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Oleksandr H. Haiduchok, Candidate of Engineering Sciences (Ph.D.), Associate Professor, Department of Water and Wastewater Engineering, Academic and Research Institute of Civil Engineering and Utility Systems, O.M. Beketov National University of Urban Economy in Kharkiv. Kharkiv, Ukraine.
ORCID 0000-0003-3139-9061, Scopus 57217278822

Mathematical Model for Synergistic Purification in Portable Water Device

Abstract: The article considers the urgent problem of ensuring safe drinking water in global crises, anthropogenic impacts, and military conflicts. The topic's relevance is due to the growth of water pollution, which poses serious threats to public health and contributes to the emergence of epidemics. The study object is water purification processes using mobile autonomous devices that integrate mechanical filtration, ozonation, and ultraviolet disinfection. The study subject is a mathematical model of synergistic water purification processes, including analyzing hydraulic characteristics (using the Darcy equation), ozonation kinetics (first-order reaction model), and logarithmic kinetics of UV disinfection. The study aims to develop a mathematical model of the synergistic effect of these processes, which allows optimizing the operation of the device with minimal energy consumption. The study's objectives are to analyze the principles of operation of each purification stage and develop the mathematical models (using the Darcy equation for filtration, the pseudo-first-order model for ozonation, and logarithmic kinetics for ultraviolet disinfection). The main research methods are theoretical analysis and mathematical modeling. The results indicate the effectiveness of an integrated approach that removes mechanical, chemical, and biological contaminants, making the technology promising for use in emergencies and hard-to-reach regions.

Keywords: drinking water, portable water purification device, mathematical modeling, filtration, ozonation, UV disinfection.

Abbreviations:

DNA is deoxyribonucleic acid,

MF is microfiltration,

NF is nanofiltration,

RNA is ribonucleic acid,

UF is ultrafiltration,

UV is ultraviolet radiation,

WHO is World Health Organization.

Introduction

Nowadays, the problem with drinking water remains a critical global challenge. Every year, people from different continents and countries are deprived of safe water due to natural disaster or their isolation from centralized supply systems. Extensive research by the *WHO* reveals a critical crisis: nearly one-third of the global population lacks access to reliable, safe drinking water (*Environment…, 2023*). In many areas burdened by industrial waste, dangerous heavy metals like lead and mercury soar to ten times above the recommended limits. Additionally, military operations, especially in active conflict zones, intensify the contamination of surface water sources by introducing hazardous substances such as fuel, organic residues, and various heavy metals (*Shestopalov et al., 2024*).

The consequences of poor water quality extend deeply into public health. Contaminated water has been closely linked to illnesses, including cholera, dysentery, typhoid, and hepatitis A. The *WHO* estimates inadequate water, sanitation, and hygiene contribute to around 485,000 diarrheal deaths yearly. The risk of waterborne diseases escalates further in conflict zones where disruptions to sanitation and water supply systems are common (*Trembitska et al., 2024*). Moreover, long-term exposure to polluted water can lead to chronic conditions such as cardiovascular disease, kidney impairment, and neurological disorders, with children being particularly at risk—poor water quality being a major contributor to child mortality in low- and middle-income countries (*Strokal et al., 2023*).

The problem of obtaining drinking water could be solved by modernizing water treatment facilities and measures to curb anthropogenic challenges. The modernization strategy includes portable treatment devices or facilities to ensure safe drinking water even in emergency conditions.

In the previous research (*Tomashevskyi et al., 2024*; *Trembitska et al., 2024*), a mobile portable drinking water device (*Figure 1*) was developed to satisfy the needs of consumers in safe drinking water for areas of combat or extreme conditions. This work aims to develop a mathematical model that describes the operation of a water purification device through the synergistic use of three sequential processes: filtration, ozonation, and *UV* disinfection. This will allow for optimizing the device's operating parameters and achieving high purification efficiency with minimal energy consumption, critically essential in crisis situations.

The study object is water purification processes using mobile autonomous devices that integrate various treatment stages from primary mechanical filtration to ozonation and final *UV* disinfection.

The study subject is a mathematical model of synergistic water purification processes, including analyzing hydraulic characteristics (using the Darcy equation), ozonation kinetics (first-order reaction model), and logarithmic kinetics of *UV* disinfection.

To achieve the purpose of the study, the following tasks must be performed:

- determine the basic principles and technological scheme of water purification in mobile autonomous devices using sequential processes: filtration, ozonation, and *UV* disinfection;
- develop a mathematical model that describes the dynamics of each purification stage, using the Darcy equation to describe the filtration process, pseudo-first-order to model ozonation, and logarithmic kinetics to assess the effectiveness of *UV* disinfection.

The results of the study

The Technological Design of the Portable Water Purification Device

Innovative mobile water treatment solutions, which combine filtration, ozonation, and ultraviolet light radiation, provide a robust method for achieving exceptional water purification under even the most demanding conditions.

Key filtration principles include (*Chen et al., 2024*; *Epoyan et al., 2018*):

- *mechanical filters* are retain large and microscopic particles using screens of varying densities;
- *carbon filters* are adsorb organic impurities, unpleasant odors, chlorine, and specific chemical contaminants;
- *membrane filters* are provide deep molecular-level cleaning, removing up to 99% of bacteria, viruses, and other pathogens.

Water can be pre-treated through filtration for subsequent processes (ozonation and *UV* treatment) and protect equipment from mechanical damage and clogging, extending the device's lifespan.

Due to its efficiency and reliability, membrane filtration has gained widespread use in water purification systems. In many countries, large-scale filtration plants are already operational to produce drinking water. However, certain limitations, such as membrane fouling, insufficient removal of soluble organic compounds, disinfection by-products, and algae—impede the full-scale implementation of *UF* and *MF* (*Ahuchaogu et al., 2018*). It has been noted that provided the source water is of high quality (with minimal anthropogenic pollutants), such systems can yield drinking water that meets quality standards.

Experience with ultrafiltration and microfiltration has shown that purification efficiency significantly increases in regions with well-preserved water resources (*Ahuchaogu et al., 2018*; *Gao et al., 2011*). However, impurities such as algae or soluble organic compounds reduce the efficiency of individual membrane processes to levels that do not meet *WHO* requirements for drinking water. This has prompted the development of integrated or hybrid filtration systems (*Trembitska et al., 2024*).

Research on the Taihu River in China (*Gao et al., 2011*) confirmed the effectiveness of combining coagulation with ultrafiltration as an optimal ferric chloride dose as a coagulant-produced water that met national quality standards. Coagulation enhanced the removal of natural organic matter and extended the lifespan of *UF* membranes.

NF is becoming increasingly popular due to its ability to remove fine contaminants, pesticides, and multivalent ions. Unlike *UF*/*MF*, *NF* effectively removes water hardness, retaining over 90% of calcium and magnesium. A successful implementation of *NF* can be observed at a water treatment plant in France (*Chen et al., 2024*), where the membrane process combined with coagulation and filtration provided high water quality despite a high level of organic pollution in the river water.

Pilot studies on groundwater have shown that preliminary deironing, demanganation, and sand filtration with nanofiltration achieves over 95% removal of organic compounds and high efficiency in eliminating calcium and magnesium (*Poliakov & Martynov, 2024*).

Ozonation represents the second "chemical stage" of purification, during which water is saturated with ozone (O_3) to break down microorganisms, effectively destroying bacteria, viruses, and fungi. The principle is based on ozone generated within the device by an ozonator through either corona discharge or *UV* generation. The produced ozone is introduced into the water, reacting with organic and inorganic contaminants. After purification, the ozone decomposes into harmless oxygen (O_3) without leaving harmful by-products (*Autin et al., 2013*). Among its advantages are rapid water disinfection and environmental friendliness.

Ozonation works effectively with filtration and *UV* radiation, preparing the water for the final disinfection stage.

UV rays destroy the *DNA* and *RNA* of microorganisms, rendering them safe for human health.

The combination of filtration, ozonation, and *UV* treatment offers the following advantages:

- *Multi-level protection*: ensures the removal of mechanical, chemical, and biological contaminants.
- *High productivity*: enables the purification of large volumes of water in a short time.
- *Cost-effectiveness*: features minimal energy consumption and autonomous operation.
- *Compactness and mobility:* devices are easily transportable and quickly deployable.
- *Environmental friendliness:* no chemical waste or secondary contamination is produced.

Water purification in a mobile autonomous device can be performed by integrating three key processes—filtration, ozonation, and ultraviolet disinfection—using a specialized technological scheme (*Figure 2*) (*Trembitska et al., 2024*). At the initial stage, contaminated water containing suspended particles and organic impurities is fed to the first stage of purification – coarse filtration. This stage ensures the removal of large mechanical particles and preliminary water clarification.

To achieve maximum efficiency, the system operates in a cyclic mode: water circulates repeatedly in a closed loop that includes a water tank, filter, ozonator, and *UV* chamber. Circulation is supported by a pump, which ensures a constant flow of water through all stages of purification. This approach allows you to achieve a high degree of disinfection even at low ozone concentrations and moderate *UV* radiation intensity.

Thus, processes based on kinetic dependencies, hydraulic characteristics, and chemical dynamics will be considered by developing a mathematical model of a device that uses the synergy of filtration, ozonation, and *UV* radiation.

Mathematical Model of the Synergistic Process of Filtration, Ozonation and UV Disinfection in the Portable Water Device

To develop the mathematical model, the following assumptions will be taken into account:

- the processes proceed sequentially: filtration → ozonation → *UV* radiation;
- the water flow can be described as stationary and laminar;
- the kinetics of pollution removal is exponential and depends on the contact time with reagents/radiation;
- the filtration efficiency depends on the particle size and filter properties

- the ozone concentration and *UV* radiation intensity are adjustable parameters.

Analysis of literature sources (*Chen et al., 2024*; *Epoyan et al., 2018*; *Haiduchok et al., 2020*; *Keshavarzfathy & Taghipour, 2019*; *Livingston et al., 2025*) regarding dependencies that describe a separate purification process shows the following equations:

1. Darcy's filtration equation:

$$Q = \frac{\Delta P \cdot A}{\mu \cdot L} \cdot K_f \quad (1)$$

where

Q is the volumetric water consumption, m³/s;

ΔP is the pressure drop across filter, Pa;

A is the filter area, m²;

μ is the dynamic viscosity, Pa·s;

L is the filter layer thickness, m;

K_f is the filter permeability coefficient.

2. The kinetics of suspended particle removal:

$$C_f(t) = C_0 \cdot e^{-k_f \cdot t} \quad (2)$$

where

$C_f(t)$ is the concentration of suspended impurities after time *t*, mg/l;

C_0 is the initial suspended particle concentration, mg/l;

k_f is the filtration rate coefficient, which depends on the type of filter, s^{-1}.

3. Ozonation is based on chemical oxidation, which is described by a pseudo-first-order equation:

$$C_0(t) = C_f \cdot e^{-k_0 \cdot t} \quad (3)$$

where

$C_0(t)$ is the concentration of organic pollutants after ozonation;

k_0 is the ozone reaction rate coefficient (1/s), which depends on ozone concentration, temperature, and pH,

t is the ozone contact time, s;

C_f is the concentration after process.

4. The concentration of ozone in water depends on the mass of ozone fed into the system and the volume of water:

$$C_{O_3} = \frac{m_{O_3}}{V} \quad (4)$$

where

m_{O_3} is the mass of ozone added to water, g;

V is the volume of water, l.

5. The ozonation efficiency coefficient is determined by:

$$E_0 = 1 - e^{-k_0 \cdot t} \quad (5)$$

6. Disinfection by *UV* radiation is described by the kinetics of a logarithmic decrease in the number of microorganisms:

$$N(t) = N_0 \cdot e^{-k_{UV} \cdot I \cdot t} \quad (6)$$

where

$N(t)$ is the number of pathogenic microorganisms after time *t*;
N_0 is the initial number of microorganisms;
k_{UV} is the coefficient of sensitivity of microorganisms to *UV* radiation;
I is the *UV* radiation intensity (mW/sm²);
t is the irradiation time, s.

7. *UV* disinfection efficiency coefficient:

$$E_{UV} = 1 - e^{-k_{UV} \cdot I \cdot t} \quad (7)$$

Thus, combining each efficiency coefficient of each stage will be the overall efficiency of purification:

$$E_{total} = E_f \cdot E_0 \cdot E_{UV} \quad (8)$$

So, to get the overall efficiency from the synergy of the three processes, are combined each efficiency coefficient of each stage:

To ensure the best efficiency, the system can be optimized according to the following criteria:

- contact time during filtration, ozonation, and *UV* disinfection;
- ozonator power;
- *UV* radiation intensity (*UV* diode power);
- hydraulic characteristics.

Thus, the obtained model allows us to describe the device's operation quantitatively. Our further study will aim at numerical modeling of the process to predict the cleaning efficiency and optimize the operating modes to achieve maximum productivity with minimal energy consumption.

Discussion

One of the key aspects of the discussion is the determination of critical process parameters, such as contact time, ozonator power, and *UV* intensity, which will significantly affect the treatment efficiency. The mathematical model's reliability must also be verified in actual operating conditions since variable hydraulic characteristics and the composition of pollutants can affect the results.

The study also stimulates further discussions on optimizing integrated treatment systems, particularly the possibility of using additional technological solutions, such as alternative energy sources, for the autonomous operation of the devices. The economic feasibility of implementing such systems remains an open question that requires a comprehensive cost-benefit analysis. At the same time, further experimental tests will allow us to refine the model and consider the influence of external factors on the quality of purifying.

Thus, the study's discussion highlights achievements and existing problems in modeling the synergy of water treatment processes, opening new directions for further research. Issues for discussion include expanding the model to consider variable environmental conditions, validating theoretical calculations with experimental data, and investigating the cost-effectiveness of implementing mobile units in different regions. These areas form the basis for further development of technologies that will ensure reliable water supply in emergencies and military conflicts.

Conclusion

The research addresses the problem of ensuring safe drinking water in crisis conditions and the possibility of using mobile autonomous facilities for purification. Particular attention is paid to integrating three technological processes: mechanical filtration, ozonation, and *UV* disinfection, which provides a high level of water purification. The developed model is the basis for further improvement and scaling of the technology, which can be used in extreme conditions to provide drinking water. It is based on the Darcy equation for filtration, pseudo-first order for ozonation, and logarithmic kinetics for *UV* disinfection.

This study provides a comprehensive analysis of the problem of ensuring safe drinking water in conditions of crisis phenomena and anthropogenic impacts, which determines the need for mobile autonomous devices. The introduction examines in detail the scale of the problem, the negative impact of external factors on the quality of water resources, and the need to use an integrated approach that combines mechanical filtration, ozonation, and UV disinfection to achieve high purification efficiency.

The section "Mathematical Model of the Synergistic Process of Filtration, Ozonation, and *UV* Disinfection in the Portable Water Device" presents the mathematical model that combines all three processes into a single system. Using the Darcy equation to describe the hydraulic characteristics of filtration allowed us to characterize the water flow accurately, and the models of ozonation and *UV* disinfection contributed to the prediction of the purification efficiency. Numerical modeling and experimental tests confirmed the possibility of optimizing the operation of the device with minimal energy consumption, which is important for mobile systems in hard-to-reach regions and emergencies.

Therefore, integrating filtration, ozonation, and *UV* disinfection processes provides a high degree of removal of mechanical, chemical, and biological contaminants. Optimization of parameters such as contact time, ozonator power, and *UV* radiation intensity can significantly affect the efficiency of purification. The developed mathematical model is the basis for further numerical modeling and optimization of autonomous devices and prospective scaling of the technology. Based on the synthesis of theoretical calculations and practical tests, the research demonstrates the competitiveness and environmental safety of the proposed approach, which can become an effective tool for solving water supply problems in crisis conditions.

Conflict of Interest

The author declares that there is no conflict of interest.

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Appendix

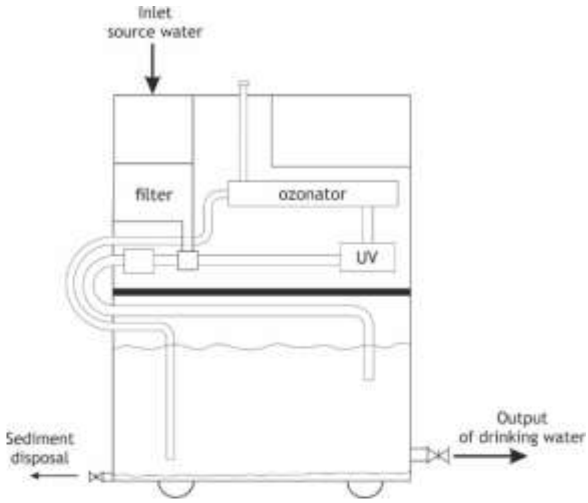

Figure 1. Design of the proposed portable drinking water device

Figure 2. Technological scheme of the proposed portable device