MEANING, RELATION, IDENTITY AND GENERALITY IN THE TRACTARIAN PHYLOSOPHY

Iulian GRIGORIU
University „Dunărea de Jos"

Abstract

In this article, I examine the theory of representation (representationalism) in *Tractatus Logico-Philosophicus*, considering it as a means of access to the Tractarian architectonics. I focus on the introduction of a new formalism specific to Wittgenstein, designed to establish conceptual writing on different foundations than those of Frege–Russell, in order to avoid inherent paradoxes and to eliminate the cumbersome techniques used to counter them. In this regard, I analyze and interpret Wittgenstein's new formalism for eliminating metalanguage and the theory of types, comparing Frege's theory of sense and reference with Wittgenstein's proposal, which separates meaning from the way an object appears and distinguishes between syntax and semantics—a critique that also applies to Russell. Wittgenstein's approach to "cutting through" these issues is to assert that the propositional sign cannot state or show anything about itself. By following the logicality of the image and the concrete way it is constituted, Wittgenstein constructs his own formalism of general forms concerning relations, operations, propositions, and logical functions. Through this framework, he defines, in a logicist manner, the concept of the general form of a proposition, logical operations, and natural numbers. All these are intensional forms, obtained by highlighting their internal structures. In this way, the Austrian philosopher exposes an entire series of metaphysical expressions associated with foundational theories of numbers, constructions, and calculative conventions, as well as with the logicist foundations of mathematics (set, function, number, conventional signs). It becomes evident that mathematics and logic share the same internal structures, and logical discoveries are reflected at the level of mathematical language. As part of this process, Wittgenstein eliminates the concept of identity as expressed by the equality sign, replaces the universal quantifier with the existential one (using negation and an additional variable), and introduces his own formal notation. Through these modifications, he attempts to eliminate the "pseudopropositions" that undermine the logicist edifice of mathematics. This article also examines how Wittgenstein reformulates Russell's concept of relation from *Principia Mathematica*, assimilating it into an empirical framework. In this case, Wittgenstein reduces the symbolism of relations to states of affairs, with the direction being from objects to relations rather than vice versa. I emphasize that Wittgenstein equates relations with the concept of logical operations, thereby ensuring the necessary generality and reconstructing the general form of the concept *aRb*. The author synthesizes the general forms presented in the *Tractatus*: relation, proposition, series of forms, natural number, and the derivation of one term from another based on the internal relation of a series of forms. The central aim of this study is a comparative reconstruction of how Wittgenstein proposes replacement theories for the formalisms of Frege and Russell/Whitehead, as well as a new reading of the *Tractatus Logico-Philosophicus* from the perspective of the theory of meaning and general forms, with a focus on the sentences contained in sections 5.53–5.54.

Keywords: representationalism, meaning, relation/logical operation, identity, internal forms and structures, general forms, Tractatus Logico-Philosophicus.

1. Representationalism – General Conception of Meaning

The concepts of meaning, relation, and identity are part of a broader theory of representation (of images or pictures) that I call representationalism.

Representationalism is a general conception or attitude toward meaning, as it was for Leibniz, Kant, Schopenhauer, and Hertz. It is also an element of continuity (between early Wittgenstein, mature Wittgenstein, and late Wittgenstein) and a unifying factor across the fundamental fields of Wittgensteinian philosophy (philosophy of logic, philosophy of mathematics, and philosophy of language).

In all fields of Wittgensteinian philosophy, the discursive-conceptual approach is based on a trans-metaphysical tension between reality and representation. Reality, or the world, tends toward symbols and concepts, and the image of the world holds explanatory power. "To understand," "to highlight," "to show," "to say," "to criticize," "to calculate"—and all that these imply in the practice of "language games"—are part of a téchné and respond to a transcendental necessity.

The possibility of representing something is transcendental; it is related to the unity of subject and object. However, for Wittgenstein, this unity does not originate from an unknowable source (as in Kant's thing-in-itself), but rather from Schopenhauer's concept of will or from a connection with the absolute, as in German idealism.

In my related works (Grigoriu, 2017, 2018, 2019), I show that representationalism is an anti-descriptive term with the simultaneous meanings of "to show," "to depict," "to generalize," and "to stage." Wittgenstein employs several expressions with this last meaning (darstellen, abbilden, vorstellen), offering an intuitive framework for the targeted fields (language, logic, and mathematics).

In the Tractatus, representationalism is linked to the similarity between thought and its logical expression; its basis is the logical space. Logic can be regarded as a representational foundation because it shows rather than describes itself—just as a play or an artistic act unfolds on a stage where the entire universe is allocated to the artistic act itself, while the rest of the world remains strictly delineated. It is not as though a spectator must step onto the stage to save Desdemona's or Juliet's life – an absurdity akin to the situation of those who take language paradoxes literally.

In Wittgenstein's philosophy after the Tractatus, representationalism becomes the manifestation and revelation of linguistic, logical, and mathematical phenomena, aiming to capture their conventional nature, techniques, rules, and preeminence, with the logical space evolving into a more complex and nuanced grammatical space.

In the equation natural language – logic – mathematics, representationalism consists of the process by which I extract, eliminate, and separate natural language from logic and mathematics so that logic and mathematics may express themselves in their own distinct languages and natures. This implies direct access to a sign language that is autonomous, consistent, coherent, and free from contradictions and paradoxes.

If, in the Tractatus, the goal of representationalism was to purify and disentangle thinking (logic and mathematics) from the distortions of natural language (4.002c), then in his later research, Wittgenstein exposes mathematical techniques as being irreversibly shaped by linguistic structures. Underneath the eccentricity and exoticism of these structures, however, lies a living body.

1.1 Tractarian representationalism

Representation theory (2.1-4.128) lays the groundwork for the philosophy of logic and mathematics in the *Tractatus* and contains certain elements of interest here:

- "We picture facts to ourselves" (2.1);
- The image, through its logical form, represents the fact (*Tatsache*), meaning that it connects it to our understanding;
- The logical form is the most general type of form; it constitutes the possibility of states of affairs (*Sachverhalten*) in the logical space (2.18, 2.201, 2.202);
- The image (logical—2.182) represents the world or an aspect of the world;
- The sense of the image (2.221) is related to the truth or falsity of the image (2.222, 2.223);
- An image is not true a priori (2.225) but only in relation to reality;

- From (3), the notion of "thought" arises: "A logical picture of facts is a thought," so that 3.12 states that the sign of thought is the proposition (*Satzzeichen*—propositional sign); hence, the theory of the sign and proposition is related to their use up to 3.5, followed by a discussion on thinking (4), language, the relation between them, and the possibility of representing reality;
- The image is a situation (*Sachlage*) in logical space, which represents the existence and non-existence of states of affairs;
- Just as there are simple elements of the world (objects), there are simple elements of the image
  in correspondence with objects, which replicate the logical (not spatial) relations between them;
- The connection between the elements of the image and the corresponding objects determines the structure of the image (2.13, 2.131, 2.15);
- The possibility of structuring the elements of the image constitutes its form of representation, which connects the image to reality (2.1511);
- The image represents reality because its elements reproduce, in their structure, a certain state of affairs; the representational relation belongs to the image (2.1513);
- The image is a fact and, therefore, part of the world (2.141). The "image fact" shares with what it represents a logical form;
- The image does not represent its own form of representation but instead shows it (2.172)—a defining element of representationalism. That is, representation terminates at the image itself;
- Therefore, the image of the "fact-image" occupies a different place in logical space than the image as such.

Observations

(a) The Non-Reproducibility of the Image

A doubt could arise from the fact that an image is a fact, and thus, it could be considered a source of another image (since it belongs to reality). However, the image of this latter fact would be an independent image, not an image of the image. It would occupy a different point in logical space than the first representation.

There is no "image of the image," just as there is no "fact of a fact" or "thought of a thought," as demonstrated by other propositions. The image as fact is not meant to be represented again, just as a play does not become a fact for a script of a script. Instead, the latter is simply another script for another fact.

An image of the image is nonsensical when it refers to the same fact represented, though it is not prohibited for the same fact to allow different images.

(b) The Economy of Logical Representation

Two logical images of the same fact should be equivalent, following the representationalist imperative that the most "economical" image (Occam's razor) prevails. The essential requirement is that any image must be logical, meaning that it must share a logical form with the represented reality. This guarantees the economy of representation.

The goal is to prevent an infinite proliferation of "an image of an image," and so on. In this sense—but somewhat indirectly—4.04 and 4.041 state that a proposition, as an image of facts, must have the same number of parts (or multiplicity) as the state of affairs it represents:

"This mathematical multiplicity, of course, cannot itself be the subject of depiction. One cannot get away from it when depicting." (4.041)

Moreover, 3.14 states that the propositional sign (the means by which thought is expressed) is itself a fact. An analogy emerges between image and proposition (logically identical): just as the elements of an image relate to each other in a determined manner (2.15), the elements of the propositional sign—the words—"stand in a determinate relation to one another" (3.14).

(c) The Counter-Descriptive Nature of the Picture

Another element of representationalism is that a picture cannot describe its own form but instead shows it. The picture is not a description of its own form but rather a manifestation. This allows us to understand:

- On the one hand, that representation is distinct from the representational;
- On the other, that a fact may be represented with varying degrees of proximity to reality, depending on how the logical form is incorporated.

Thus, we are assured that logical pictures can represent the world (2.19). However, it can be suggested that sensory pictures make representation problematic. In any case, the logic of pictures must be rationally grasped through representation techniques:

- Logical representation is infallible and serves as the criterion for other representations.
- That is, any picture is logical (2.182), meaning it represents reality and has no alternative.

(d) The Metaphysical Nature of Statement 2.182

Statement 2.182, which asserts that "any picture is logical, but not necessarily spatial," is metaphysical for the following reasons:

- It presupposes that any picture carries within it a symbolic or conceptual logic, which implies that logical representation pre-exists the object itself, prior to any thought or sensation. (For Kant, this would correspond to pure intuition or apperception, which does not belong to sensory perception.)
- This could also suggest that the sensible world can be reduced to the intelligible world and possesses an intellectual character. That is, the (logical) picture relates to proposition and negation in its relation to the world, whereas space does not: "Not every picture is spatial" (2.182), even though mathematical analysis, algebra, and geometry operate in such a way that they structure the mathematical world as Hilbert Space, Vector Space, Phase Space, etc.

This statement pertains to Logical Space, a dimensionless, a priori unfolded space containing the foundation of significance for any possible logical universe (in analogy with the ideality of Cartesian orthogonal geometric space, where concepts such as "logical place" and "logical coordinates" are discussed).

The meaning of statement 2.182 is analogous to saying: "It is not necessary for these people to speak to each other; they understand each other merely by glances, intuiting each other's statements, questions, and answers."

2. Between Frege and Russell

In the *Tractatus*, Wittgenstein makes both explicit and implicit references to Frege's and Russell/Whitehead's theories of meaning. In some cases, he adopts their concepts, while in others, he discusses them in his own terms. Although the main propositions of the *Tractatus* (not only those denoted by integers) express his own theory, certain considerations in the sub-propositions appear abrupt and radical, requiring further explanation for the reader.

Wittgenstein sometimes criticizes Frege and Russell collectively, while at other times addressing them separately. Adopting their terminology does not imply agreement. When Wittgenstein refers to them implicitly, it can generally be inferred that he has Frege in mind when discussing thought, sign, sense, symbol, meaning, object, and concept, whereas he refers to Russell when discussing propositional atoms, propositional and functional meaning, relation, identity, truth values, and, of course, the theory of types and the axioms of infinity and reducibility.

The common concepts of name, object, relation, identity, equality, sign, symbol, meaning, and sense acquire different roles in Wittgensteinian representationalism.

Wittgenstein adopts the concept of a proposition as a function of the names and expressions it contains (3.318), as well as the idea of a conceptual notation meant to eliminate the confusions and paradoxes of ordinary language. The paradoxes that Frege and Russell encountered in their foundational work on class theory arise from the misuse of general terms (*object, function, number*, etc.), which should be represented in a conceptual notation through variables rather than functions or classes, as their predecessors had done (4.1272).

The establishment of basic logical operations is merely a matter of notation, reflecting a certain mathematical multiplicity and rule, but they are not fundamental in themselves (5.474, 5.475, 5.476).

Thus, it is impossible to introduce both objects belonging to a formal concept and the formal concept itself as primitive ideas. For example, it is impossible to introduce both the concept of a function and specific functions, as Russell does, or the concept of number and particular numbers (4.12721).

Logic is a domain of a priori thought, admitting totally generalized forms that stand in contrast to generalized forms of an empirical type and to the theory of representation, where names are allocated to objects (cf. 5.526).

Wittgenstein rejects certain key elements from *Principia Mathematica*, including symbolism, formalism, the logic of predicates, the concepts of relation and identity, and the idea of logical objects and constants (5.32; 5.4).

Wittgenstein appears to be critical of a certain style of theorizing in *Principia Mathematica*, where Russell and Whitehead introduce definitions and even fundamental laws without fully understanding their consequences or providing the necessary justifications. As a result, the validity of such principles is questionable. Wittgenstein asserts that logic does not require proof, precisely because logic is a priori, and, in fact, we cannot think illogically (5.4731).

A key element of Wittgenstein's argument against classical logic—and in anticipation of the new vision of language in *Philosophical Investigations*—is the idea that any proposition has meaning, provided that the names it contains have been assigned meaning. This stands in contrast to Frege, for whom meaning is determined by the correct composition of the proposition.

For Wittgenstein, logic is a priori and does not require laws to constitute it, but rather a process of deciphering, in order to conform to the inherent logic of any structured language.

Wittgenstein criticizes his predecessors for their attitude toward logic, particularly their tendency to treat evidence as a mark of simplicity[1]. This is problematic, especially when it comes to fundamental laws. Either logic has a single fundamental law, or, if it has multiple fundamental laws, they can be combined into a single principle through logical conjunction. This final formulation of the logical principle may be highly complex, but that does not disqualify it as fundamental.

To illustrate the primacy of logic, Wittgenstein offers a metaphor: "Logic is not a body of doctrine, but a mirror-image of the world. Logic is transcendental." (6.13)

When analyzing Russell’s theory of types and descriptions (which subsumes Frege's theory of sense and reference), Wittgenstein separates the meaning of a name or sentence from the way the name appears in a proposition.

The rules governing the signs of a proposition (syntax) should not refer to their meaning (their truth value relative to an object in reality). Wittgenstein thus distinguishes the two domains of classical logic without integrating them into a mutual foundation, thereby exposing Russell's error in the theory of types:

---

[1] From this perspective, neither *modus ponens* nor *the rule of substitution* works automatically, because they use in hypotheses symbols that will appear in the conclusion.

"It can be seen that Russell must be wrong, because he had to mention the meaning of signs when establishing the rules for them." (3.331)

"No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the 'theory of types')." (3.332)

Classical logic permits paradoxes because it allows a propositional function to become its own argument. However, the same does not apply to logical operations, which can become their own arguments without generating contradictions. This distinction enables Wittgenstein to advance a series of logical forms, culminating in the generalization of the propositional sign and the discovery of the general form of number.

When Frege defines the concept of a natural number, he relies on an entire structure of logical and mathematical notions, including object, meaning, reference, concept, class, property, proper name, common name, identity, equality, discernibility, and substitutability.

Frege's definition of natural numbers is structured as follows:

"The number of concept F is the extension of the concept equinumeric with the concept F."

"The number that belongs to the concept F is the extension of the concept 'equal to the concept.'" (Frege, 1950/1960, §68)

The process of defining particular numbers is connected to the possibility of these extensions being identical or arranged in an ordered structure. With this in place, Frege defines natural numbers:

"0 is the number that belongs to the concept 'non-identical with itself.'" (Ibid., §74)

The concept "non-identical with itself" does not correspond to any object. Frege argues that no object falls under a concept that contains a logical contradiction. If "a" falls under the concept of "non-identical with itself," then "a" is not identical to "a", which defines the number 0.

Frege then defines the successor function, from which the number 1 and, consequently, the sequence of natural numbers is derived.

In *Principia Mathematica*, the cardinal number "1" is defined as "the class of all unitary classes" (*Russell/Whitehead, 1925-27/1968, (I), p. 363*). The definition of number thus requires set theory supplemented with the theory of types, in order to avoid semantic paradoxes.

Wittgenstein's critique of Russell unfolds in three dimensions:

1. The theory of types and the axiom of reducibility are unnecessary.
2. The theory of types still generates paradoxes.
3. Wittgenstein develops his own formalism of general forms (relation, operation, proposition, logical function), allowing him to define, in a logicist manner, his own concepts of function and natural number.

3. Interpretation of meaning in Principia and reaction from the Tractatus

Russell introduces the concept of meaning in the context of discussing propositional functions, which must avoid vicious circles through an appropriate definition. The concept of a function is well defined when its values are well defined, meaning that they do not refer to the function itself—thus avoiding vicious circles and the emergence of paradoxes.

The concept of a function itself remains ambiguous. As Russell states, a function ambiguously denotes the totality of its values, and those that do not refer to the function itself constitute the allowed values of the function (*Ibid.*, p. 38 sqq).

Russell introduces two notations , $\emptyset$(x) which denotes the indeterminate value of the function, or the function in general and $\emptyset(\hat{x})$, the function as such, that is, the concrete values of the function; here $\emptyset(\hat{x})$ ambiguously signifies its indefinite value $\emptyset$(x).
Thus $\emptyset(\hat{x})$ ambiguously signifies the $\emptyset$x, and $\emptyset$(x) is ambiguously denoted by $\emptyset(\hat{x})$.
Russell also refers tothe arguments "x" for which $\emptyset(\hat{x})$ has value, "the possible values of x. It can be said that: (x) $\emptyset$(x) assumes $\emptyset$(x) and (Ex) $\emptyset$(x) assumes $\emptyset(\hat{x})$.

It is therefore notable that the concrete values of the function, $\emptyset(\hat{x})$ play an overlapping role in the process of meaning: on the one hand $\emptyset(\hat{x})$ ambiguously signifies $\emptyset$(x), on the other hand, it signifies the argument "x", if it admits values for the respective arguments. (cf. *Ibid.*)
However, Russell does not stop there. He further argues that $\emptyset(\hat{x})$ does not have to express a proposition, as it is not signifying - meaning that it does not express anything. This is because its values are all propositions of the form $\emptyset$(x), and this prevents the emergence of a vicious circle. For this reasons, the types of truth that belong to a proposition may differ from those of a particular proposition.
Consider the following statements: "x is a proposition" (1) and "x is a propositional function" (2);

Statement (1) is an ambiguous statement about the values of the propositional function (where the ambiguity pertains to the mode of denotation), while statement (2) is a statement about an ambiguity (since the function itself is ambiguous). Thus, the proposition is true regardless of the value of "x", and it ambiguously denotes the values of "x."

To avoid the vicious circle $\emptyset(\hat{x})$ whose values are propositions of the form $\emptyset$(x), must not express a proposition. It is not signifying, meaning that it does not express anything. In this way, the type of falsity that can be attributed to a general proposition differs from that which can be assigned to a particular proposition.

Through this extremely ingenious approach, Russell develops his theory of "a-functions", which are divided into types. The theory of types requires the introduction of the axiom of reducibility, as well as other techniques and concepts—which he later abandons in the second edition of *Principia Mathematica* (partly due to Wittgenstein's influence, as Wittgenstein considers them pseudo-concepts).

However, the Austrian philosopher does not need to follow the entire structure of *Principia Mathematica*, because, in his view, this entire scaffolding can be cut off at the root. Nonetheless, this requires a preliminary agreement in principle. Regarding the general issue of the meaning of language and philosophy, Wittgenstein adopts the critical and analytical stance of his predecessors and credits Russell with good intentions:

> All philosophy is a 'critique of language'... It was Russell who performed the service of showing that the apparent logical form of a proposition need not be its real one. (4.0031)

However, the way in which Russell formulates his program of eliminating misleading appearances leads Wittgenstein to pursue his own path, rather than following Russell. For Wittgenstein, Russell's problems and theories are meaningless if they can be avoided altogether.

A first reaction in the *Tractatus* against confusing aspects of the theory of meaning appears in 3.325, where Wittgenstein asserts that within logical syntax: a sign cannot be used for different symbols, nor can it symbolize differently. This is precisely what Frege and Russell attempt to prevent in their conceptual notation, but which they fail to eliminate entirely.

The result of subordinating the unique meaning of the sign, syntax or rules in the logical-linguistic system will lead to the elimination of meaning from the syntax. As a first stage Wittgenstein eliminates the meaning from the logical syntax, because the sign has meaning by itself, i.e. by its use. A sign is uniquely determined by its meaning, which occurs naturally, through use, or even conventionally. A sign cannot have two meanings, nor can two signs signify the same thing. The rules of a system of signs constitute the logical syntax in which each sign already has its own meaning subordinated to the syntax; it no longer has to play an additional role in logical syntax. (cf. 3.33). There is no superposition between the sign, its role in syntax and syntax as such. This is an attribute of Wittgenstein's representationalism. The idea of the uniqueness of each object (logical, when viewed logically, not spatio-temporally, for example, or linguistic object – name, expression ,proposition, propositional function) is already an established fact and is repeated or reinforced in (5.53), which will avoid the unjustified multiplication of Russellian types.

Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs (5.53)

The problem is not that we must have a unique sign for each object, but that each object must be analyzed in order to be represented. Since objects are infinite in number, it is inevitable that different objects in language may have the same sign, or that the same sign may stand for different meanings. For the classification of objects and the avoidance of paradoxes there are needed either classes, sets, type theory, as Frege and Russell do, or definitions and intensional forms, which Wittgenstein does. His solution is to identify objects, in general, through their internal logical structure and using their own formalism, as will see in a moment.

4.The question of identity in *Principia* and *Tractatus* and the elimination of the equal sign

The problem of meaning entails the problem of identity, along with its associated formal symbolism. In *5.53*, Wittgenstein addresses the issue of identity within formal sign language. He states that 'identity is not a relation between objects' (*5.5301*) and provides the following sentence as an example:

(x): fx. ⊃. x = a'. What this proposition says is simply that only a satisfies the function f , and not that only things that have a certain relation to a satisfy the function f. (5.5301)

That is, "a" has the quality of satisfying "f", without being in relation to the variable "x", or the variable "x" identifies with "a" when it is the argument of "f", but it is not in relation to "a". What is sought to be avoided here is the proliferation of the meaning of "x", as the variable of the predicate (f, x) and as the object variable (x, a). But identity is not a relation for entities, because "x" is only "x", (cf. 5.53).

I add that these "x" may appear as arguments in other functions "g" for example without the danger of the vicious circle appearing, when the function "f" is the argument of "g", such as for example "g (f, x) = b".

To avoid vicious circles, Russell introduces limitations to ensure that functions do not refer to all their own values. However, when defining the equality of two functions or two sets, we must consider the coincidence of all their values. To maintain the generality of the definition of identity and prevent undesirable situations, Russell develops the "Theory of Types," supported by the "Axiom of Infinity." In summary, Russell does the following:

The definition of identity "x = y" implies "$\emptyset(x) \supset \emptyset(y)$", whatever the function $\emptyset$ variable; the function shows that anything true about "x" is true for "y" too. Being a function of $\emptyset$ it must be limited so that it does not refer to all the values of $\emptyset$. In this hypothesis, if it is considered that

"x = a" is a value of ∅ from the above formula, then "x equals a" implies "y equal to a" so "x equals y". At the same time, we have "x equal to a" and "x equal to y" (from "∅(x) ⊃∅(y)"), then "y = a".
So "∅(x) ⊃∅(y)" which is a value of Fix, vicious circle.
In this hypothesis, if it is considered that "x = a" is a value of ∅ from the above formula, then "x equals a" implies "y equal to a" so "x equals y".
At the same time, we have "x equal to a" and "x equal to y" (from "∅(x) ⊃∅(y)"), then "y = a".
So "∅(x) ⊃∅(y)" which is a value of ∅(x), vicious circle.

Other limitations are imposed for ∅, Russell says, but other limitations imposed on ∅ could lead to the ∅(x) being true and ∅(y) false so ∅(x) no longer involves ∅(y), which would mean that the identity between "x" and "y" is no longer reached.

To save the definition of identity, Russell will introduce the theory of types and the axiom of reducibility. In this endeavor he will use the equal sign, "=" in the construction and partitioning of types, which is an abuse, in Wittgenstein's opinion.

Wittgenstein's reaction appears at 5.5302 where he states:

Russell's definition of '=' is inadequate, because according to it we cannot say that two objects have all their properties in common. (Even if this proposition is never correct, it still has sense.)

With the corollary:

Roughly speaking, to say that two things are identical is nonsense, and to say that one thing is identical with itself is to say nothing at all. (5.5303)

This is the moment when Wittgenstein demonstrates how to remove the sign of equality from conceptual writing. To this end, he introduces alternative notations in conceptual writing.

I write that not 'f (a,b) . a = b' but "f (a,a)" (or " f (b,b)"). And not "f(a,b) . . ~a = b",
but "f(a,b). (5.531)

So when Wittgenstein writes a function with the arguments "a" and " b", "b" represents „~ a",and, understandably, in a function with three arguments "f (a, b, c)", "c is .~ a. ∧ . ~ b ", confirmed by the general form of the function of truth or the generalized propositional sign „$[\bar{p}, \bar{\xi}, N(\bar{\xi})]$" (6).

Also, when "f", "g", "h" represent propositional functions as arguments of the operation "Ω", which can be applied to them, then
"Ω(f, g, h)" is "Ω(Ω(f,g), h)", writing that leads to the definition of the natural number according to the occurrences of the operation "Ω".
It seems natural that functions, more precisely logical operations, should be composed in pairs using the logical operation 'Ω,' which leads to the idea of developing the one-dimensional logical space of truth functions first at the two-dimensional level, resulting in a 'two-dimensional logic,' and then extending this process to an n-dimensional 'logic universe.' (Grigoriu, 2019, 2).

Because you can't use different symbols for the same object, "f (a,b)" with "a = b" in Russell's writing becomes at Wittgenstein either "f (a,a)", or "f(b,b)" (cf. 5.531).
Same motivation for predicate formulas at 5.532:

	Russell's writing	Wittgenstein's writing

a. Predicates with equal variables	‘(∃x, y). f (x, y) . x = y’	‘(∃x). f (x, x)’
b. Predicates with opposite variables	‘(∃x, y). f (x, y) . ~x = y’	‘(∃x, y). f (x, y)’
c. Predicates with distinct variables	‘(∃x, y). f (x, y)’	‘(∃x, y). f (x, y) .V. (∃x). f (x, x)’

a. Because equality is not a relation between the extensions of objects, Wittgenstein proposes a notation that explicitly denotes identity. More precisely, identity is a relation in which the extensional appearance of the external coincidence of two objects transforms into an internal relation of intrinsic properties—that is, it belongs to the internal structure of objects, which is formally determined by the predicate.
The fundamental structure of objects is logical and internal, and it is determinative for each logical object.
b. In conjunction with 5.53, in a predicate with two variables, if the first is "x", the second, "y", designates „~x ".
c. In order to express a predicate function of two variables, Russell’s notation allows for the possibility that one variable refers to or identifies with the other—a case that Wittgenstein explicitly addresses (similar to the process described in the first point).
In the last phrase "y" designates „~x", the disjunction ("x. V. ~x ") does not change the truth value of the predicate function, being a tautology.

5. Elimination of the universal quantifier

A typical innovation occurs in 5.5321, where one can observe the interaction between universal and existential quantifiers in the context of negation. Wittgenstein eliminates the universal quantifier over the variable 'x' by introducing another variable, 'y', which designates, as we have seen, '~x', and thereby increases the order of the predicate 'f'."
In short, "(x)" is replaced by "∃x and ~ (∃x, y), where "y" stands for „~x ".

5.5321:

	Russell's writing	Wittgenstein's writing
a. „Only a satisfies the function ( )f’ (removal of quantifier Universal and "=" sign)	‘(x): f x ⊃ x = a’	‘(∃x). f x . ⊃. fa : ~ (∃x, y). f x . f y’
b. „Only one x satisfies f( )”	‘(x): f x ⊃ x = a’[2]	‘(∃x). f x : ~ (∃x, y). f x . f y’

I think at 5.5321 Wittgenstein shows that in PM there is no difference between "only a satisfies the function f" " and "only one x satisfies f". To eliminate this possibility of confusion, Wittgenstein eliminates the universal quantification and the "=" sign of identity. The compromise which is made consists of the tacit designation of "any" other variable, "y" say, with „~x". That is, there is "x" that checks "f", and no other variable "y" verifies f".[3] Sentence 5,533 concludes the avoidance of equality in conceptual writing.

---

[2] This writing is implied in *Principia Mathematic*a.
[3] The negation of quantifiers leaves such situations unresolved to this day.

6. Elimination of other language habits

a. Sentences 5.534 and 5.535 (with the sub-propositions annotated) become consequences of the above: Expressions such as "a =a"; the relation of transitivity with extensional allure"a = b . b=c. imply a equal c", are called "pseudo-propositions" that must be removed from conceptual writing; also cause confusion expressions of the following form:
"(x) . x = x", "(∃x). x = a" (cf. 5.534);

Wittgenstein believes that all problems related to the axiom of *Infinity* are solved and reframes the axiom naturally "... there are infinitely many names with different meanings" (cf. 5.535)

b. It is considered a formal abuse to use the assertorical sign "⊢" in front of a proposition, in order to ensure that it is a proposition, and that the seats allocated to variables are occupied by propositions; so are the expressions "a = a" and "p ⊃ p" (cf. 5.5351).

c. At 5.5352 is shown the ambiguity of the pseudoproposition "There are no things" with the writing "~ (∃x). x = x" that could also mean that there are things that are non-identical to oneself. Ironically or not, the very principle of identity and, perhaps, the level of its reality is targeted here. General logical forms express the processual nature of the facts of the world, rather than merely an inventory of things.

The above pseudo-expression would imply that there are no f-things, regardless of the predicate 'f', as universal quantification in this domain raises insurmountable difficulties. Wittgenstein, starting with the *Tractatus*, exposes a broad spectrum of metaphysical formulations, ranging from 'vidences' to conceptual constructions (such as those of cardinal and real numbers), as well as conventions and calculative constructions.

d. Because mathematics and logic have the same general internal bases, (as will be shown below), the identity with the sign of equality is removed from the syntax of mathematics.
"It is impossible to assert the identity of meaning of two (mathematical) expressions" (6.2322).

In case of solving an equation Q = 0, the equation passes through several equivalent forms (Q' = 0, Q" = 0,....), the meaning or the logical form (6.23) remains the same, only the sense of intermediate expressions changes. The identity of meanings as forms does not show the meaning that I find but after solving the equation, so I cannot indicate it in the course of solving it. That is why it can be stated that "Calculation is not an experiment" (6.2331).

7. General Form of Relation in *Principia* and *Tractatus*

Russell conceives the concept of relations as an empirical one, grounded in perception, where, in addition to the objects in relation, other elements—such as the mind—are involved. He distinguishes between the judgment of perception, which consists of four terms ('a', 'R', 'b', and the perceiver'), and perception itself, which consists of two terms ('aRb' and the perceiver').
In contrast to this 'complex' vision, Wittgenstein reduces the symbolism of relation to the state of affairs, stating that

Instead of 'The complex sign 'aRb' says that a stands to b in the relation R', we ought to put, 'That "a" stands to "b" in a certain relation says that aRb.' (3.1432)

Wittgenstein moves from objects to Relation, rather than the other way around, because he does not conceive of relation as a sensory complex or an empirical entity that ultimately obscures the objects involved in the relation (*Beziehung*).

The concept of Relation provides Russell with an opportunity to introduce, in Aristotelian fashion, the concepts of Truth and Falsity relative to a judgment of perception. Specifically, 'aRb' is true when an 'aRb' complex exists and false when this complex does not exist.

Building upon the concept of relation, Russell first defines general judgments and then particular judgments, distinguishes between the different truth values of elementary and complex general propositions, and emphasizes that even seemingly elementary propositions are connected to complex relations that must be discerned through logical analysis.

Through the techniques discussed above, he seeks to prevent the paradoxes of language by ensuring that neither a proposition nor any part of it refers to the totality or generality of which it is a part.

For Russell, the relation 'R' is a general relation of representation when it is expressed as an incomplete symbol (i.e., one that lacks meaning in its own right).

He gives examples in which different types of propositions—universal or particular—appear: "a in relation R with b" or "a having the quality q", or "a, b and c are in S relation". [4]

Wittgenstein states that primitive signs (∧,∨,⊃, etc.), because they can express themselves through each other, are not primitive and are not relation (cf. 5.42). In order to find a "primitive" framework well anchored in generality, Wittgenstein will assimilate the "relation" with the "logical operation" in general, of transitioning from one form to another, however, using an operator with concrete meaning, the "N" symbol of multiple negation.

To Wittgenstein, according to 3.1432, the "R" of "aRb" cannot represent all relatios, it does not represent the general form of the relations. This is evident from 4.1273 where I believe that Wittgenstein discovers the vicious circle in the so-called general form "aRb", regardless of the content of "R" (at 4.1273 "R" is custom as denoting the succession relation). In this case, if to the string of forms "aRb, (∃x) : aRx. xRb, (∃x, y) : aRx. xRy. yRb. etc" we associate the general term of "aRb", then we see that it appears in the initial string, so the general form of the relation would appear as a term in a particular sequence, which leads to a vicious circle. Such preoccupations will be represented throughout the Wittgensteinian philosophy of mathematics, such as the problem of induction, the demonstrations by recursion where the same "vicious" practices are discovered, the fact that the transition from the so-called general to the particular forms is not justified, the demonstrations with general forms at most justify their particular applications as rules or definitions, and are pseudopropositions outside of the truth and the falsehood. Just as Wittgenstein refuses to write "a = a", he will also refuse to write "aRa". It also follows that identity is not a relation in that it does not imply a relation between objects, as the relation implies, but between the object and itself, which is absurd and therefore the equal sign does not make sense, neither between different objects, because they are different, nor between an object and itself (I can't make of an object, two, just to put them in relation).

[4] It must be recognized that all this approach is one in which logic develops, the foundations of the logic of predicates are laid, the relation between universal and existential quantifiers is defined, the negation and disjunction of predicate forms, prenex forms are obtained, etc.

8. General Forms in *Tractatus*: Relation, Proposition, Series of Forms, Natural Number

Wittgenstein does not give up the general form of relation, but this does not emerge from a perception analysis, it is not the external relation between objects, but an internal one, which is not a proper relation; Internal relations are related to the internal structure of the states of affairs, cannot be affirmed by propositions, they are shown (4.122). Therefore, Wittgenstein will represent the general form of the relation as a variable, part of the formula of a formal concept (4.1273, 5.2522), the relation being assumed by the logical operation "O".

The internal relation by which a series is ordered, is equivalent to the operation that produces one term from another. (5.232)

Formal representations of generality

a.	The general term of the series of forms (as variable) (5.2522)	$[a, x, O'x]$
b.	The general form (variable) of a proposition (6)	$[\bar{p}, \bar{\xi}, N(\bar{\xi})]$
c.	The general form of action of an operation (6.01)	$\Omega'(\bar{\eta}) = [\bar{\xi}, N(\bar{\xi})]'(\bar{\eta}) = [\bar{\xi}, \bar{\eta}, N(\bar{\xi})]$
d.	A general form of some series of forms (cf. 4.1273 – 6.02)	$[x, \xi, \Omega'\xi]$
e.	The general term of the resulting series After applying the operation Ω' on the propositional basis ξ (cf. 6.02)	$[\Omega^{0'} x, \Omega^{\nu'} x, \Omega^{\nu+1'} x]$
f.	The general form of an integers (6.03)	$[0, \xi, \xi + 1]$

The general form of any series (a, d) (cf. explanations of 4.1273 and of the formalism introduced up to 6.02) is a variable that must contain a starting point, a law of action and a result: the internal structure of the series has the expression $[x, \xi, \Omega'\xi]$, where "x" represents the given elementary propositions, "ξ" the proposition variable resulting from the application of the operation "Ω'" on x, which results in the string $[\Omega^{0'} x, \Omega^{\nu'} x, \Omega^{\nu+1'} x]$ (6.02).

The formal analogy between the general term of a series (a,d), the general propositional form (b), the result of the action of an operation (c,e) and the general form of the integer (f) shall be observed. The general form of the Relation is the general form of the operation as a transition from one proposition to another; the relation, the operation, the proposition are united as general forms and represented in a single concept that shows everything that all logical propositions have in common and this is the only logical constant. Here we have, Wittgenstein argues, the essence of the proposition or the essence of the world. (5.47, 5.471, 5.4711)

The operation as a general logical form has as an argument the elementary propositions in a Logical Space and results in a propositional function. The way of defining the operation allows the successive application of an operation to its own results (5.3), which results in "advancing in a series of forms" (5.25) because the operation can become its own argument, unlike functions (5.251) and thus avoid the paradoxes of set theory and type theory. If we were to wonder by what internal mechanism such an "alchemy" occurs, the answer would be that it is the negation of the totality of propositional functions (as an argument of elementary propositions) in the operation argument, which still raises problems of interpretation.[5]

However, it comes down to defining (intuitively, as a rule of signs) the natural number as the exponent (or index) of a logical operation (6.021).

That is, from the general form $[\Omega^{0'}x, \Omega^{\nu'}x, \Omega^{\nu+1'}x]$ the string is extracted from the exponents of the operation Ω: $0', \nu', \nu+1'$ and so on. This was one of the most important aims of *Tractatus*.

8.1. "Mathematics is a method of Logic"

The statement "Mathematics is a method of Logic" (*6.234*) indicates not only that mathematics is founded on logic, but also that they share a common foundation—that is, common internal structures that do not contain "statements about" or "assert themselves through sentences," but instead show, manifest, and reveal themselves through a natural symbolism.
This is a logical symbolism that exists beyond logic itself, as it is not grounded and does not obey the very laws that it generates.

> ... logic is not a field in which *we* express what we wish with the help of signs, but rather one in which the nature of the absolutely necessary signs speaks for itself. If we know the logical syntax of any sign-language, then we have already been given all the propositions of logic.
> (6.124).

The result is Representationalism, Realism (where structures are discovered), and Intensionalism (where structures pertain to the internal properties of objects).
Mathematics is a method, but it is not reducible to logic, as they have different meanings and occupy distinct domains of meaning.
The representation of general logical forms is preserved in mathematical forms; their common internal structures are given, and they share the same logical form. What differs, however, is meaning—the 'outer shell', the manner in which they are materialized in current symbolism (*6.031, 6.11, 6.111, 6.1232*).
Thus, even if Mathematics and Logic share the same general forms, they do not share the same functions and do not engage in the same general philosophical inquiry.

Conclusions

In the *Tractatus*, meaning is eliminated from logical syntax, just as the concept of identity and the sign of equality are removed from conceptual writing. The universal quantifier over 'x' is replaced by the existential quantifier and an additional variable, 'y', which does not satisfy the predicate of 'x'.

For Wittgenstein, identity is intrinsic to the object, and the '=' sign is avoided through alternative notation.

Signification is logical and immanent (*representationalism*), and if it is not unique, it is related to logical interpretations, uses, or differing conventions (*immanently logical*).

Relation pertains to the internal logical structure of things and is not an analyzable external sensory complex (*as it is in Russell's framework*).

---

[5] The idea is that in a given Logical Space, the process of successively applying one or more operations to the respective bases stops, the process has no more signification (see Grigoriu 2018)

Negation is connected to the deepest structural intimacy of the internal proposition and, together with the symbol of totality, leads to a process that formally exhibits the structure of the propositional sign.

Logic and mathematics are their own representations.

Additionally, I note that in the *Tractatus*, it is not stated that there is only one logic of language, but rather that there is only one logic of symbolic language, to which language can or should generally be reduced. When this reduction is not possible, we step into the realm of silence.

Bibliografy

FREGE, Gotlob (1948), *Sense and Reference*, translated by Max Black, in „the Philosophical Review", Volume 57, Issue 3 (May), 209-230.

1950/1960, *The Foundations of Arithmetic*, translated by J. L. Austin, Harper, New York.

GRIGORIU, Iulian. (2017a). „Tractatus Logico-Philosophicus – internal and external contradictions, and translation issues" in *Probleme de Logică*, volume XX Romanian Academy Publishing House, Bucharest, 67-92.

GRIGORIU, Iulian. (2017b). *Wittgenstein – Prolegomene pentru o filosofie a matematicii / Prolegomena for a philosophy of mathematic,* Eikon Publishing House, Bucharest.

GRIGORIU, Iulian. (2018). „Philosophy of Logic and Logicist Failure in the Tractatus Logico-Philosophicus", in *Probleme de Logică*, volume XXI, Romanian Academy Publishing House, Bucharest, 67-95.

GRIGORIU, Iulian. (2019a). „Wittgenstein's philosophy elements of mathematics in the ideal space of the thirties", in *Revista de Filosofie*, volume LXVI, issue 5, sept – nov, Romanian Academy Publishing House, Bucharest, 615-649.

GRIGORIU, Iulian. (2017b). „Logic Universe Generator. The general form of a proposition. Bidimensional logic", in *Probleme de Logică*, volume. XXII, Romanian Academy Publishing House, Bucharest, 151-180.

RUSSEL, Bertrand; WHITEHEAD, Norbert (1925-27/1968). *Principia Mathematica,* (1), Cambridge University Press, second edition.

WITTGENSTEIN, Ludwig. (1969). *Tractatus Logico-Philosophicus*, in Wittgenstein Schriften I, Suhrkamp Verlag, Frankfurt am Main, 11-83.

WITTGENSTEIN, Ludwig. (1961, 1974). *Tractatus Logico-Philosophicus*, first published in Routledge Classics, 2001, translated by D.F. Pears and B.F. Mc Guinness whit an Introduction by Bertrand Russell Routledge & Kegan Paul, London.