations of mathematical logic, optimisation, statistics and decision theory. Their organisational missions differed from those of universities, particularly in the balance between concentrated research activity and formal teaching obligations. Yet these differences could become productive when doctoral training, joint seminars and collaborative projects connected the institutions. The contribution of Academy research is not adequately represented by its direct applications or its annual publication output alone. Its ability to protect continuity in demanding foundational problems also has long-term significance for the national research system. Institutional diversity therefore constitutes an asset when responsibilities are

coordinated and research communities remain open to exchange. The Czech experience indicates that a resilient mathematical ecosystem depends on cooperation between complementary organisations rather than uniformity of their functions.

The evolution of mathematical analysis demonstrates why fundamental research retained a central position throughout the period. Investigations of function spaces, operator theory, inequalities and infinite-dimensional structures addressed questions that arise within mathematics itself. Such work may not promise immediate practical results, but it establishes rigorous methods for studying equations and models used in other disciplines. Czech contributions in functional analysis and the theory of differential equations illustrate this movement between abstract formulation and wider application. The mathematical treatment of existence, uniqueness, regularity and stability provides necessary foundations for understanding evolving physical systems. Without these properties, numerical solutions may be difficult to interpret and model predictions may lack a secure theoretical basis. The value of foundational mathematics therefore extends beyond direct citation in technological projects. Its influence is often transmitted indirectly through concepts that become indispensable to later theoretical and computational developments. A responsible assessment of Czech scientific achievement must recognise these less immediately visible forms of intellectual impact. This distinction remains important when evaluating the period as a whole.

The work of researchers associated with mathematical fluid mechanics provides a particularly revealing connection between theory and physical reality. Investigations by Eduard Feireisl and collaborators helped establish rigorous mathematical approaches to compressible-fluid equations and related evolutionary systems. Studies of nonlinear constitutive behaviour, including research associated with Josef Málek and Tomáš Roubíček, likewise demonstrated the need for sophisticated analysis in material modelling. These programmes required precise distinctions between the existence of a weak solution, its regularity and its possible uniqueness. They also illustrated how experimentally motivated questions could generate problems of independent mathematical importance. Such relationships do not eliminate the distinction between pure and applied research, because mathematical rigor and physical adequacy remain separate criteria. Rather, they show that productive contact between the disciplines depends upon maintaining both criteria simultaneously. The resulting achievements are best interpreted as sustained research programmes rather than isolated practical solutions. Their historical significance lies in the creation of methods that other researchers can test, refine and extend.

Discrete mathematics became another area in which Czech research acquired distinctive international visibility. Work associated with Jaroslav Nešetřil and his collaborators connected structural combinatorics, graph homomorphisms, Ramsey theory and the theory of sparse graph classes. The resulting research did not merely produce techniques for handling larger collections of finite objects. It changed the conceptual vocabulary available for describing structural restrictions and algorithmic behaviour across broad families of graphs. The monograph on sparsity by Nešetřil and Patrice Ossona de Mendez exemplified the synthesis of deep theoretical results with questions relevant to algorithms and logic. Their recognition with the 2025 Nerode Prize confirmed the international significance of this long-running research direction. Such recognition is meaningful because it reflects the evaluation of specific scientific contributions

by a specialist community. It should not, however, be mistaken for evidence that every achievement in discrete mathematics is associated with a single school or individual. Czech strength in this field emerged from productive connections between researchers, institutions and international collaborators. This distinction remains important when evaluating the period as a whole.

Mathematical logic and algebraic structures reveal a different form of continuity between abstract thought and computation. Petr Hájek's work on the metamathematics of fuzzy logic demonstrated how questions about graded truth could be treated with the precision of formal mathematical systems. Research in proof theory and foundations, including contributions associated with Pavel Pudlák, addressed limits and structures of formal reasoning. In algebra and category theory, scholars such as Jiří Adámek and Jiří Rosický developed frameworks used well beyond any single class of mathematical objects. Later work on algebraic approaches to constraint satisfaction illustrated the relevance of structural theory to the complexity of computational problems. These examples show that the boundary between mathematics and theoretical informatics is historically contingent rather than conceptually fixed. Both disciplines rely upon exact definitions, formal arguments and the identification of structures that govern what can be computed or proved. Mathematical ideas may acquire computational importance many years after their initial formulation. Investment in abstract research is therefore not a retreat from innovation but one of its possible foundations.

Geometry and topology remained important precisely because their modern development incorporated multiple interpretations of structure. The geometric traditions of Brno continued through work on natural differential operators, connections and parabolic geometries. Investigations associated with Ivan Kolář and Jan Slovák exemplified the sustained value of sophisticated differential-geometric techniques. Related studies of geometric operators and global structures also intersected with mathematical physics and analysis. Here, innovation frequently involved recognising equivalences between descriptions that initially appeared unrelated. A new invariant or geometric framework could clarify an entire family of problems without producing a recognisable technological artefact. The scientific contribution was the reorganisation of mathematical understanding itself. This form of progress demands careful exposition because its importance may be obscured when evaluated solely through industrial applications. The Czech record illustrates why geometrical research deserves an independent place within assessments of national scientific innovation. This distinction remains important when evaluating the period as a whole.

Mathematical physics demonstrates another productive meeting point between abstract theory and experimental motivation. Pavel Exner's work and his international collaborations advanced the mathematical investigation of quantum graphs, waveguides and spectral phenomena. Questions concerning operator spectra, geometrically induced bound states and resonances required advanced analytical tools. The research offered mathematically controlled models for studying situations in which geometry can modify physical behaviour. It also reinforced the importance of relationships between domestic research groups and specialised international networks. A rigorous model does not reproduce every detail of the system it represents, nor does it automatically imply an imminent commercial application. Its value lies in isolating

mechanisms, specifying assumptions and allowing conclusions to be assessed through proof. Such discipline is essential when mathematical models are used to interpret increasingly complex scientific processes. The prominence of Czech work in mathematical physics therefore connects conceptual originality with a broader culture of methodological precision.

Numerical mathematics broadened the range of questions that could be approached beyond the limits of analytical solutions. Czech research in finite-element techniques, discontinuous Galerkin methods, computational fluid dynamics and numerical linear algebra combined error analysis with algorithmic implementation. Work associated with Miloslav Feistauer and Vít Dolejší illustrates the close relationship between mathematical discretisation and physical modelling. The reliability of a computation depends upon choices concerning approximation, stability, convergence and the representation of uncertainty. Faster machines cannot compensate for an algorithm whose mathematical behaviour is poorly understood. Conversely, an elegant theoretical scheme has limited practical effect if it cannot be implemented efficiently for realistic problem sizes. The strongest computational programmes therefore unite analysis, software engineering and domain-specific scientific knowledge. Czech universities and research centres developed these capabilities through cooperation between mathematicians, engineers and computational specialists. This cooperation gradually made numerical methods a shared research language rather than an isolated technical service. This distinction remains important when evaluating the period as a whole.

The establishment and expansion of national supercomputing infrastructure altered the institutional scale of scientific computation. IT4Innovations in Ostrava introduced Anselm in 2013, followed by Salomon in 2015 and Barbora in 2019. The installation of Karolina in 2021 further extended the capacity available for simulation, data analysis and artificial-intelligence workloads. These successive systems represented more than increased processing speed, because each generation introduced new requirements for parallel algorithms and computational workflows. Large numerical problems demanded expertise in data movement, memory organisation, scalability and reproducible research practice. Access through European programmes also brought Czech researchers into wider communities sharing hardware and technical standards. Such infrastructure is valuable when scientific users can formulate meaningful questions and select appropriate mathematical tools. It cannot replace the education of researchers able to distinguish a plausible computational result from a reliable one. The long-term contribution of the Ostrava centre consequently depends as much on skills, open collaboration and methodological innovation as on the machines themselves.

The expansion of statistical and probabilistic methods provided a further connection between mathematical theory and contemporary scientific practice. Czech work on robust inference, structural change, multivariate analysis and stochastic processes addressed the difficulty of drawing defensible conclusions from imperfect information. Researchers including Jana Jurečková, Marie Hušková and Milan Studený contributed to fields that connect rigorous mathematical assumptions with the interpretation of observed data. Bayesian approaches to inference and decision-making offered additional ways of representing uncertainty within complex systems. These methods became relevant to biological research, engineering, economic modelling and information technologies. Nevertheless, the availability of larger

datasets does not remove problems of bias, dependence, measurement error or model misspecification. Mathematical statistics remains indispensable because it provides a disciplined account of what evidence can legitimately establish. Czech research in these areas therefore illustrates continuity between classical inference and the new demands of data-intensive science. It also challenges the notion that mathematical innovation necessarily consists of replacing older methods with computationally fashionable alternatives. This distinction remains important when evaluating the period as a whole.

Optimisation and operations research became increasingly prominent as Czech institutions addressed more complex industrial and organisational questions. Advances in nonlinear programming, equilibrium-constrained optimisation and semidefinite methods connected abstract mathematical structures with computational procedures. Research associated with Jiří Outrata and Michal Kočvara contributed to the mathematical understanding and practical treatment of difficult optimisation problems. Related developments in decision theory and stochastic optimisation provided methods for choosing under uncertainty rather than merely calculating a deterministic optimum. Their applications could extend to production systems, transport planning, energy management and resource allocation. Yet practical modelling inevitably includes judgements about objectives, constraints and what information is sufficiently reliable. A mathematically optimal solution is not necessarily socially desirable or operationally feasible when important considerations lie outside the model. Responsible application therefore requires continuing dialogue between mathematicians and specialists familiar with the underlying institutions. The Czech experience shows how mathematical rigour can improve decisions without pretending to eliminate the need for human judgement.

Artificial intelligence created new opportunities for interaction between mathematics, statistics and computer science during the final years of the period. Mathematical questions about approximation, optimisation, generalisation and uncertainty underlie the performance and limitations of machine-learning systems. Czech researchers contributed to these foundations through work on neural-network approximation, formal methods, probabilistic modelling and algorithmic structures. Increasing computational capacity allowed more demanding experiments with data-driven systems, but it also made careful validation more important. The explanatory strength of a model must be distinguished from its predictive performance on a particular dataset. Issues of robustness, reproducibility and interpretability cannot be solved by computational scale alone. The strongest contributions arise when statistical reasoning, theoretical analysis and algorithmic experimentation inform one another. In this sense, the development of artificial intelligence did not supersede classical mathematics but created new settings in which its principles were tested. The future scientific value of these systems will depend heavily on the continued development of their mathematical foundations. This distinction remains important when evaluating the period as a whole.

The Czech mathematical landscape became more geographically varied without ceasing to be uneven. Prague maintained an exceptionally dense concentration of universities, Academy institutes, senior researchers and international professional connections. Brno preserved major strengths in algebra, geometry, analysis and engineering-oriented mathematical research. Olomouc, Ostrava, Plzeň and České Budějovice cultivated distinct forms of expertise in areas

ranging from dynamics and mathematical physics to numerical methods and mathematical biology. These centres should not be treated simply as peripheral extensions of the capital. Their intellectual importance depends upon particular research programmes and the networks they sustain. At the same time, differences in institutional resources and recruitment capacity shape what smaller departments can undertake. Internationally successful specialisation may offer a more realistic strategy than reproducing every research discipline in every region. A productive national ecosystem requires mechanisms that connect these specialised strengths while respecting their local identities.

European integration transformed the practical geography of mathematical collaboration. Accession to the European Union in 2004 facilitated participation in research frameworks, cross-border projects and mobility arrangements. Czech institutions increasingly engaged with the wider European Research Area, where joint proposals and shared evaluation procedures became familiar aspects of scientific work. The importance of this change was not limited to the additional resources available through particular programmes. European networks could broaden the audiences for new results and introduce researchers to techniques developing elsewhere. They also enabled young mathematicians to pursue international experience without necessarily severing their relationships with Czech departments. Such circulation can support excellence when domestic institutions remain attractive places in which to conduct independent research. If opportunities abroad are not matched by credible pathways at home, however, mobility may intensify difficulties in retaining scientific talent. European integration therefore provided a framework of possibilities whose benefits depended upon the quality of national institutions and career conditions. This distinction remains important when evaluating the period as a whole.

Scientific distinction is particularly difficult to attribute within increasingly collaborative mathematical research. A paper with authors from several countries may incorporate crucial ideas produced through seminars, visits and exchanges that cross institutional boundaries. It would be misleading to assign the entire achievement to one country merely because an author held a Czech affiliation. It would be equally misleading to overlook Czech contributions simply because the final result emerged within an international team. A sound historical assessment must identify the individual contribution, the institutional setting and the collaborative process where the evidence permits. Such care is especially necessary when discussing prizes, influential theorems and internationally recognised monographs. Public recognition offers useful evidence of esteem but cannot substitute for analysis of the mathematical result itself. The Czech experience suggests that scientific identity and international collaboration are not competing categories. National research capacity is strengthened when researchers are able to contribute substantively to shared intellectual endeavours.

The development of funding arrangements was one of the most consequential structural changes after independence. Competitive grants increased the importance of clearly stated research questions, external evaluation and transparent project responsibilities. Czech Science Foundation programmes became a central element of this environment alongside institutional support and European funding opportunities. Competition can encourage originality and expose weaker assumptions in proposed research plans. Nevertheless, it cannot remove the

uncertainty inherent in fundamental mathematical discovery. A major breakthrough may emerge only after years of exploration with no predetermined sequence of deliverables. Frequent application cycles and short funding horizons can therefore impose costs that are less visible than the direct financial expenditure. Stable institutional financing and competitive grants should be understood as complementary instruments rather than mutually exclusive alternatives. Their appropriate balance is especially significant for mathematics, where intellectual continuity often matters more than access to expensive physical apparatus. This distinction remains important when evaluating the period as a whole.

The sustainability of scientific careers presents an equally important challenge. Doctoral students and postdoctoral researchers constitute the foundation from which future mathematical schools will emerge. They require not only technical training but also opportunities to formulate questions, present arguments and receive informed criticism. Short-term appointments may provide valuable mobility yet can make it difficult to maintain ambitious research programmes or plan a career. International salary differences and the attraction of other professional sectors can intensify competition for talented graduates. The growth of data science and artificial intelligence has created additional employment opportunities for mathematically trained specialists. Such opportunities benefit society but may leave universities struggling to recruit experienced teachers and researchers. Institutions must therefore offer credible professional development rather than rely entirely on the intellectual appeal of mathematics. The continued strength of Czech research will depend upon the conditions under which capable young scholars can build sustainable careers.

The relationship between mathematical education and scientific research deserves sustained attention in any assessment of future capacity. Advanced research depends upon strong preparation in proof, abstraction, computation and critical reasoning. University curricula must provide these foundations without treating newer fields as alternatives to mathematical depth. Students who understand linear algebra, analysis, probability and discrete structures are better prepared to evaluate changing computational technologies. Equally, exposure to genuine research problems can make foundational courses intellectually meaningful rather than merely procedural. International doctoral networks and visiting scholars broaden the scientific experience available within domestic programmes. Research seminars and specialist journals preserve forms of scholarly conversation that cannot be replaced by examination requirements or software tutorials. Modernisation of mathematical education should consequently combine rigorous disciplinary content with openness to interdisciplinary methods. The central objective is to prepare researchers capable of developing new knowledge rather than only applying established techniques. This distinction remains important when evaluating the period as a whole.

Interdisciplinary research has changed the social role of mathematicians without dissolving their disciplinary identity. Problems in climate science, medicine, materials research, transport and economics increasingly require formal models and quantitative analysis. Mathematical contributions can clarify assumptions, identify limiting cases and expose contradictions that may be hidden within informal explanations. They can also reveal when the available data are insufficient to support the predictions demanded of a model. Such work is most effective when

domain specialists participate throughout the research process rather than merely supplying data at its beginning. Mathematicians, in turn, must remain willing to revise idealised assumptions when those assumptions misrepresent the relevant physical or social process. This collaboration does not make all research immediately applicable or deprive foundational inquiry of independent value. It instead enlarges the range of questions that mathematical thinking can approach responsibly. Czech research institutions have increasingly demonstrated this potential through joint projects linking analysis, computation and substantive scientific problems.

The expansion of research infrastructure also raises questions about equitable access and the stewardship of public resources. Supercomputers require continuing investment in electricity, technical expertise, software and replacement equipment long after initial construction. Their effectiveness depends on accessible allocation procedures, support for inexperienced users and reliable research-data practices. Smaller academic groups may need additional training to convert theoretical questions into computational projects suited to large systems. Otherwise, expensive facilities risk benefiting chiefly those teams already possessing advanced technical capacity. European cooperation can alleviate some limitations by sharing expertise and resources across national borders. Yet international access arrangements do not remove the need for strong domestic scientific communities. The decisive question is whether infrastructure produces lasting capabilities in mathematics, computation and neighbouring disciplines. Investment should therefore be evaluated through the quality of research and skills it enables, not only the peak performance of installed machines. This distinction remains important when evaluating the period as a whole.

An accurate account of mathematical innovation must distinguish conceptual originality from methodological adoption and technological implementation. The proof of a new theorem can transform the understanding of a mathematical field without generating a device or a commercial product. An algorithm may provide a different kind of advance by making a previously infeasible calculation practical. A modelling framework can introduce valuable knowledge by relating disparate observations within an explicit structure of assumptions. Finally, institutional innovations may permit forms of collaboration or computation that were previously unavailable. These achievements differ in their evidence, time horizons and modes of influence. Treating them as equivalent would obscure the characteristic contribution of mathematics to scientific development. Czech examples across graph theory, nonlinear analysis, spectral theory, optimisation and high-performance computing reveal the importance of maintaining such distinctions. The historical achievement lies in building a system capable of supporting several forms of innovation simultaneously.

The years 2020 to 2025 brought the relevance of mathematical reasoning into particularly sharp public focus. Pandemic modelling exposed both the practical usefulness and the limitations of quantitative scenarios under rapidly changing conditions. Expanding artificial-intelligence applications increased demand for mathematical explanations of reliability, uncertainty and generalisation. Growing computational resources widened the scope of simulations while creating more demanding questions about verification and energy use. These developments placed pressure on researchers to communicate their findings to audiences unfamiliar with the

assumptions of advanced models. Such communication is an intellectual responsibility rather than a secondary public-relations exercise. It requires distinguishing established results from conjectures, model-based projections and provisional computational observations. The strength of mathematical reasoning lies partly in its capacity to make these distinctions explicit. Czech mathematics entered the mid-2020s with a mature foundation from which to respond to these challenges, although future achievements remained necessarily uncertain. This distinction remains important when evaluating the period as a whole.

The three decades examined in this volume reveal neither uninterrupted progress nor a simple transition from theoretical scholarship to technological utility. Different areas of mathematical research developed at different speeds, and their opportunities were shaped by unequal institutional resources. Some fields attracted wider attention through prominent international results, whereas others progressed through quieter accumulation of theorems and techniques. Large computational infrastructures transformed selected methods more dramatically than they changed research in abstract algebra or geometry. European programmes expanded collaboration while also exposing institutions to intensified competition and administrative burdens. The concentration of expertise in established centres could foster excellence yet create obstacles for smaller regional departments. These tensions belong to the history of Czech mathematics and should not be edited out of an account devoted to innovation. A serious evaluation must recognise both achievements and the structural conditions required to sustain them. The overall picture is therefore one of productive development shaped by persistent institutional choices rather than automatic modernisation.

The defining strength of Czech mathematics between 1993 and 2025 was its capacity to sustain foundational intellectual traditions while engaging new international and computational environments. Research in analysis, geometry, logic and algebra continued alongside substantial achievements in combinatorics, probability, optimisation and mathematical physics. Academy institutes and universities supported different but interconnected forms of scientific work. Regional specialisation broadened the national research landscape, while European collaboration placed Czech scholars within larger intellectual networks. Developments in numerical modelling, statistics and high-performance computing extended the practical reach of mathematical methods. These transformations did not make fundamental mathematics subordinate to application, because each depends upon the other in ways that cannot be reduced to a single direction of influence. The most durable innovations emerged where conceptual depth, methodological rigour and institutional continuity reinforced one another. Maintaining this relationship will require reliable support for long-term research, attractive careers for younger scholars and sustained openness to international exchange. Czech mathematics thus offers an instructive example of how a relatively compact national scientific community can contribute original knowledge while forming an integral part of the contemporary European research ecosystem. This distinction remains important when evaluating the period as a whole.

Chronological Timeline 1993–2025

1993, Institutional reorganisation after independence

On 1 January the Czech Republic became an independent state and the Mathematical Institute entered the new Academy of Sciences of the Czech Republic. Academic departments and journals preserved research continuity across the dissolution of Czechoslovakia. The new national framework gradually strengthened competitive funding and international evaluation.

1994, Category theory and mathematical structures

Jiří Adámek and Jiří Rosický published *Locally Presentable and Accessible Categories* with Cambridge University Press. The book systematically related category theory to universal algebra, logic and theoretical computer science, illustrating the continuing international reach of the Brno mathematical school.

1995, Consolidation of established research programmes

Mathematical groups in Prague and Brno continued analytical, geometric and algebraic research within the new national grant environment. The historical significance of the mid-1990s rests more in continuity of research institutions and publication outlets than in one single national breakthrough assigned to this year.

1996, Formation of an international discrete-mathematics network

The DIMATIA initiative emerged at Charles University as a framework linking discrete mathematics and theoretical informatics with international research partners. Its institutional development prepared a more regular programme of seminars, visiting scholars and collaborative activities beginning the following year.

1997, DIMATIA and EQUADIFF 9

DIMATIA began regular activity in January, and its offices were formally inaugurated in Prague in May. Brno hosted the ninth EQUADIFF conference, sustaining an international forum for differential equations, dynamical systems and their mathematical methods.

1998, A landmark in mathematical fuzzy logic

Petr Hájek published *Metamathematics of Fuzzy Logic*, a major systematic treatment of formal deduction in many-valued logics. It helped establish the mathematical foundations of reasoning with vagueness and reinforced the international visibility of Prague research in logic.

1999, Continuation of international integration

Czech mathematical researchers continued to expand journal publishing, exchanges and participation in international scientific meetings. This entry records a gradual transformation rather than an independently dated institutional foundation or mathematical discovery.

2000, Growth of cross-disciplinary mathematical research

At the turn of the century, mathematical analysis, differential equations and numerical methods became increasingly connected with engineering computation and mathematical informatics. Existing academic programmes and specialist journals provided continuity while research questions became more interdisciplinary.

2001, EQUADIFF 10 in Prague

Prague hosted the tenth EQUADIFF conference. Its published proceedings document Czech participation in international research discussions on differential equations and applied mathematical analysis.

2002, Fifty years of the Mathematical Institute

A historical article marking the Mathematical Institute's fiftieth anniversary documented its development and scientific leadership, including the transition into the Czech Academy in 1993. The anniversary provided a contemporary record of institutional continuity extending from the Czechoslovak to the independent Czech period.

2003, Pre-accession consolidation

Czech mathematicians and their institutions prepared for deeper participation in European research while maintaining established international scholarly networks. This was a period of incremental organisational integration rather than a single dated mathematical event.

2004, European Union accession and graph homomorphisms

The Czech Republic entered the European Union on 1 May, expanding opportunities for cross-border research cooperation. Pavol Hell and Jaroslav Nešetřil published "Graphs and Homomorphisms" with Oxford University Press, helping consolidate a central framework of modern structural graph theory.

2005, Legal framework for public research institutions

Act No. 341/2005 Coll. established the legislative framework for public research institutions. Its adoption was important for the subsequent legal and administrative organisation of Academy institutes, including institutions supporting mathematics.

2006, Institutional and research consolidation

Research groups continued to develop programmes in geometry, functional analysis, mathematical physics, discrete mathematics and applications. The developing public-research-institution framework was part of the broader transformation in scientific governance.

2007, Public research institution model

The post-2005 legislative framework underpinned the transition of Academy institutes to the public research institution model. This period strengthened formal administrative arrangements for fundamental research and long-term scientific programmes.

2008, European collaborative research becomes routine

Participation in multinational research partnerships and interdisciplinary programmes was increasingly embedded in Czech academic mathematics. The wider period also saw sustained cooperation linking mathematical methods to computer science and engineering.

2009, EQUADIFF 12 in Brno

Brno hosted the twelfth EQUADIFF conference, documented in the Czech Digital Mathematics Library. The meeting reinforced the position of the Czech mathematical community in the international study of differential equations and related dynamics.

2010, Research infrastructure preparation

As the European framework for larger-scale research facilities developed, Czech scientific organisations increasingly planned shared computational resources. Numerical simulation and mathematical modelling were among the fields positioned to benefit from this infrastructure-led approach.

2011, IT4Innovations begins

The IT4Innovations centre was established in Ostrava, providing a new institutional basis for high-performance computing and data-intensive research. Its origins demonstrate how

mathematical modelling, algorithms and scientific computing became increasingly connected to national infrastructure policy.

2012, Sparsity as a unified research programme

Jaroslav Nešetřil and Patrice Ossona de Mendez published “Sparsity: Graphs, Structures, and Algorithms”. The monograph assembled a powerful framework for sparse graphs and relational structures with applications to algorithm design, complexity and mathematical logic.

2013, Anselm and EQUADIFF 13

The Anselm supercomputer was made available at IT4Innovations, enabling larger-scale numerical simulations and computational studies. Prague also hosted EQUADIFF 13, sustaining Czech participation in an established international mathematical conference series.

2014, New national computing facilities

IT4Innovations formally opened its dedicated research building in Ostrava. The centre’s development increasingly connected parallel algorithms, numerical computation and simulation with researchers in materials science, engineering and other disciplines.

2015, Launch of Salomon

The Salomon supercomputer entered operation at IT4Innovations. Its capacity expanded the scale of computations available to research teams using numerical models, algorithmic methods and high-volume scientific data.

2016, European high-performance computing networks

IT4Innovations became involved in the European Technology Platform for High Performance Computing (ETP4HPC). Such engagement connected Czech scientific computing to European technical strategies and collaborative research networks.

2017, Consolidation of numerical and data-intensive research

Czech researchers increasingly integrated simulation, optimisation and data analysis into interdisciplinary research programmes. This continuing trend is recorded as a process, rather than attributed to one specific national discovery.

2018, Entry into the EuroHPC framework

The Czech Republic joined the European High Performance Computing Joint Undertaking (EuroHPC JU), and IT4Innovations participated in its activities. This move strengthened the institutional context for European-scale computational research and future joint infrastructure.

2019, Barbora, artificial intelligence hardware and European agreements

IT4Innovations launched the Barbora supercomputer and installed a specialised NVIDIA DGX-2 system supporting artificial intelligence computation. An agreement was signed to host a future EuroHPC co-funded petascale supercomputer in Ostrava.

2020, Computational infrastructure during a disruptive year

IT4Innovations developed its national competence activities in high-performance computing. The COVID-19 period placed renewed emphasis on computational modelling, data analysis and collaborative digital research, while its effects varied across fields.

2021, Karolina enters operation

The Karolina supercomputer began operating at IT4Innovations. It substantially enlarged the computational resources available for numerical mathematics, machine learning and multidisciplinary simulation, while earlier systems Anselm and Salomon reached the end of their operation.

2022, EQUADIFF 15 and the LUMI infrastructure

Brno hosted EQUADIFF 15, documented in the Czech Digital Mathematics Library. The international LUMI supercomputer was inaugurated in Finland, strengthening a European computing ecosystem in which Czech institutions participated.

2023, Maturing European computational collaboration

Czech mathematical and computational research continued to develop through national and European infrastructures, open research access and interdisciplinary numerical projects. This entry describes the institutional trajectory rather than a singular dated theorem or award.

2024, Continuing convergence of mathematics and computation

Mathematical optimisation, scientific computing, probabilistic methods and data-driven algorithms remained increasingly relevant to research programmes in Czechia. Long-standing

areas such as pure analysis, geometry and structural combinatorics continued alongside these applications.

2025, International recognition and a quantum-computing milestone

Jaroslav Nešetřil and Patrice Ossona de Mendez received the EATCS Nerode Prize for pioneering research on graph sparsity. On 23 September, the VLQ quantum computer was inaugurated at IT4Innovations in Ostrava, introducing an important new experimental platform for quantum algorithms and optimisation. The prize recognises mathematical and algorithmic achievement; the computing inauguration is an infrastructure milestone, not a claim of a new mathematical theorem.

Major Czech Mathematical Institutions and Research Centres, 1993–2025

Institutes of the Czech Academy of Sciences

Institute of Mathematics of the Czech Academy of Sciences

Location. Prague; additional mathematical research activity in regional academic networks.

Research specialisation. Mathematical analysis, differential equations, functional spaces, geometry, topology, numerical mathematics and mathematical physics.

Role in the Czech research system. A principal institution for fundamental mathematical research, continuity of specialist schools and university collaboration.

Institute of Computer Science of the Czech Academy of Sciences

Location. Prague.

Research specialisation. Theoretical computer science, mathematical foundations of computation, artificial intelligence, statistics, computer models and interdisciplinary data science.

Role in the Czech research system. Connects foundational mathematics with algorithms, computational systems and statistical modelling across academic disciplines.

Institute of Information Theory and Automation of the Czech Academy of Sciences

Location. Prague.

Research specialisation. Probability, mathematical statistics, optimisation, system science, information theory, decision-making, image analysis and artificial intelligence.

Role in the Czech research system. Provides a bridge between theoretical quantitative methods, control systems and research applications in engineering and society.

Prague University Schools and Specialist Units

Charles University — Faculty of Mathematics and Physics

Location. Prague.

Research specialisation. Mathematical analysis, algebra, geometry, mathematical statistics, numerical analysis, discrete mathematics and theoretical computer science.

Role in the Czech research system. A leading national centre of fundamental mathematics and doctoral training, linking university departments with international research communities.

Mathematical Institute of Charles University

Location. Prague (Karlín).

Research specialisation. Analysis and geometry, mathematical modelling, continuum mechanics, and mathematical aspects of fluid flow.

Role in the Czech research system. A constituent institute of Charles University that sustains analytical and geometric research traditions and specialist international meetings.

Charles University — Department of Applied Mathematics and associated theoretical-computer-science groups

Location. Prague (Malá Strana).

Research specialisation. Graph theory, combinatorics, algorithm design, optimisation, geometry and topological methods.

Role in the Czech research system. A university research environment associated with the Prague school of discrete mathematics and international work on algorithms.

Czech Technical University in Prague — Faculty of Nuclear Sciences and Physical Engineering

Location. Prague.

Research specialisation. Mathematical physics, applied mathematics, mathematical modelling and mathematical methods for physics and engineering.

Role in the Czech research system. Trains mathematical scientists whose work connects operator methods, quantum models and numerical techniques with experimental sciences.

Czech Technical University in Prague — mathematical and computational engineering departments

Location. Prague.

Research specialisation. Numerical methods, applied analysis, modelling of structures and engineering systems, optimisation and computational mechanics.

Role in the Czech research system. An institution-wide bridge between mathematical research and engineering design, simulation and scientific computation.

Doppler Institute for Mathematical Physics and Applied Mathematics

Location. Prague, associated with the Czech Technical University.

Research specialisation. Spectral and operator methods, mathematical physics, quantum systems and associated seminars and research projects.

Role in the Czech research system. A specialised research framework for interaction between mathematical physics, Czech universities and international groups.

Brno University Schools

Masaryk University — Faculty of Science, Department of Mathematics and Statistics

Location. Brno.

Research specialisation. Pure and applied mathematics, algebra, geometry, mathematical analysis, differential equations and statistics.

Role in the Czech research system. A principal university centre of the Brno mathematical tradition, combining research, doctoral training and international scholarly exchange.

Masaryk University — Faculty of Informatics

Location. Brno.

Research specialisation. Theoretical informatics, formal methods, algorithms, logic, data and computational methods.

Role in the Czech research system. Connects mathematical structures and algorithmic research to modern computer science and interdisciplinary training.

Brno University of Technology — Institute of Mathematics, Faculty of Mechanical Engineering

Location. Brno.

Research specialisation. Finite-element methods, numerical differential equations, optimisation, applied algebra, discrete mathematics and image processing.

Role in the Czech research system. A specialised applied-mathematics institute linking mathematical theory with mechanical engineering and industrial processes.

Brno University of Technology — NETME Centre

Location. Brno.

Research specialisation. Mathematical modelling in engineering, mechanics, computational analysis and applied research for industry.

Role in the Czech research system. An interdisciplinary technology centre with mathematical and mechanics divisions, illustrating university–industry research translation.

CERIT-SC, Masaryk University

Location. Brno.

Research specialisation. Research computing, scientific software, complex data processing and computational support for interdisciplinary projects.

Role in the Czech research system. A regional computing pillar connected with the national e-INFRA CZ infrastructure.

Regional Mathematical Schools

Palacký University Olomouc — Faculty of Science

Location. Olomouc.

Research specialisation. Algebraic structures, geometry, mathematical analysis, applied mathematics, dynamical systems and mathematics education.

Role in the Czech research system. An important Moravian centre of mathematical teaching and research with identifiable algebra, geometry and analysis departments.

University of Ostrava — Faculty of Science, Department of Mathematics

Location. Ostrava.

Research specialisation. Mathematics, geometrical and theoretical investigations, mathematical modelling and fuzzy or non-classical approaches.

Role in the Czech research system. A regional university school supporting research projects, mathematical conferences and graduate training in north-eastern Czechia.

University of West Bohemia — Faculty of Applied Sciences, Department of Mathematics

Location. Plzeň.

Research specialisation. Applied mathematics, computational mathematics, numerical techniques, mathematical structures and mathematics for engineering.

Role in the Czech research system. Connects mathematics with the technical and industrial research traditions of western Bohemia.

NTIS — New Technologies for the Information Society Research Centre

Location. Plzeň, University of West Bohemia.

Research specialisation. Mathematical structures, modelling, mechanics, cybernetics, informatics and geospatial data methods.

Role in the Czech research system. An interdisciplinary centre established within the university research system, enabling joint mathematics-and-engineering projects.

University of South Bohemia in České Budějovice — Faculty of Science

Location. České Budějovice.

Research specialisation. Mathematical methods in biology, ecological modelling, quantitative science and teaching of mathematics.

Role in the Czech research system. Supports the application of mathematical models in life and environmental sciences within a regionally specialised university.

Silesian University in Opava — Mathematical Institute and related faculty groups

Location. Opava.

Research specialisation. Mathematics and mathematical methods supporting interdisciplinary and theoretical research.

Role in the Czech research system. Contributes to the regional network of mathematical education and scholarship in Czech Silesia.

National Computational and Collaborative Infrastructure

IT4Innovations National Supercomputing Center, VSB — Technical University of Ostrava

Location. Ostrava.

Research specialisation. High-performance computing, scalable numerical algorithms, simulation, data analysis, artificial intelligence and quantum computing as developed by 2025.

Role in the Czech research system. A national user infrastructure which broadened the feasible scale of computational mathematics and enabled Czech participation in European computing programmes.

CESNET Association and the e-INFRA CZ research infrastructure

Location. Prague and national network; partners in Brno and Ostrava.

Research specialisation. Research networking, distributed computing, storage, data services and access to shared scientific computational resources.

Role in the Czech research system. Provides the digital infrastructure linking institutions and supporting mathematical and computational research at a national scale.

References

Adam, L., & Branda, M. (2021). Risk-aversion in data envelopment analysis models with diversification. *Omega*, *102*, Article 102338.

Adam, L., Branda, M., Heitsch, H., & Henrion, R. (2020). Solving joint chance constrained problems using regularization and Benders' decomposition. *Annals of Operations Research*, *292*, 683–709.

Adámek, J., & Rosický, J. (1993). On injectivity in locally presentable categories. *Transactions of the American Mathematical Society*, *336*(2), 785–804.

Adámek, J., & Rosický, J. (1994). *Locally presentable and accessible categories*. Cambridge University Press.

Adámek, J., & Rosický, J. (1994). *Locally presentable and accessible categories*. Cambridge University Press.

Adámek, J., Herrlich, H., & Strecker, G. E. (1990). *Abstract and concrete categories*. Wiley.

Adámek, J., Herrlich, H., & Strecker, G. E. (2006). Abstract and concrete categories: The joy of cats. *Reprints in Theory and Applications of Categories*, *17*, 1–507.

Andrle, M., Hlédik, T., Kameník, O., & Vlček, J. (2009). *Implementing the new structural model of the Czech National Bank* (Working Paper No. 2/2009). Czech National Bank.

Antoch, J., & Hušková, M. (2001). Permutation tests in change point analysis. *Statistics & Probability Letters*, *53*(1), 37–46.

Artificial Intelligence Center, Czech Technical University in Prague. (n.d.). Optimization.

Barto, L., & Kozik, M. (2012). Absorbing subalgebras, cyclic terms, and the constraint satisfaction problem. *Logical Methods in Computer Science*, *8*(1), Article 7.

Barto, L., & Kozik, M. (2014). Constraint satisfaction problems solvable by local consistency methods. *Journal of the ACM*, *61*(1), Article 3.

Barto, L., & Pinsker, M. (2016). The algebraic dichotomy conjecture for infinite domain constraint satisfaction problems. *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science*, 615–622.

Brand, C., Koutecký, M., & Ordyniak, S. (2021). Parameterized algorithms for MILPs with small treedepth. *Proceedings of the AAAI Conference on Artificial Intelligence*, *35*(14), 12249–12257.

Branda, M. (2018). Distributionally robust fixed interval scheduling on parallel identical machines under uncertain finishing times. *Computers & Operations Research*, *98*, 231–239.

Branda, M., & Matoušková, M. (2024). A Lagrangian relaxation algorithm for stochastic fixed interval scheduling problem with non-identical machines and job classes. *Computers & Operations Research*, *164*, Article 106542.

Branda, M., Henrion, R., & Pištěk, M. (2023). Value at risk approach to producer's best response in electricity market with uncertain demand. *Optimization*, *72*(11), 2745–2767.

Brno University of Technology. (n.d.). AdMaS centre. Faculty of Civil Engineering.

Brno University of Technology. (n.d.). Centre of Sensor, Information and Communication Systems.

Brno University of Technology. (n.d.). Institute of Mathematics. Faculty of Mechanical Engineering.

Brno University of Technology. (n.d.). Operational Programme Research and Development for Innovation: IT4Innovations Centre of Excellence.

Brno University of Technology. (n.d.). Operational programme Research and Development for Innovation.

Brno University of Technology. (n.d.). Research centres. Faculty of Electrical Engineering and Communication.

Brzeźniak, Z., Maslowski, B. & Seidler, J. Stochastic nonlinear beam equations. *Probability Theory and Related Fields*, *132*, 119–149

Bulíček, M., Gwiazda, P., Málek, J., & Świerczewska-Gwiazda, A. (2012). On unsteady flows of implicitly constituted incompressible fluids. *SIAM Journal on Mathematical Analysis*, *25*, Article 72.

Bulíček, M., Málek, J., & Maringová, E. (2023). On unsteady internal flows of incompressible fluids characterized by implicit constitutive equations in the bulk and on the boundary. *Journal of Mathematical Fluid Mechanics*, *25*, Article 72.

Calderbank, D. M. J., Slovák, J., & Souček, V. (2021). Subriemannian metrics and the metrizability of parabolic geometries. *Journal of Geometric Analysis*, *31*, 1671–1702.

Čap, A., & Slovák, J. (2003). Weyl structures for parabolic geometries. *Mathematica Scandinavica*, *93*(1), 53–90.

Čap, A., & Slovák, J. (2009). *Parabolic geometries I: Background and general theory* (Mathematical Surveys and Monographs, Vol. 154). American Mathematical Society.

Čap, A., & Slovák, J. (2009). Parabolic geometries I: Background and general theory. *American Mathematical Society*, *154*.

Čap, A., Slovák, J., & Žádník, V. (2004). On distinguished curves in parabolic geometries. *Transformation Groups*, *9*(2), 143–166.

Central European Institute of Technology. (n.d.). CEITEC BUT.

Centre of Sensor, Information and Communication Systems. (n.d.). History.

Centre of Sensor, Information and Communication Systems. (n.d.). Research groups.

CESNET. (2023). Annual report 2023.

Chamber of Deputies of the Parliament of the Czech Republic. (1993). Report on the first round of the Czech Grant Agency competition (Parliamentary print 774).

Charles University, Faculty of Mathematics and Physics. (1999). KMA preprint series: 1999.

Charles University, Faculty of Mathematics and Physics. (2025). Prestižní Nerodova cena za průkopnickou práci v teorii grafů [Prestigious Nerode Prize for pioneering work in graph theory].

Charles University, Faculty of Mathematics and Physics. (n.d.). Department of Applied Mathematics: About the department.

Charles University, Faculty of Mathematics and Physics. (n.d.). Department of Probability and Mathematical Statistics: About the department.

Charles University, Faculty of Mathematics and Physics. (n.d.). DIMATIA: Personnel in 1996.

Charles University, Faculty of Mathematics and Physics. (n.d.). DIMATIA: Year 1997.

Charles University, Faculty of Mathematics and Physics. (n.d.). Institute for Theoretical Computer Science: About the institute.

Charles University, Faculty of Mathematics and Physics. (n.d.). Jaroslav Nešetřil.

Charles University, Faculty of Mathematics and Physics. (n.d.). Mathematical Institute of Charles University.

Charles University, Faculty of Mathematics and Physics. (n.d.). The Prague seminar on function spaces: Archive of lectures.

Charles University, Faculty of Mathematics and Physics. (n.d.). Theoretical computer science.

Charles University, Faculty of Science. (n.d.). The Faculty of Science since foundation to 1989.

Charles University. (n.d.). Key dates in the history of Charles University.

Charles University. (n.d.). *Regularity of solutions of Navier–Stokes equations* [Course description]. Charles University Press.

Charles University. (n.d.). Theoretical computer science and combinatorics.

Cheon, T., & Exner, P. (2004). An approximation to δ' couplings on graphs. *Journal of Physics A: Mathematical and General*, *37*(29), L329–L335.

Cianchi, A., & Pick, L. (2003). Sobolev embeddings into spaces of Campanato, Morrey, and Hölder type. Journal of Mathematical Analysis and Applications, 282(1), 128–150.

Cianchi, A., Kerman, R., Opic, B., & Pick, L. (2000). A sharp rearrangement inequality for the fractional maximal operator. *Studia Mathematica*, *138*(3), 277–284.

Cintula, P., & Noguera, C. (2021). *Logic and implication: An introduction to the general algebraic study of non-classical logics*. Springer.

Cintula, P., Hájek, P., & Noguera, C. (Eds.). (2011). *Handbook of mathematical fuzzy logic* (Vols. 1–2). College Publications.

City of Brno. (n.d.). Scientific research centres in Brno.

Czech Academy of Sciences. (n.d.). Act No. 283/1992 Coll., on the CAS.

Czech Academy of Sciences. (n.d.). History overview.

Czech Academy of Sciences. (n.d.). Representatives of the CAS and its predecessors.

Czech Digital Mathematics Library. (n.d.). EQUADIFF proceedings collection.

Czech Digital Mathematics Library. (n.d.). Probability theory and stochastic processes: Subject bibliography.

Czech Institute of Informatics, Robotics and Cybernetics. (n.d.). Automated reasoning group.

Czech Institute of Informatics, Robotics and Cybernetics. (n.d.). Machine learning group.

Czech Institute of Informatics, Robotics and Cybernetics. (n.d.). Members of the machine learning group.

Czech Republic. (1998). Act No. 111/1998 Coll., on higher education institutions (original text).

Czech Republic. (2005). Act No. 341/2005 Coll., on public research institutions. Collection of Laws of the Czech Republic.

Czech Science Foundation. (2023, September 1). Czech Science Foundation celebrates 30th anniversary.

Czech Science Foundation. (n.d.). Annual reports.

Czech Science Foundation. (n.d.). GACR in profile.

Czech Science Foundation. (n.d.). Types of grant projects.

Czech Technical University in Prague, Faculty of Civil Engineering. (n.d.). Department of Mathematics.

Czech Technical University in Prague, Faculty of Mechanical Engineering. (n.d.). Research activities, Department of Technical Mathematics.

Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering. (n.d.). Department of Mathematics.

Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering. (n.d.). Research fields.

Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering. (n.d.). Research centres.

Czech Technical University in Prague, Faculty of Nuclear Sciences and Physical Engineering, Mathematical Modelling Group. (n.d.). Publications.

DIMATIA. (n.d.). *Centre for Discrete Mathematics, Theoretical Computer Science and Applications*. Charles University.

DIMATIA. (n.d.). Personnel of DIMATIA. Charles University.

DIMATIA. (n.d.). Year 1997: Overview of events. Charles University.

Dodge, Y., & Jurečková, J. (2000). *Adaptive regression*. Springer.

Dolejší, V., & Feistauer, M. (2015). *Discontinuous Galerkin method: Analysis and applications to compressible flow*. Springer.

Doppler Institute for Mathematical Physics and Applied Mathematics. (n.d.). About Doppler Institute.

Doppler Institute for Mathematical Physics and Applied Mathematics. (n.d.). Publications.

Doubrov, B., & Slovák, J. (2010). Inclusions between parabolic geometries. *Pure and Applied Mathematics Quarterly*, *6*(3), 755–780.

Drábek, P., & Simader, C. G. (2002). On general solvability properties of p-Laplacian-like equations. *Mathematica Bohemica*, *127*(1), 103–122.

Edmunds, D. E., Kerman, R., & Pick, L. (2000). Optimal Sobolev imbeddings involving rearrangement-invariant quasinorms. *Journal of Functional Analysis*, *170*(2), 307–355.

EuroHPC Joint Undertaking. (2024). About EuroHPC JU.

European Commission. (2007). Commission staff working document accompanying the Green Paper “The European Research Area: New perspectives” (SEC(2007) 412).

European Commission. (2007). The European Research Area: New perspectives (Green Paper, COM(2007) 161 final).

European Commission. (2021). Horizon Europe: The EU research and innovation programme (2021–2027).

European Commission. (2024, April 30). 20 years together: EU celebrates 2004 enlargement.

European Commission. (n.d.). European Research Area.

European Commission. (n.d.). Support for research and innovation in the Czech Republic.

European High Performance Computing Joint Undertaking. (n.d.). European high performance computing.

European Mathematical Society. (2021). Institute of Mathematics of the Czech Academy of Sciences. EMS Magazine.

European Parliament and Council of the European Union. (2006). Decision No 1982/2006/EC concerning the Seventh Framework Programme of the European Community for research, technological development and demonstration activities (2007–2013). *Official Journal of the European Union*, *L 412*, 1–41.

European Union. (2003). Treaty concerning the accession of the Czech Republic and nine other states to the European Union. Official Journal of the European Union, L 236.

Exner, P. (n.d.). Doppler Institute (historical archive, 1993–2005). Czech Technical University in Prague.

Exner, P. (n.d.). *List of scientific publications*. Czech Technical University in Prague.

Exner, P., & Ichinose, T. (2005). A product formula related to quantum Zeno dynamics. *Annales Henri Poincaré*, *6*, 195–215.

Exner, P., & Kovařík, H. (2015). *Quantum waveguides*. Springer.

Exner, P., & Post, O. (2005). Convergence of spectra of graph-like thin manifolds. *Journal of Geometry and Physics*, *54*(1), 77–115.

Exner, P., & Šeba, P. (1989). Bound states in curved quantum waveguides. *Journal of Mathematical Physics*, *30*(11), 2574–2580.

Exner, P., Kostenko, A., Malamud, M., & Neidhardt, H. (2018). Spectral theory of infinite quantum graphs. *Annales Henri Poincaré*, *19*, 3457–3510.

Faculty of Mathematics and Physics, Charles University. (2022). 70 years of the Faculty of Mathematics and Physics.

Faculty of Mathematics and Physics, Charles University. (2025, September 24). Prestižní Nerodova cena za průkopnickou práci v teorii grafů [Prestigious Nerode Prize for pioneering work in graph theory].

Feireisl, E. (2004). *Dynamics of viscous compressible fluids*. Oxford University Press.

Feireisl, E. (2009). On compactness of the velocity field in the incompressible limit of the full Navier–Stokes–Fourier system on large domains. *Mathematical Methods in the Applied Sciences*, *32*(10), 1269–1286.

Feireisl, E., & Málek, J. (2006). On the Navier–Stokes equations with temperature-dependent transport coefficients. *Differential Equations and Nonlinear Mechanics*, Article 90616.

Feireisl, E., & Novotný, A. (2009). *Singular limits in thermodynamics of viscous fluids*. Birkhäuser.

Feireisl, E., Lu, Y., & Málek, J. (2016). On PDE analysis of flows of quasi-incompressible fluids. *ZAMM — Journal of Applied Mathematics and Mechanics*, *96*(4), 491–508.

Feireisl, E., Novotný, A., & Sun, Y. (2014). *A regularity criterion for the weak solutions to the Navier–Stokes–Fourier system*. [Research publication]. Charles University Research Information System.

Feistauer, M. (1993). *Mathematical methods in fluid dynamics*. Longman Scientific & Technical.

Feistauer, M. (n.d.). *Publications since 1990*. Charles University.

Feistauer, M., Felcman, J., & Straškraba, I. (2003). *Mathematical and computational methods for compressible flow*. Oxford University Press.

Foltyn, L., Lukáš, D., & Peterek, I. (2020). Domain decomposition methods coupled with parareal for the transient heat equation in 1 and 2 spatial dimensions. *Applications of Mathematics*, *65*(2), 173–190.

Gfrerer, H., & Outrata, J. V. (2021). On a semismooth' Newton method for solving generalized equations. *SIAM Journal on Optimization*, 31(1), 489–517.

Goldys, B., & Maslowski, B. (2001). Uniform exponential ergodicity of stochastic dissipative systems. *Czechoslovak Mathematical Journal*, *51*(4), 745–762.

Gover, A. R., & Slovák, J. (2022). Non-holonomic equations for the normal extremals in geometric control theory. *Journal of Geometry and Physics*, *171*, Article 104395.

Hájek, P. (1998). *Metamathematics of fuzzy logic*. Kluwer Academic Publishers.

Hájek, P. (2006). What is mathematical fuzzy logic. *Fuzzy Sets and Systems*, *157*(5), 597–603.

Hájek, P., & Holeňa, M. (2003). Formal logics of discovery and hypothesis formation by machine. *Theoretical Computer Science*, *292*, 345–357.

Hájek, P., & Pudlák, P. (1993). *Metamathematics of first-order arithmetic*. Springer.

Hell, P., & Nešetřil, J. (2004). *Graphs and homomorphisms*. Oxford University Press.

Holický, P. (2001). Generalized analytic spaces, completeness and fragmentability. *Czechoslovak Mathematical Journal*, *51*(4), 791–818.

Hrušková, P., Dostál, Z., Vlach, O., & Vodstrčil, P. (2025). On multipoint constraints in FETI methods. *Applications of Mathematics*, *70*(1), 47–64.

Hubička, J., & Nešetřil, J. (2019). All those Ramsey classes (Ramsey classes with closures and forbidden homomorphisms). *Advances in Mathematics*, *356*, Article 106791.

Hušková, M., & Kirch, C. (2008). Bootstrapping confidence intervals for the change-point of time series. *Journal of Time Series Analysis*, *29*(6), 947–972.

Hušková, M., Kirch, C., Prášková, Z., & Steinebach, J. (2008). On the detection of changes in autoregressive time series, II: Resampling procedures. *Journal of Statistical Planning and Inference*, *138*(6), 1697–1721.

Institute of Computer Science of the CAS. (n.d.). About us.

Institute of Computer Science of the CAS. (n.d.). Department of artificial intelligence.

Institute of Computer Science of the CAS. (n.d.). Department of machine learning.

Institute of Computer Science of the CAS. (n.d.). Department of theoretical computer science.

Institute of Computer Science of the CAS. (n.d.). Research groups and selected projects.

Institute of Computer Science of the CAS. (n.d.). Research.

Institute of Computer Science of the Czech Academy of Sciences. (n.d.). Department of theoretical computer science.

Institute of Computer Science of the Czech Academy of Sciences. (n.d.). Petr Hájek: Selected publications.

Institute of Computer Science of the Czech Academy of Sciences. (n.d.). Books by institute authors: 2010–2014.

Institute of Experimental Botany of the Czech Academy of Sciences. (n.d.). History of IEB: New opportunities, new challenges (1990–2000).

Institute of Information Theory and Automation of the CAS. (n.d.). Research areas.

Institute of Information Theory and Automation of the CAS. (n.d.). Organisational structure.

Institute of Information Theory and Automation of the CAS. (n.d.). Department of pattern recognition.

Institute of Information Theory and Automation of the CAS. (n.d.). Miroslav Kárný.

Institute of Information Theory and Automation of the Czech Academy of Sciences. (n.d.). Research areas and institutional profile.
Institute of Information Theory and Automation, Department of Pattern Recognition. (n.d.). Research profile.
Institute of Information Theory and Automation. (n.d.). About us. Czech Academy of Sciences.
Institute of Information Theory and Automation. (n.d.). Department of Decision-Making Theory.
Institute of Information Theory and Automation. (n.d.). Jan Seidler: Research interests and publications. CAS.
Institute of Information Theory and Automation. (n.d.). Miroslav Kárný: Research interests and publications. CAS.
Institute of Information Theory and Automation. (n.d.). Research areas.
Institute of Mathematics of the CAS. (2022). Annual report 2021.
Institute of Mathematics of the CAS. (2024). Annual report 2023.
Institute of Mathematics of the CAS. (2025). Annual report 2024.
Institute of Mathematics of the CAS. (n.d.). Eduard Feireisl: Research profile.
Institute of Mathematics of the CAS. (n.d.). Human resources strategy and progress.
Institute of Mathematics of the CAS. (2020). Institute of Mathematics of the Czech Academy of Sciences. European Mathematical Society Magazine.
Institute of Mathematics of the CAS. (2023). Annual report 2023.
Institute of Mathematics of the CAS. (n.d.). Eduard Feireisl: Research profile.
Institute of Theoretical Computer Science, Charles University. (n.d.). DIMATIA: Centre for Discrete Mathematics, Theoretical Computer Science and Applications.
Institute of Theoretical Computer Science, Charles University. (n.d.). Jaroslav Nešetřil: Research interests and academic appointments.
IT4Innovations National Supercomputing Center. (2019). The new supercomputer in Ostrava is called Barbora.
IT4Innovations National Supercomputing Center. (2020). Review 2020.
IT4Innovations National Supercomputing Center. (2021). Karolina.
IT4Innovations National Supercomputing Center. (2022). Review of IT4Innovations 2022.
IT4Innovations National Supercomputing Center. (n.d.). About IT4Innovations: History and milestones.
IT4Innovations National Supercomputing Center. (n.d.). About IT4Innovations.
IT4Innovations National Supercomputing Center. (n.d.). About IT4I: History and mission.
IT4Innovations National Supercomputing Center. (n.d.). Infoservice.
IT4Innovations National Supercomputing Center. (n.d.). Karolina.
IT4Innovations National Supercomputing Center. (n.d.). Organisation.
IT4Innovations National Supercomputing Center. (n.d.). Our research activities.
IT4Innovations National Supercomputing Center. (n.d.). Our supercomputers.
IT4Innovations National Supercomputing Center. (n.d.). Research laboratories.
IT4Innovations. (n.d.). About IT4Innovations.
Janyška, J. (2019). Remarks on natural differential operators with tensor fields. *Archivum Mathematicum*, *55*(5), 289–308.

Janyška, J., & Modugno, M. (2017). Quantum potential in covariant quantum mechanics. *Differential Geometry and Its Applications*, *54*, 175–193.

Janyška, J., Kolář, I., & Slovák, J. (Eds.). (1996). *Differential geometry and applications: Proceedings of the Conference, Brno 1995*. Masaryk University.

Jurečková, J., & Picek, J. (2006). *Robust statistical methods with R*. Chapman & Hall/CRC.

Jurečková, J., & Sen, P. K. (1996). *Robust statistical procedures: Asymptotics and interrelations*. Wiley.

Jurečková, J., Picek, J., & Sen, P. K. (2013). *Methodology in robust and nonparametric statistics*. Chapman & Hall/CRC.

Kalenda, O. F. K. (2004). *Classes of compact spaces in functional analysis* [Habilitation thesis, Charles University]. Charles University Press.

Kalenda, O. F. K. (n.d.). *Publications and research*. Charles University.

Kalina, J. (2012). On multivariate methods in robust econometrics. *Prague Economic Papers*, *21*(1), 69–82.

Kalina, J. (2025). A robust coefficient of determination based on implicit weighting. *Applications of Mathematics*, *70*(5), 647–670.

Kárný, M. (2020). Fully probabilistic design unifies and supports dynamic decision making under uncertainty. *Information Sciences*, *509*, 104–118.

Kárný, M., & Guy, T. V. (2006). Fully probabilistic control design. *Systems & Control Letters*, *55*(4), 259–265.

Kočvara, M., & Stingl, M. (2003). PENNON: A code for convex nonlinear and semidefinite programming. *Optimization Methods and Software*, *18*(3), 317–333.

Kolář, I., Michor, P. W., & Slovák, J. (1993). *Natural operations in differential geometry*. Springer.

Koutecký, M. (2024). *Parameterized algorithms for block structured integer programs* [Habilitation thesis, Charles University]. Charles University Press.

LUMI. (n.d.). LUMI supercomputer: European collaboration.

Málek, J., & Rajagopal, K. R. (2007). Mathematical issues concerning the Navier–Stokes equations and some of their generalizations. In P. G. Ciarlet & J.-L. Lions (Eds.), *Handbook of differential equations: Evolutionary equations* (Vol. 2, pp. 371–459). Elsevier.

Mandl, P. (1986). K tradicím a perspektivám teorie pravděpodobnosti a matematické statistiky. *Pokroky matematiky, fyziky a astronomie*, *31*(5), 260–263.

Masaryk Institute and Archives of the CAS. (n.d.). Fonds of the Czechoslovak Academy of Sciences and the Czech Academy of Sciences.

Masaryk Institute and Archives of the Czech Academy of Sciences. (n.d.). Fonds of the Czechoslovak Academy of Sciences (CSAS) and the Czech Academy of Sciences (CAS).

Masaryk University. (1998). Preface: EQUADIFF 9, Brno, 25–29 August 1997. Archivum Mathematicum.

Masaryk University. (2022). EQUADIFF 15: Conference information and proceedings.

Masaryk University. (2022). Ivo Vrkoč. Significant mathematicians in the Czech lands.

Masaryk University. (n.d.). Archivum Mathematicum: Journal information.

Masaryk University. (n.d.). Archivum Mathematicum: Volumes.

Masaryk University. (n.d.). Department of Mathematics and Statistics.

Masaryk University. (n.d.). Jan Slovák: Doctoral supervision.

Masaryk University. (n.d.). Jan Slovák: Publications and research profile.

Masaryk University. (n.d.). Jiří Rosický: Publications.

Masaryk University. (n.d.). Mathematical structures of algebra and geometry: Publications.

Maslowski, B., & Seidler, J. (1998). Invariant measures for nonlinear SPDE's: Uniqueness and stability. *Archivum Mathematicum*, *34*(1), 153–172.

Maslowski, B., & Seidler, J. (2000). Probabilistic approach to the strong Feller property. *Probability Theory and Related Fields*, *118*, 187–210.

Maslowski, B., & Šnupárková, J. (2018). Stochastic affine evolution equations with multiplicative fractional noise. *Applications of Mathematics*, *63*(1), 7–35.

Mathematics and Physics Faculty of Charles University. (2025). Prestigious Nerode Prize for pioneering work on sparsity.

Matoušek, J. (2002). *Lectures on discrete geometry*. Springer.

Mielke, A., & Roubíček, T. (2015). *Rate-independent systems: Theory and application*. Springer.

Ministry for Regional Development of the Czech Republic. (n.d.). Operational Programme Research and Development for Innovations, 2007–2013.

Ministry for Regional Development of the Czech Republic. (n.d.). Programmes 2007–2013.

Ministry of Industry and Trade of the Czech Republic. (2007). The programmes of the Operational Programme Enterprise and Innovation.

Modelling the first wave of the COVID-19 epidemic in the Czech Republic and the role of government interventions. (2020). medRxiv [Preprint].

Nešetřil, J. (2005). Ramsey classes and homogeneous structures. *Combinatorics, Probability and Computing*, 14(1–2), 171–189.

Nešetřil, J., & Ossona de Mendez, P. (2012). Sparsity: Graphs, structures, and algorithms. Springer.

Nešetřil, J., & Ossona de Mendez, P. (2014). First order properties on nowhere dense structures. *The Journal of Symbolic Logic*.

Nešetřil, J., & Ossona de Mendez, P. (2016). Modeling limits in hereditary classes: Reduction and application to trees. *The Electronic Journal of Combinatorics*, *23*(2), Article P2.52.

Nešetřil, J., & Ossona de Mendez, P. (2016). Structural sparsity. *Russian Mathematical Surveys*, *71*(1), 79–107.

Outrata, J., Kočvara, M., & Zowe, J. (1998). *Nonsmooth approach to optimization problems with equilibrium constraints: Theory, applications and numerical results*. Kluwer Academic Publishers.

Palacký University Olomouc. (n.d.). Acta Universitatis Palackianae Olomucensis, Facultas Rerum Naturalium, Mathematica.

Palacký University Olomouc. (n.d.). Department of mathematical analysis and applications of mathematics.

Palacký University Olomouc. (n.d.). Mathematics and computer science. Faculty of Science.

Partnership for Advanced Computing in Europe. (n.d.). PRACE research infrastructure.

Post, O. (2012). *Spectral analysis on graph-like spaces*. Springer.

Pudlák, P. (2013). *Logical foundations of mathematics and computational complexity: A gentle introduction*. Springer.

Pudlák, P. (n.d.). *Bibliography of Pavel Pudlák*. Institute of Mathematics of the Czech Academy of Sciences.

Rosický, J. (2024). Discrete equational theories. *Mathematical Structures in Computer Science*, *34*(2).

Roubíček, T. (2013). *Nonlinear partial differential equations with applications* (2nd ed.). Birkhäuser.

Seidler, J., & Sobukawa, T. (2003). Exponential integrability of stochastic convolutions. *Journal of the London Mathematical Society*, *67*.

Springer Nature. (n.d.). Metamathematics of first-order arithmetic [Publisher catalogue].

Studený, M. (2005). *Probabilistic conditional independence structures*. Springer.

Technology Agency of the Czech Republic. (n.d.). AdMaS UP — Advanced Building Materials, Structures and Technologies [Project record LO1408]. STARFOS.

The Czech Science Foundation. (n.d.). Annual reports.

The European High Performance Computing Joint Undertaking. (2025). VLQ: Europe's next leap towards quantum-powered research and innovation.

TOP500. (2015). TOP500 list, June 2015.

University of Ostrava. (n.d.). Algebra, geometry and mathematical physics. Department of Mathematics.

University of Ostrava. (n.d.). Department of Mathematics.

University of South Bohemia. (n.d.). Centre for Mathematical Biology.

University of South Bohemia. (n.d.). Department of Mathematics. Faculty of Science.

University of South Bohemia. (n.d.). Luděk Berec: Research profile. Centre for Mathematical Biology.

University of South Bohemia. (n.d.). Research. Centre for Mathematical Biology.

University of West Bohemia. (n.d.). About NTIS.

University of West Bohemia. (n.d.). Department of Mathematics.

University of West Bohemia. (n.d.). Mathematical models. NTIS Research Centre.

University of West Bohemia. (n.d.). Modern methods of geometric modelling. NTIS Research Centre.

Víšek, J. Á. (2006). The least trimmed squares. Part I: Consistency. *Kybernetika*, *42*(1), 1–36.

Víšek, J. Á. (2006). The least trimmed squares. Part II: √n-consistency. *Kybernetika*, *42*(2), 181–202.

Víšek, J. Á. (2006). The least trimmed squares. Part III: Asymptotic normality. *Kybernetika*, *42*(2), 203–224.

Vomlel, J., & Studený, M. (2008). Graphical and algebraic representatives of conditional independence models. In P. J. Lucas, J. A. Gámez, & A. Salmerón (Eds.), *Advances in probabilistic graphical models* (pp. 55–80). Springer.

VSB–Technical University of Ostrava. (n.d.). IT4Innovations National Supercomputing Center.

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