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Study of Thermal Rectification in a Bi-Material with Interfacial Inhomogeneities

Abstract: Thermal rectification is a phenomenon of heat exchange that allows heat to be transferred in one direction to a lesser extent than in the other. The study of this process enables the control of temperature, energy conversion, and the creation of potentially new materials. Studies of the thermal rectification effect using nanofiber compounds are known in the literature, in which the heat flow causes different heat transfer in both directions by changing the operating temperature, and thus, in one direction, the connected structure plays the role of a dielectric, in the other — a conductor. In this article, the phenomenon of thermal rectification is studied based on a model of thermos-elastic behaviour of a bi-material with interfacial inhomogeneities. It is assumed that the interfacial inhomogeneities are filled with some substance and are characterised by a given thermal conductivity coefficient. It is demonstrated that inhomogeneities along the interfacial line of the bi-material result in varying heat transfer depending on the direction of the heat flow and the properties of the bi-material components. The phenomenon of thermal rectification for such a model consists in increasing the difference between the values of effective thermal resistance for two opposite directions of heat flow. It has been established that the lower the thermal conductivity of the inhomogeneities and the greater the density of their location, the more pronounced the phenomenon of thermal rectification becomes.

Keywords: bi-material, interfacial inhomogeneities, thermal conductivity, thermal rectification.



Developing novel materials with predictable properties such as thermal conductivity, electrical conductivity, and mechanical compliance is crucial for modern technologies. These properties can be imparted through various methods, including combining different materials, creating geometric and physical surface inhomogeneities, doping, and applying influencing factors like electric and magnetic fields. A key application of such material engineering is in thermal management. For instance, experimental studies on a thin film of vanadium dioxide deposited on a silicon wafer have demonstrated a radiative thermal rectifier (*Ito, K. et al., 2014*). In this system, the operating temperatures are precisely adjusted by doping the film with tungsten.

Thermal rectification is a phenomenon that is often used to regulate the electrical conductivity of a material by changing the temperature. For example, mass-loaded carbon and

boron nitride nanotubes exhibit asymmetric axial thermal conductance, with greater heat flow occurring in the direction of decreasing mass density (*Wang et al., 2014*). Similarly, the thermal rectification coefficient is enhanced by increasing the geometric asymmetry in nanostructures such as graphene (*Wang et al., 2017*). To investigate these principles, experimental studies on defect-engineered graphene have utilized focused ion beam methods to create defects with precisely controlled sizes and locations (*Nobakht et al., 2018*). In some materials, this temperature-dependent phenomenon is also coupled with changes in electrical conductivity, offering a potential route for its regulation.

Thermal rectification occurs in bi-material structures in the presence of surface roughness at the material contact points, a thermal potential barrier between the material contacts, and a difference in the temperature dependence of thermal conductivity between different materials at the contact point (Roberts et al., 2011; Chumak et al., 2012). Nanoscale bi-material thermal rectification induced by a bi-material interface has been experimentally verified and its underlying mechanism investigated using molecular dynamics simulations (Ye et al., 2017). The thermal diode consists of polyamide (PA) and silicon (Si) nanowires in contact with each other. It has been found that temperature has a negligible effect on the rectification coefficient, while decreasing the contact length or increasing the temperature difference can enhance the rectification coefficient. These results are related to the development of solid-state thermal diodes based on the interface. The thermal rectification effect of a solid is demonstrated in the article (Shrestha et al., 2020), using the heterogeneous transition of "irradiated-pure" polyethylene from nanofibers using electron irradiation. For irradiated nanofiber samples, it is shown that the heat flux can be rectified by changing the operating temperature, and the average thermal rectification coefficient is significantly higher than the experimental values obtained in previous studies.

However, achieving a large and controllable rectification effect remains a significant challenge, as it requires either a macroscale or a substantial temperature shift, and experimental methods are typically quite expensive. This article proposes a theoretical approach to study the phenomenon of thermal rectification based on a bi-material endowed with interfacial inhomogeneities, which cause the appearance of thermal resistance under the action of a heat flow.

Materials and Methods

Consider the bi-material, which consists of two isotropic materials D_1 and D_2 with different thermomechanical properties. A periodical system crack is located at the bi-material interface (Figure 1). The height of the crack is given by $h_0(x)$, the length is 2a, period of the location of the cracks is d. At infinity, the bi-material is subjected to uniformly distributed tensile and shear forces p, S_1 , S_2 , respectively, and a stationary homogeneous heat flow q. Under the action of the load the cracks are opening the resulting height of the gaps will be h(x),. The cracks are filled with a substance that penetrates the crack from the external environment or material by diffusion or filtration. We assume that the crack filler does not resist deformation of the body and is characterised by a thermal conductivity coefficient λ_c . When heat and force are transferred across a bi-material interface, imperfect thermal contact occurs between faces of the cracks.

Outside the cracks, mechanical and thermal contact is ideal.

The heat transfer between the faces of the cracks is modelled by the thermal resistance and the longitudinal thermal conductivity of the filler (*Martynyak & Serednytska*, 2017). The thermal resistance is directly proportional to the thermal conductivity coefficient of the filler and inversely proportional to the height of the gap formed during the loading process. The longitudinal thermal conductivity is equal to the product of the thermal conductivity coefficient of the filler and the height of the crack. We assume that the heat flow between the faces of the cracks in the transverse direction is continuous.

The thermal and mechanical boundary conditions on the crack are as follows:

 $x \in (-a + kd, a + kd)$:

$$\tau_{xy}^+(x) = \tau_{xy}^-(x) = 0, \; \sigma_y^+(x) = \sigma_y^-(x) = 0$$

$$\lambda_c h(x) \frac{\sigma^2}{\sigma x^2} \left(T^-(x) - T^+(x) \right) + 12 q_y^{\ +}(x) - 12 \frac{\lambda_c}{h(x)} \left(T^-(x) - T^+(x) \right) = 0, \ q_y^-(x) = q_y^+(x)$$
 (1)

where the superscript "+" and "-" denote the boundary values of temperature T, normal components of heat flow q_y , normal and tangential stress components τ_{xy} and σ_y on the x-axis in the upper and lower half-plane, respectively.

Using the method of complex potentials (*Chumak et al.*, 2012) and contact boundary conditions (1), the formulated problem is reduced to a nonlinear system of singular integraodifferential equations with a Hilbert kernel with respect to functions that have the physical meaning of the temperature jump $\gamma(x)$ between the crack surfaces and the height h(x) of the formed gaps:

$$\frac{1}{d} \int_{-a}^{a} h(t) \operatorname{ctg}\left(\frac{\pi(t-x)}{d}\right) dt + \frac{\lambda \eta}{2} \left(\gamma(x) - \gamma_{ef}\right) = -\frac{G^*}{2} p + \frac{1}{d} \int_{-a}^{a} h'_0(t) \operatorname{ctg}\left(\frac{\pi(t-x)}{d}\right) dt, h(a) = 0,$$

$$|x| < a.$$

$$\lambda_c h \gamma''(x) + \frac{6\lambda}{\pi} \int_{-a}^a \gamma'(t) \, ctg\left(\frac{\pi(t-x)}{d}\right) dt - \frac{12\lambda_c}{h(x)} \gamma(x) = -12q, \ \gamma(a) = 0, |x| < a, \tag{2}$$

where

$$\gamma_{ef} = \frac{1}{d} \int_{-a}^{a} \gamma(x) \, dx, \ G^* = \frac{(1-k_1k)}{G_1(1-k_2)}, \ G_1(1-k_2) = G_2(1-k_1), \ k = 3-4v_n,$$

$$\lambda = \frac{2\lambda_1\lambda_2}{(\lambda_1+\lambda_2)}$$
, $\lambda_2 = \lambda_1$, $\eta = \eta_2 - \eta_1$, $\eta_n = \frac{\alpha_n(1+\nu_n)}{\lambda}$, $n = 1, 2$; ν_1, ν_2 are Poisson's ratios,

 G_1 , G_2 are shear moduli,

 α_1 , α_2 are coefficients of linear thermal expansion,

 λ_1 , λ_2 are thermal conductivities of materials D_1 and D_2 .

The resulting system (2) was solved using a modified analytical-numerical procedure (Serednytska et al. 2019) based on the methods of collocations and successive approximations.

Effective temperature jump γ_{ef} is determined by the solution of the system (2). The value of the temperature jump γ_{ef} characterises the additional temperature distribution caused by the periodic system of cracks. The effective temperature jump is a function of the heat flow $\gamma_{ef}(q)$.

Effective thermal resistance is a parameter that characterises the interfacial contact thermal resistance of a bi-material and is defined as differentiation of effective temperature jump function with respect heat flow $R_{ef}(q) = \frac{\delta \lambda(q)}{\delta q}$.

The effective temperature jump γ_{ef} and the effective thermal resistance R_{ef} depend on the density and direction of the heat flow.

The phenomenon of thermal rectification consists of increasing the difference between the values of effective thermal resistance for two opposite directions of heat flow.

The direction of the heat flow is determined by the parameter η , which is the difference in thermal distortivity of materials. The coefficient of thermal distortivity of a material $\eta_n(n=1,2)$ characterises the curvature of a rectilinear element due to a heat flow q of unit intensity perpendicular to this element. The coefficient of thermal distortivity is determined by the thermomechanical properties of the material. It is believed that if the difference in thermal distortions is positive $\eta > 0$, then the heat flow is directed to the material with a greater thermal distortivity. If the difference in thermal distortivity is negative $\eta < 0$, then the heat flow is directed to the material with a lower thermal distortivity.

The thermal rectification parameter δ determines the relative error between two values of effective thermal resistance R_{ef}^+ and R_{ef}^- for two cases of heat flow direction towards the material with greater thermal distortivity and towards the material with less thermal distortivity, respectively $\delta = \left|\frac{R_{ef}^+ - R_{ef}^-}{R_{ef}^+}\right| \times 100\%$.

Thus, to assess the phenomenon of thermal rectification, it is sufficient to analyse the change in the parameter δ . The larger the parameter δ , the greater the thermal rectification of the bi-material.

Literature Review

The phenomenon of thermal rectification has become a subject of growing interest due to its potential applications in thermal management, energy conversion, and the creation of materials with direction-dependent heat transfer. The fundamental principle of thermal rectification—where heat is transmitted more efficiently in one direction than the other—was first observed in solid-state systems and later extended to nanoscale structures and bi-material interfaces (Roberts & Walker, 2011). These early studies established the basis for understanding asymmetric heat transfer arising from differences in interfacial thermal resistance and the temperature dependence of material properties.

Further development of the concept was achieved through the study of thermoelastic interactions in bi-material systems. Chumak and Martynyak (2012) demonstrated that thermal rectification could occur between two thermoelastic solids due to the presence of rough zones at the interface. Their model revealed that the degree of rectification is influenced by the periodicity and amplitude of surface irregularities, which act as heat barriers. This theoretical foundation led to a growing number of studies exploring how geometric asymmetry and interface imperfections can affect the efficiency of heat transfer in bi-material structures.

Experimental confirmation of the effect was obtained through studies of nanostructured materials, where the manipulation of lattice configurations and defect distributions allows for precise control over thermal conductivity. For instance, Wang et al. (2014) reported that phonon confinement in asymmetric nanostructures can induce a substantial rectification effect even in single-material systems, thus proving that geometric asymmetry alone can serve as a driving factor. Similarly, Wang et al. (2017) performed experiments on suspended monolayer graphene and demonstrated that heat flux asymmetry can be tuned by altering the structural geometry and

defect density. These results confirmed that thermal rectification could be achieved through nanoscale engineering of structural asymmetry.

In the domain of graphene-based materials, Nobakht et al. (2018) revealed that introducing asymmetric structural defects significantly alters phonon transport, resulting in enhanced rectification. Their research underlined the importance of defect engineering for modulating thermal flow in two-dimensional materials. Such studies paved the way for understanding how localized interfacial inhomogeneities—including cracks, voids, or fillers—can modify effective thermal resistance and facilitate rectifying behaviour in composite systems.

At the same time, Ito et al. (2014) introduced a novel radiative thermal rectifier using a vanadium dioxide (VO₂) thin film on a silicon substrate. The film's radiative properties were tuned by tungsten doping, enabling controlled transition between conducting and insulating states depending on temperature. This experiment demonstrated that radiative heat transfer could also exhibit rectifying behaviour, expanding the concept beyond solid conduction mechanisms.

Theoretical studies of bi-material interfaces further advanced understanding of how heat transfer can be influenced by the thermomechanical properties of component materials. Martynyak and Serednytska (2017) developed an analytical framework for contact problems of thermoelasticity in bi-materials with interface cracks, demonstrating that the presence of filled or partially filled cracks introduces additional temperature jumps that modify the local heat flux. Their findings emphasised that the interaction between mechanical stress, temperature gradients, and interface morphology governs the resulting effective thermal resistance.

Later research by Ye and Cao (2017) provided molecular dynamics simulations of bimaterial nanocontacts, proving that nanoscale interfaces between materials with dissimilar thermal conductivities can yield measurable rectification effects. Their results indicated that decreasing contact length or increasing the temperature gradient enhances the rectification coefficient. This confirmed that both contact geometry and temperature-dependent conductivity are key determinants in achieving controllable rectification.

More recently, Shrestha et al. (2020) examined solid-state dual-mode rectifiers based on heterogeneous polyethylene nanofibres, showing that electron irradiation can induce transitions between irradiated and pure states that strongly affect the heat flux directionality. Their experiments produced significantly higher rectification coefficients than those reported in previous studies, highlighting the role of material irradiation and phase transition as tools for thermal control.

Finally, Serednytska, Martynyak, and Chumak (2019) extended these findings by analysing the thermoelastic state of a bi-material with an open gas-filled interface crack. Their analytical-numerical approach provided insights into how interfacial inhomogeneities contribute to heat transfer asymmetry. Building upon these theoretical models, current research focuses on bi-materials with periodic interfacial inhomogeneities, where the density, thermal conductivity, and deformation properties of the filler substance determine the degree of rectification. The cumulative evidence from these studies demonstrates that the interplay of structural geometry, material properties, and external stress enables precise tuning of heat transport characteristics in advanced functional materials.

Results

The main results of the thermal rectification study are illustrated in the appendix (*Figure 2*; *Figure 3*; *Figure 4*; *Figure 5*; *Figure 6*). Based on the solution of system (2), the value of the effective thermal resistance for the considered bi-material was determined. The difference between the values of the thermal resistance for the two directions of the heat flow was analysed.

Numerical calculations were performed for a bi-material characterized by the following dimensionless quantities:

$$\begin{split} \bar{x} &= \frac{x}{d}, \ \bar{a} = \frac{a}{d}, \ \overline{h_0} = \frac{h_0}{d}, \ \bar{q} = q d \eta, \ \bar{p} = p G^*, \ \bar{\lambda}_c = \frac{\lambda_c}{\lambda}, \ \bar{\gamma}_{ef} = \gamma_{ef} \lambda \eta, \ \bar{R}_{ef} = \frac{R_{ef} d}{\lambda}, \ \bar{k}_t = 2 \bar{a}, \\ \bar{\delta} &= \left| \frac{R_{ef}^+ - R_{ef}^-}{R_{ef}^+} \right| \times 100\%, \ \bar{h}_0(\bar{x}) = 0.001(1 - \bar{x}^2)^{3/2}. \end{split}$$

The change in the effective thermal resistance of the bi-material \bar{R}_{ef} depending on the direction and density of the heat flow \bar{q} for different fixed values of the forces \bar{p} , the coefficient of thermal conductivity $\bar{\lambda}_c$ and the coefficient of interface heterogeneity \bar{k}_t . Note that the coefficient of interface heterogeneity \bar{k}_t determines the density of the cracks' location. The larger the value of the coefficient \bar{k}_t , the more densely the cracks are located, the smaller the value \bar{k}_t , the further the cracks are from each other. The direction of the heat flow depends on the value \bar{q} : if $\bar{q}>0$, then the heat flow is directed to the material with greater thermal distortivity, if $\bar{q}<$ 0, then the heat flux is directed to the material with less thermal distortivity. The dependence of the thermal rectification parameter on the heat flow is determined by the relative error between the values of the dimensionless effective thermal resistance for the two directions of the heat flow. Figure 2 shows the dependence of the effective thermal resistance \bar{R}_{ef} on the heat flow \bar{q} for different values of the thermal conductivity $\bar{\lambda}_c$ (Figure 2). It can be seen that with increasing heat flow density, thermal resistance increases in the case of a flow directed to a material with greater thermal distortivity and decreases in the case of a flow directed to a material with less thermal distortivity. An increase in the thermal conductivity $\bar{\lambda}_c$ leads to a decrease in thermal resistance \bar{R}_{ef} . The effect of tensile forces on the change in effective thermal resistance \bar{R}_{ef} is proportional to the intensity of the forces \bar{p} (Figure 3). Thermal resistance \bar{R}_{ef} increases with increasing the forces \bar{p} for both directions of the heat flow. With increasing \bar{k}_t , the effective thermal resistance R_{ef} increases and the difference between the values of the thermal resistance for two opposite directions of the heat flow increases (Figure 4). The effective thermal resistance acquires greater values in the case of a flow directed to a material with greater thermal distortivity.

A nonlinear dependence of the thermal rectification parameter $\bar{\delta}$ on the heat flow density $|\bar{q}|$ was found (Figure 5) for different values of the thermal conductivity of the crack filler $\bar{\lambda}_c$. An increase in the heat flux density leads to an increase in the thermal rectification parameter. With an increase in thermal conductivity $\bar{\lambda}_c$, the value of the parameter $\bar{\delta}$ decreases, which means a decrease in thermal rectification. The thermal rectification parameter simultaneously increases with an increase in the density of the gaps and a decrease in the intensity of tensile forces (Figure 6). The dependence of thermal rectification on the density of the cracks' location, which indicates a different thermal distortivity of the bi-material depending on the number of cracks located along a unit length of the interface.

Therefore, an increase in the values of the effective thermal resistance and the thermal

rectification parameter of the bi-material manifests itself with an increase in the heat flow density and a decrease in the resistance to heat transfer through the interface. Accordingly, the ability of the interface to transmit heat is characterised by the presence of inhomogeneities such as cracks, their density of location, and the thermal conductivity of the substance that fills them.

Discussion

The phenomenon of thermal rectification demonstrates how, by altering the temperature, it is possible to control the electrical conductivity of materials. Alternatively, by adjusting the dimensions and geometry of the material structure, it is also possible to achieve the desired heat transfer. This effect can be used in engineering developments to create new devices or materials that can convert thermal energy into valuable electrical energy. Experimental studies on various types of nanostructured materials enable us to gradually increase the thermal distortion parameter, confirming the effect itself and refining existing methods for converting thermal motion into directed current. Theoretical approaches to studying the phenomenon of thermal rectification can facilitate the development of a mathematical model for a device based on a material with controllable properties, such as thermal conductivity and electrical conductivity. Therefore, theoretical studies of thermal rectification for bi-materials can logically be continued for electrically conductive interface inhomogeneities and piezoelectric bi-materials.

Conclusion

The phenomenon of thermal rectification is studied based on a model of a bi-material consisting of components characterised by different thermomechanical properties, and at the interface of which there is a periodic system of heat-conducting inhomogeneities, such as cracks. Interface inhomogeneities introduce an additional temperature difference, resulting in a change in thermal resistance during heat transfer through the interface, which depends on the direction of the heat flow. The problem of thermoelasticity for a bi-material with interfacial cracks is reduced to a system of nonlinear singular integro-differential equations concerning the functions of crack opening and the temperature jump between their surfaces. Using analytical-numerical methods for solving such systems, the additional temperature distribution as a function of the heat flow, as well as the effective interfacial thermal resistance, are determined. The thermal rectification parameter is analysed based on the values of the effective thermal resistance determined for two opposite directions of the heat flow. It is shown that changing the direction of the heat flow leads to qualitatively different values of thermal resistance, and the difference between these values increases with increasing heat flow and significantly depends on the thermal conductivity of the crack filler and the parameter characterising the density of the crack location, as well as on the influence of the force load. The thermal rectification parameter determines the relative error in percentage between the values of the effective thermal resistance for two opposite directions of the heat flow and increases with increasing flow. Thus, the phenomenon of thermal rectification in a bi-material with different coefficients of thermal distortivity of its components is manifested to a greater extent at high values of the heat flow density, low thermal conductivity of the interface and low intensity of tensile forces. The results obtained show a qualitative change in the thermal rectification parameter and can be used as a model for engineering calculations.

Conflict of Interest

The author declares that there is no conflict of interest.

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Appendix

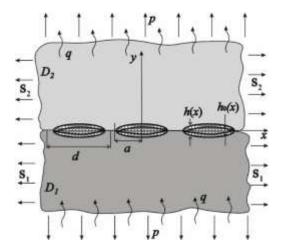


Figure 1. Shema of bi-material

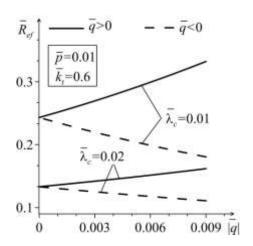


Figure 2. Dependencies of the thermal resistance \bar{R}_{ef} on the heat flow \bar{q} for different values of the thermal conductivity $\bar{\lambda}_c$.

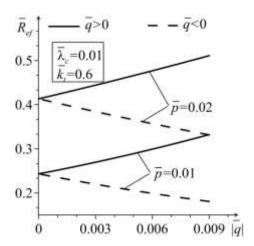


Figure 3. Dependencies of the thermal resistance \bar{R}_{ef} on the heat flow \bar{q} for different values of the intensity of forces \bar{p} .

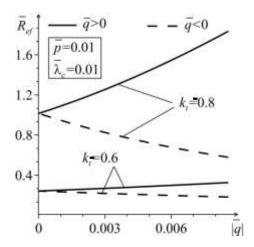


Figure 4. Dependencies of the thermal resistance \bar{R}_{ef} on the heat flow \bar{q} for different values of parameter \bar{k}_t .

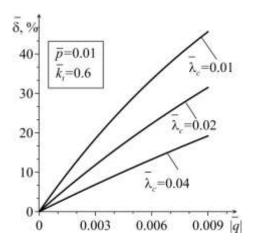


Figure 5. Dependencies of parameter of thermal rectification $\bar{\delta}$ on the heat flow density $|\bar{q}|$ for different values of the thermal conductivity $\bar{\lambda}_c$.

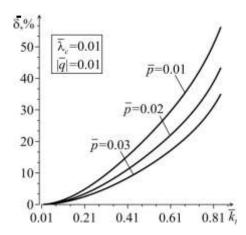


Figure 6. Dependencies of parameter of thermal rectification $\bar{\delta}$ on the coefficient of interface heterogeneity \bar{k}_t for different values of the intensity of forces \bar{p} .